2006 International Conference on Topology and its Applications,

June 23-26, 2006, Aegion, Greece.

ABSTRACTS

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Contents

Abstracts

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Continuous Stalk Functions

Abstract. Let X be a nonvoid set, I=[0,1] and a $A \subset X \times I$, defined by the functions i,s from $X \to I$, such that $0 \leq i(x) < s(x) \leq 1$. Then a generalised fuzzy topological space (X, A, T^A) is defined as a family of functions u_α from $X \to I$, $i(x) \leq u_\alpha(ix) \leq s(x)$ satisfying the usual conditions. A s talk function is defined as a function from the set X^* to the set Y^* satisfying some conditions, where X^* and Y^* are the sets of points of the fuzzy set sets X and Y respectively.

If $F : X^* \to Y^*$ is a given s talk function, some properties of forward and backward powerset operators are proved. The continuity of s talk- functions as well as openess and fuzzy homeomorphism are studied too.

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Equivariant hyperspaces and their orbit spaces

Abstract. For a metric space X we denote by $\exp X$ the hyperspace of all nonempty compact subsets of X endowed with the Hausdorff metric topology. If a compact group G acts on X then $\exp X$, equipped with the natural induced action of G, becomes a G-space. In this talk we study $\exp X$ and its orbit space $(\exp X)/G$ from the point of view of the equivariant theory of retracts and Q-manifolds.

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Homotopy characterizations of Equivariant ANRś

Abstract. We shall present here some results from the equivariant theory of retracts which constitute part of the Ph.D. thesis of A. Soria-Pérez written under the supervision of S.A. Antonyan.

Let G be a compact Lie group. It is well known that each G-ANR is locally G-contractible but not viceversa. It turns out that just as in the non-equivariant case, local G-contractibility together with the equivariant Homotopy Extension Property characterizes G-ANRś. Local characterizations of G-ANRś will also be presented.

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A proposal for the completion problem of quasi-uniform spaces

Abstract. We give a new completion (the τ-completion) for quasi-uniform spaces. We prove that every T_0 quasi-uniform space has a τ-completion, which fulfils many of the desired requirements for a "good" completion and coincides with the classical completions in the literature.

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Hahn-Mazurkiewicz revisited: A generalization.

Abstract. We generalize Hahn-Mazurkiewicz theorem from Peano continua to generalized Peano continua (spaces which are locally compact, locally connected, connected and metrizable).

We characterize when a generalized Peano continua is the perfect continuous image of another generalized Peano continua by means of an internal property and by means of the existence of a finite compactification.

A new approach from fractal structures is used.

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Representations of the braid group infinite groups : an experime,tal approach.

Abstract. In their work, D. Silver and S. Williams exploited the structure of the kernel subgroup K of an epimorphism $\chi : G \to \mathbb{Z}$, where G is a finitely presented group, to show that the set $Hom\,(K, \Sigma)$ of representations of K into a finite group Σ has a structure of a subshift of finite type (SFT), a symbolic dynamical system described by a graph Γ; namely, there is a one to one correspondence between representations $\rho : K \to \Sigma$ and bi-infinite paths in Γ. We apply this method to the group B_n of braids with n-strands, with χ being the abelianization homomorphism and Σ the symmetric group S_r of degree r or the special

linear group $SL(2, \mathbf{F})$, where $\mathbf{F}$ is a finite field. The subgroup $K_n = \ker \chi$ is then the commutator subgroup $D(B_n)$ of B_n. The group $SL(2, \mathbf{F})$ has been chosen because it is highly non abelian, since $D(SL(2, \mathbf{F})) = SL(2, \mathbf{F})$. In fact, when Σ is abelian, then $Hom(K_n, \Sigma)$ will be trivial as soon as n is greater than 4. As for S_r, it is a well known fact that for a given group K, there is a finite to one correspondence between its subgroups of index no greater than r and representations $\rho : K \to S_r$. This will allow us to draw some conclusions about the subgroups of finite index of K_n. We give an algorithm to compute $Hom\,(K_n, \Sigma)$, for $n \geq 5$ and $r \geq n$. Since every representation in $Hom\,(B_n, \Sigma)$ restricts to an element of $Hom\,(K_n, \Sigma)$, we enhance the given algorithm in order to compute $Hom\,(B_n, \Sigma)$.

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Two over rings of C(X)

Abstract. Two rings T(X) and $T'(X)$ that contains C(X) and each ring is a family of real-valued functions are introduced. New characterization of Hausdorff, locally compact or metrizable, almost discrete space is given. Also some algebraic and topological characterization of crowed open-hereditarily irresolvable space X is given.

Key words: Without isolated point; Almost discrete topological space; Locally compact; Metrizable space; irresolvable space.

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Approximation by continuous functions in the Fell topology

Abstract. $\mathrm{CL}(X \times Y)$ denotes the set of non-empty closed subsets of $X \times Y$. If $F \in \mathrm{CL}(X \times Y)$ we put $F(x) = \{y \in Y : (x, y) \in F\}$. In this way the elements of $\mathrm{CL}(X \times Y)$ are multifunctions with closed graph. $\mathrm{CL}^*(X \times Y)$ is the subset of the elements such that $F(x) \neq \emptyset\ \forall x \in X$.

The following terminology is used for multifunctions $F \in \mathrm{CL}(X \times Y)$:

usc	= upper semicontinuous
cusc	= usc + connected values
usco	= usc + non-empty compact values
cusco	= cusc + usco

1. Theorem (Holá, McCoy, Pelant 2005) X countably paracompact normal space. If $F \in \mathrm{CL}^*(X \times \mathbb{R})$ is cusco and F maps isolated points into singletons, then F belong to the closure of $C(X)$ in the locally finite topology.

2. Theorem (same authors) X countably paracompact normal space. The following are equivalent:

• $\dim X = 0$

• the closure of $C(X)$ in the Vietoris topology consists of all $F \in \mathrm{CL}(X \times \mathbb{R})$ such that $F(x) \neq \emptyset\ \forall x \in X$ and F maps isolated points into singletons.

We provide the analogous of Theorem 1 for the Fell topology:

3. Theorem. Let X be a Tychonoff space. If $F \in \mathrm{CL}^*(X \times \mathbb{R})$ is the graph of a cusc map which maps isolated points into singletons, then F belongs to the closure of $C(X)$ in the Fell topology.

4. Remark. A cusco multifunction which maps isolated points into singletons is not necessarily approximable by $C(X)$ in the Vietoris topology.

5. Proposition. If X is locally connected and locally compact and F is in the closure of $C(X)$ in the Fell topology, then F is cusc if $F(x) \neq \emptyset$.

The zero-dimensional case.

6. Theorem. Let X be a zero-dimensional space and let Y be a Hausdorff space.

- If Y is compact, then the closure of $C(X, Y)$ in $\mathrm{CL}(X \times Y)$ with the Fell topology is the set of all F $in \mathrm{CL}^*(X \times Y)$ which take a single value at every isolated point $x \in X$.
- If Y is not compact, then the closure of $C(X, Y)$ in $\mathrm{CL}(X \times Y)$ with the Fell topology is the set of all $F \in \mathrm{CL}(X \times Y)$ such that $|F(x)| \leq 1$ for every isolated point $x \in X$.

Remark. If X is locally compact and $Y = \{0, 1\}$ or $[0, 1]$ then the first statement of Theorem 6 ensures that X is zero-dimensional.

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Between open and omega-open sets

Abstract. In this talk, we introduce the relatively new notion of ω^o-open subset which is strictly stronger than ω -open and weaker than open. We prove that the collection of all ω^o-open subsets of a space forms a topology that is finer than the original one. Several characterizations and properties of this class are also given as well as connections to other well-known generalized open subsets. Moreover, we introduce what we call ω^o-continuity and ω^o_X-continuity and we give several characterizations and two decompositions of ω^o-continuity. Finally, new decompositions of continuity are provided.

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On Uniform Stratifiable Spaces

Abstract. In the present paper we prove a uniform analogue of the well-known Dugundji theorem concerning the stratifiable spaces (i.e. apparently the same right for mappings into any nuclear locally convex space). Moreover, I also give some new statements, which refer to the UANR (stratifiable) and UANE (stratifiable) of the stratifiable spaces.

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Dranishnikovś resolution does preserve Z-sets

Abstract. We prove that Dranishnikovś k-dimensional resolution $d_k\colon \mu^k \to Q$ is a UV^{n-1}-divider of Chigogidześ k-dimensional resolution c_k, i.e. $\nu^k \hookrightarrow \mu^k \in$ UV^{k-1}, and c_k is a restriction of d_k on ν^k. This fact implies that d_k^{-1} preserves Z-sets. The further development of the concept of UV^{n-1}-dividers permits us to find sufficient conditions for $d_k^{-1}(A)$ to be homeomorphic to the Nöbeling space ν^k or the universal pseudoboundary σ^k.

Theorem 1. If complete subset $Z \in$AE(k) *is strongly k-universal with respect to Polish spaces and* $Z \hookrightarrow Q \in$UV^{k-1}, then $d_k^{-1}(Z) \cong \nu^k$.

Theorem 2. If σ-compact subset $Z \in$AE (k) is discretely I^k-approximate and strongly k-universal with respect to compacta, and also $Z \hookrightarrow Q \in$UV^{k-1}, then $d_k^{-1}(Z) \cong \sigma^k$.

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Construction of exotic smooth structures on manifolds with small Euler characteristic

Abstract. We provide a new technique that yields infinitely many distinct smooth structures on manifolds with small Euler characteristics. Our construction uses the combinations of knot and rational blowdown surgeries of Fintushel-Stern. We use Seiberg-Witten invariants to distinguish the smooth manifolds constructed.

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Isolated invariant sets and ANR topologies

Abstract. The complexity of the flow in the region of attraction W of an isolated invariant set A of a flow on a separable, locally compact, metrizable space M can be measured by the instability depth of W. It is an ordinal, which measures how far A is from being asymptotically stable within W and is defined through the intrinsic topology of W. This is a finer topology than the subspace topology of W inherited from M and is separable, locally compact and metrizable. The flow in W remains continuous in the intrinsic topology, and then A becomes asymptotically stable in W with respect to that topology. If A is a 1-dimensional continuum, which contains no fixed point of the flow and W is an ANR with respect to the intrinsic topology, then A is a periodic orbit.

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A note on the pure Morse complex of a graph

Abstract. The goal of this work is to study the structure of the pure Morse complex of a graph, that is, the simplicial complex given by the set of all possible class of discrete Morse functions (in Formanś sense) defined on a given graph. We shall start this study by considering trees and we shall prove that the pure Morse complex of a tree is collapsible. In order to study the general case, we shall

consider all the spanning trees included in our given graph G, and we shall express the pure Morse complex of G as the union of all pure Morse complexes corresponding to such trees

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Uniform and proximity convergence structures for function spaces on the members of filter family

Abstract. The purpose of this paper is to define a proximity convergence structure (*p.c.s.*) on a functions set. In part 1 we give another definition of concept of "proximity convergence structures" due to E. Reed . My definition and E. Reed definition are compared. Additional we investigate the connection between ours concept of proximity convergence structure and the uniform convergence structures (*u.c.s.)* of Cook and Fischer and the convergence structures of Fischer. In part 2 we suggest to define an uniform and a proximity convergence structure on the set Y^X of all functions from X to Y taking on X a family of filters Σ and on Y a proximity convergence structure $\mathcal{P}$. So if $\Gamma_\Sigma(\mathcal{P})$ denote the u.c.s on $Y^X$of proximal convergence on the members of Σ, $\Pi_\Sigma(\mathcal{P})$ denote the p.c.s. on Y^X of proximal convergence on the members of Σ then $\Pi_{\Gamma_\Sigma(\mathcal{P})} = \Pi_\Sigma(\mathcal{P})$, where $\Pi_{\Gamma_\Sigma(\mathcal{P})}$ denote the p.c.s.on Y^X induced by $\Gamma_\Sigma(\mathcal{P})$. If γ_Σ^u denote the convergence structure induced by the u.c.s. $\Gamma_\Sigma(\mathcal{P})$ and γ_Σ^pdenote the convergence structure induced by the p.c.s. $\Pi_\Sigma(\mathcal{P})$, then $\gamma_\Sigma^u = \gamma_\Sigma^p$.

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On shapes and (co)homologies of compactifications

Abstract. The motivation of report is the classical problems of topology published in papers [1], [2], [3] and [4]. Problem. Find necessary and sufficient conditions under which a space of some given class has a compactification (remainder) with given topological property. This problem is interesting for the following topological properties: (i) n-dimensional (co)homology group, (co)homotopy groups of compactification (remainder) is a given group; (ii) shape of compactification (remainder) is shape of given space; (iii) cohomological and shape dimensions of compactification (remainder) is given number. The aim of the report is the characterization of shapes and description of (co)homology groups of compactifications and remainders of topological spaces. The report deals with a precompact shape theory. The theorems which give conditions for a uniform space to have an ARU-resolution will be given. In particular, a finitistic uniform space admits an ARU-resolution if and only if it has trivial uniform shape or it is an absolute uniform shape retract. Necessary and sufficient conditions are found for which the precompact shapes of remainders are coinsided. An intrinsically characterization of cech (co)homology groups of remainders is given. Border cohomological dimension, dimAX and coefficient of border cyclicity, c, AX are defined and the inequality dimAX; dimA(cX) and the equality c, AX =c,A(cX)are proved for a space X normally adjoined to its remainder. Border homology and cohomology groups of pairs of uniform spaces are defined and studied. These groups give an intrinsic

characterization of cech type homology groups of the remainder of a completion. We construct uniform Alexander-Spanier cohomology functor from the category of pairs of uniform spaces to the category of abelian groups. This functor satisfies the Steenrod-Eilenbergs [5] type axioms, is uniform shape [6] invariant and intrinsically, in terms of uniform structures, describes the Alexander-Spanier cohomology groups [7] of compactifications of completely regular space.

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On Birkhoff Property for Continua of low dimension

Abstract. We say a continuum X has the Birkhoff property (B-property) if $\overline{P(f)} = \overline{R(f)}$ for all continuous maps from X into itself. $P(f)$ and $R(f)$ denote respectively the set of periodic and recurrent points of f.

B-property indicates that in some sense, all the interesting dynamical behavior of f occurs on $\overline{P(f)}$, for example, every minimal set is contained in it, $h(f) = h(f|\overline{P(f)})$ where h denotes the topological entropy and $\mu(\overline{P(f)}) = 1$ for every normalized invariant measure and no smaller closed invariant subset has this property. Besides B-property is connected with other properties: the *depth of the center*, the *expansiveness* or the set of *ω-limit points* of maps.

Currently it is known that all *compact intervals* of real line and all finite *trees* have B-property but it is not the case for all dendrites (some of them have the property and other not depending on the map considered) and the same for other one-dimensional continua: *graphs, arc-like, tree-like.*

In two-dimension continua with non-empty interior we have the same dichotomy. Using results on one-dimension continua we obtain positive and negative results. Also we obtain the dependence of the validity of B-property on the subclass of continuous maps we have.

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On dimension of inverse limits with upper semicontinuous set-valued bonding functions

Abstract. In this talk we give results about the dimension of continua, obtained by combining inverse limits of inverse sequences of continua and one-valued bonding maps with inverse limits of inverse sequences of continua and upper semicontinuous set-valued bonding functions ([3], [4]). In particular, we consider the inverse sequences of arbitrary continua X_n and upper semicontinuous multi-valued bonding functions $\tilde{f}_n : X_{n+1} \to X_n$, obtained from one-valued maps f_n: the graph $\Gamma(\tilde{f}_n)$ is the union of $\Gamma(f_n)$ and the product $A_{n+1} \times X_n$, for a closed subset A_{n+1} of X_{n+1} ([1], [2]).

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Generalisations of braid groups, lower central series and finite type invariants

Abstract. In this talk we show some combinatoric properties of topological and algebraical generalisations of classical braid groups, like as Artin-Tits groups, surface braid groups and virtual braid groups. We will outline briefly the analogies with the case of braid groups and we will focus on the new interesting features that appear when we consider surface and virtual braid groups. After recalling the relations between lower central series and finite type invariants for classical braids and we will show some results and applications on finite type invariants for surface and virtual braids.

This talk is based on three distinct works in collaboration with L. Funar (C.R.A.S. **338** (2004)), with J. Guaschi and S.Gervais (math/GT/0512155) and with V. Bardakov (in progress).

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Geometry of mappings into Euclidean spaces

Abstract. We consider problems related to the following result obtained by this author jointly with V.Valov.

Let $f\colon X \to Y$ be a perfect map between metrizable spaces such that $\dim f \leq n$ and $\dim Y \leq 0$. Then, for every $m \geq n+1$, $C^*(X, \mathbb{R}^m)$ contains a dense G_δ-subset $\mathcal{K}$ of maps g such that, for any integers d, t, T with $0 \leq t \leq$

$d \leq T \leq m$ and $d \leq m-n-1$ and any d-dimensional plane $\Pi^d \subset \mathbb{R}^m$ parallel to some coordinate planes $\Pi^t \subset \Pi^T \subset \mathbb{R}^m$, each set $f^{-1}(y) \cap g^{-1}(\Pi^d)$, $y \in Y$, has at most q points, where $q = d+1-t+\frac{n+(n+T-m)(d-t)}{m-n-d}$ if $n \geq (m-n-T)(d-t)$ and $q = 1 + \frac{n}{m-n-T}$ otherwise.

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Uniformity of the Tychonoff space absolute.

Abstract. Let (X, U) be the uniform space. We denote $\theta_U(\mathrm{X})$ be the set of all ultrafilters on X, consisting of open sets.

We construct on the $\theta_U(\mathrm{X})$ the S.Iliadis and V.Fomin topology ([3]), namely: let $O_A = \{p \in \theta_U(\mathrm{X}) : A \in p\}$ be collection of all ultrafilters, containing A as element, where$A$ be some open set of X. Family of O_A forming the topology on $\theta_U(\mathrm{X})$ and $\theta_U(\mathrm{X})$ is extremally disconnected compact.

We denote $x(r)$ be the set of all ultrafilters $p$of space (X, U) in space $\theta_U(\mathrm{X})$, which containing all neighborhoods of point r, i.e. $x(r) = \{p \in \theta_U(\mathrm{X}) : B(r) \subset p\}$, where $B(r)$ be the filter of the neighborhoods of point $r \in X$. Then we set $w_U(X) = \bigcup\limits_{rinX} x(r)$. Then the space $w_U(\mathrm{X})$ is everywhere dense in $\theta_U(\mathrm{X})$.

We will describe the uniformity on $w_U(\mathrm{X})$. Let α be an arbitrary uniform covering, then we set $\hat{\alpha} = \{\mathrm{O_A} : \mathrm{A} \in \alpha\}$ and $\mathrm{O_A} = \{p \in w_U(\mathrm{X}) : \mathrm{A} \in p\}$. We will set the uniform covering α consists of open sets. Every $\hat{\alpha}$ is the covering of $w_U(\mathrm{X})$. The system of coverings

$sum = \{\hat{\alpha} : \alpha \in U\}$ forming the base of the some pseudouniformity $\hat{U}$ on w_U (X) ([2]).

Thus, the system $\sum$ is the base of the some pseudouniformity $\hat{U}$ on the space w_U (X). We set $w(U) = \sup\left\{\hat{U}_p, \hat{U}\right\}$, where $\hat{U}_p$ is the maximal precompact uniformity on w_U (X), induced by θ_U (X). It is clear, the $w(U)$ is the uniformity on w_U (X), generating the same topology on the w_U (X) and containing the maximal precompact uniformity $\hat{U}_p$ of the space w_U (X) ([1]).

Theorem 1. The mapping $\pi : (w_U(X), w(U)) \to (X, U)$ of the uniform space $(w_U(X), w(U))$ to the uniform space (X, U) is the uniformly continuous uniformly perfect irreducible mapping.

Theorem 2. The uniform space $(w_U(X), w(U))$ is the uniformly homeomorphic to (X, U) iff the uniform space $(X, U)$is uniformly extremally disconnected.

Theorem 3. The uniform space $(w_U(X), w(U))$ and the absolute $(\dot{X}, \dot{U})$ are uniformly homeomophic.

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On Ricci Tensor of Weakly Ricci symmetric Kahler spaces

Abstract. In this paper, the author studies some properties of a weakly Ricci symmetric Kahler space. The notions of weakly symmetric and weakly Ricci symmetric spaces were introduced by L. Tamássy and T.Q. Binh in [1], [2]. A non-flat Riemannian space is called weakly Ricci symmetric and denoted by $(WRS)_n$ if the Ricci tensor is non-zero and satisfies the condition $R_{ij,k} = a_k R_{ij} + b_i R_{kj} + c_j R_{ik}$ where a,b,c are 1-forms (non-zero simultaneously). The present paper deals with a type of 2-dimensional $(n \neq 1, 2)$ Kahler space which satisfies the above condition, R_{ij} is the Ricci tensor, "," denotes the operation of the covariant differentiation with respect to the Riemannian metric of the space. Such a space shall be called weakly Ricci recurrent Kahler space and a,b,c shall be called its 1-forms.

Let F_i^h be the structure tensor and g_{ij} be the positive definite Riemannian metric a real 2n-dimensional $(n \neq 1, 2)$ Kahler space. Then, we have $F_j^r F_r^i = -\delta_j^i$, $g_{rm} F_j^r F_i^m = g_{ji}$, $F_{ji} = g_{mi} F_j^m = -F_{ij}$, $F^{ji} = g^{jm} F_m^i = -F^{ij}$, $F_{jik}^i = 0$, $g_{ij,k} = 0$.

In the second part of this paper, weakly Ricci symmetric Kahler spaces with definite metric are examined.

In the third part of it, the following theorems are proved:

Theorem. Conformally flat weakly Ricci symmetric Kahler space does not exist.

Theorem. In a conformally symmetric weakly Ricci symmetric Kahler space, T^k is orthogonal to $R_{,k}$ where $R_{ij,k} - R_{ik,j} = T_k R_{ij} - T_j R_{ik}$ and $T_j = a_j - c_j$.

Theorem. In a weakly Ricci symmetric Kahler space of definite metric if $\lambda_i \neq 0$ then

(i) the scalar curvature R is non-zero and the tensor H_{ij} is of the form

$$H_{ij} = R\theta_i\theta_j F^i_m.$$

(ii) $R_{,k}$ is orthogonal to θ_k but $R_{,k}$ is never orthogonal to T_k and the Ricci form $R_{ij}T^iT^j$ is always definite.

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On selectively and Frechet separable spaces

Abstract. A space X is selectively separable if for any family $\{D_n : n < \omega\}$ of dense subsets of X it is possible to choose finite sets $F_n \subseteq D_n$ in such a way that $\bigcup\{F_n : n < \omega\}$ is dense in X. If the finite sets F_n can be chosen in such a way that each non-empty open set of X intersects all but finitely many of them, then we say that the space is Frechet separable.

Among other things, we have:

1. The smallest weight of a countable non-selectively separable space is $\mathfrak{d}$;

2. The smallest weight of a countable non-Frechet space is $\mathfrak{b}$.

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A non-precompactness test for topological groups

Abstract. Let G be a topological group. We show that the following are equivalent:

(i) G is not precompact;

(ii) there are two sequences $(s_n)_{n\in N}$, $(t_n)_{n\in N}$ in G and a neighborhhod V of the unit of G, such that:

$$(\{s_m t_n : m > n\}V) \cap \{s_m t_n : m \leq n\} = \emptyset.$$

We also present a class $\mathcal{C}$ of topological groups, including all locally precompact groups, for which the left proximity in (ii) can be replaced by the lower proximity.

This result is applied to obtain, with a simplified proof, the following theorem by Megrelishvili, Pestov and Uspenski: G is precompcat if and only if every bounded real-valued left uniformly continuous function on G is weakly almost periodic. Furthermore, if G belongs to the class $\mathcal{C}$, then this theorem remains true if left uniformly continuous functions on G are replaced by lower uniformly continuous ones. This gives an extension of a well-known result by Granirer.

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Simultaneous linear extensions of uniformly continuous functions and pseudometrics

Abstract. Dugundji (1951) proved that if A is a closed subset of a metric space X then there is a continuous linear extension operator $F : C(A, R) \to C(X, R)$. Bessaga (1992) asked for a similar result for pseudometrics. This was provided by a number of mathematicians over the next decade.

Kunzi and Shapiro (1997) proved a version of Dungundji's theorem with variable domain. Let PCc(X,R) be the union of all C(A,R) where A is a compact subset of the metric space X and where the distance between two members of PCc(X,R) is the Hausdorff distance between their graphs.

Theorem (Kunzi-Shapiro): There is a continuous ,linear extension operator F:PCc(X,R) to C(X,R). In 2004 Tymchatyn and Zarichnyi proved the Kunzi-Shapiro theorem for pseudometrics but with X compact metric. In this preliminary report I will discuss extensions of these results for uniformly continuous functions and pseudometrics on a bounded metric space (X,d).

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Root closed algebras

Abstract. We discuss the property of an algebra A to be (approximately) n-th root closed or (approximately) algebraically closed. In particular, we are interested in relations between (approximate) algebraic closedness of the algebra of continuous functions on a space X and topological properties of X. We construct higher-dimensional examples of compacta with root closed function algebras. We also provide a characterization of approximate root closedness.

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Sequential Normality of Ditopological Texture Spaces and Dimetrizability

Abstract. The notion of sequential normality was defined earlier by the speaker for bitopological spaces. Unlike full binormality, sequential normality is a property that is possessed by all p-q-metrizable bitopological spaces, and is involved in a bitopological analogue of the Bing metrization theorem.

In this talk, the concept of sequential normality is considered in the much more general framework of ditopological texture spaces, and its relationship with the dimetrizability of such spaces is considered.

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On Equivariant Strong Shape

Abstract. The purpose of this talk is to define the strong shape category for compact metrizable G-spaces. In its construction we follow the method of F.Cathey given in [2] for the non-equivariant case. One of the crucial points of this method is the existence of fibrant extensions for metrizable compacta. We generalize the way of obtaining such fibrant extensions showing that for every G-ANR-resolution of a given compact metrizable G-space X there corresponds an equivariant fibrant extension of X, namely the cotelescope of this G-resolution. The definition of cotelescopes in the non-equivariant case as well as some its basic properties can be found, for instance, in [1].

We define the category of equivariant strong shape for compact metrizable G-spaces in a quite usual way as the full image of some functor-reflector. In our case, it is the reflector of the equivariant homotopy category of compact G-spaces in the equivariant homotopy category of fibrant G-spaces.

We also give a description of equivariant strong shape equivalences of the constructed category in terms of weak homotopy equivalences of appropriate function spaces providing an equivariant version of the approach to strong shape given in [3]. Taking advantage of this description, we prove that equivariant strong shape equivalences induce ordinary ones of orbit spaces. Equivariant function spaces are essentially used in this approach.

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On a weak form of countable compactness

Abstract. A star covering property which is equivalent to countable compactness for regular spaces and weaker than countable compactness for Hausdorff spaces is introduced and considered. Various kinds of irregularity of topological spaces are discussed.

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Totally preordered topological spaces and their utility representation

Abstract. In the framework of totally preordered topological spaces, we study the satisfaction of some topological properties and the relationship with the existence of a continuous or semicontinuous utility representation (that is, a real-valued function f with the property that $x \precsim y$ if and only if $f(x) \leq f(y)$).

We present a survey of the main already known results, and we present some new results and open questions in this direction.

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Einstein Spaces With Semi-Symmetric

Abstract. Semi-symmetric metric connection was introduced by Hayden , and further developments were made by Yano based on the previous studies. In this work, Einstein spaces with semi-symmetric metric connection are demacrned by considering the symmetric part of the Ricci tensor due to the semi-symmetric metric connection as a multiple of the metric tensor g and some results for two and three dimensional cases are obtained. Furthermore, an example for such spaces of dimension greater then three is established.

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On Generalized Recurrent Kahlerian Weyl Spaces

Abstract. In an earlier paper [1], Canfes examined Generalized recurrent Weyl spaces. In [2], Ozdemir and Yıldırım considered conformally recurrent Kahlerian Weyl spaces on which some pure and hybrid tensors are defined. In the present work, a $2n$-dimensional $(n \neq 1, 2)$ generalized recurrent Kahlerian Weyl spaces are examined and some theorems about general recurrency are proved.

Key words Kahlerian Weyl Spaces, Generalized Recurrent Weyl spaces, Conformally recurrency.

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More on Λ-closed sets in Topological Spaces

Abstract. H. Maki [6] introduced the notion of Λ-sets in topological spaces. A Λ-set is a set A which is equal to its kernel(= saturated set), i.e. to the intersection of all open supersets of A. F. G. Arenas, J. Dontchev and M. Ganster [1] introduced and investigated the notion of Λ-closed sets and Λ-open sets by involving Λ-sets and closed sets. This enabled them to obtain some nice results. In this paper, for these sets, we introduce the notions of Λ-derived, Λ-border, Λ-frontier and Λ-exterior of a set and show that some of their properties are analogous to those for open sets. Also, we give some additional properties of Λ-closure operator. Moreover, we offer and study new separation axioms by utilizing the notions of Λ-open sets and Λ-closure operator. In this connection we have shown that the digital line is Λ-T_1. It is well-known that the set of all primitive ideals of a C^*-algebra $\mathcal{A}$, denoted by Prim

$\mathcal{A}$, plays a very important role in noncommutative spaces and its relation to particle physics (see for example [5]). We have shown that Prim $\mathcal{A}$ is also a Λ-T_1-space. S. Jafari [3] has shown that T_1-spaces are precisely those which are both R_0 and Λ-T_1. Recall that a topological space (X, τ) is said to be an R_0 space [2] if every open set contains the closure of each of its singletons.

I. Kupka [4] introduced firm continuity in order to study compactness. Kupka inspired by a number of characterizations of UC spaces (called also Atsuji spaces) [7] to characterize compact spaces. In doing this, he asked the question that what kind of continuity should replace uniform to be sufficiently strong to characterize compact spaces. He answered to this question by introducing a new type of continuity between topological spaces called firm continuity. He obtained several characterizations of compact spaces. In the same spirit, we introduce and investigate the notion of Λ-Kupka continuity to study Λ-compactness.

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When is a Volterra space Baire?

Abstract. In this paper, we study the problem when a Volterra space is Baire. It is shown that every stratifiable Volterra space is Baire. This answers affirmatively a question of Gruenhage and Lutzer in 2000. Further, it is established that a locally convex topological vector space is Volterra if and only if it is Baire; and a topological vector space in its weak topology fails to be Baire if its dual contains an infinite linearly independent pointwise bounded subset.

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Debreu-like properties of utility representations

Abstract. A *utility representation* of a linear order $(X, \prec)$ is a linear order $(U, \prec)$ and an injective order-preserving function $f\colon X \to U$ (i.e., for each $x, y \in X$, $x \prec y$ implies $f(x) \prec f(y)$). In *Utility Theory*, a branch of Mathematical Economics, the codomain U has been traditionally chosen

to be $\mathbb{R}$. Recently, some authors (see [3]) have sharply argued about the possibility of choosing a different codomain for utility representations. To this aim, they have introduced the notion of a Debreu chain and studied some of their properties. A linear order X is a *Debreu chain* if for each subchain $Y \subseteq X$, there exists an injective order-preserving function $f: Y \to X$, which is continuous if both spaces are endowed with the order topology. Debreu chains are exactly those linear orders U such that for each linear order X, if X can be order-embedded into U, then it can also be continuously embedded into U.

Very few chains are Debreu. Thus it is natural to examine some Debreu-like properties, which are desirable from the point of view of utility representations, but are not so unlikely to hold as the Debreu property. We call a linear order $(X, \prec)$ a *locally Debreu chain* if for each $Y \subseteq X$ and $y \in Y$, there exists an injective order-preserving function $f_y: Y \to X$, which is continuous at y. Further, we say that X is a *pointwise Debreu chain* if for each $Y \subseteq X$ and $y \in Y$, there exists an order-homomorphism $f_y: Y \to X$, which is continuous at y and is such that the preimage of $f(y)$ is the singleton $\{y\}$. We analyze some properties of these types of linear orders, with particular attention to lexicographic products of linear orders. Our study is linked to the so-called *Lexicographic Utility Theory*, which uses lexicographic powers of linear orders for the representation of a chain (see [1] and [2]).

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Kelley continua.

Abstract. We will discuss results, examples and problems on Kelley continua.

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A note on the Brouwer dimension of chainable spaces.

Abstract. Brouwer's *dimensiongrad*, Dg, is defined as follows. $DgX = -1$ iff $X = \emptyset$ and, for $n = 0, 1, 2, ...$, $DgX \leq n$ iff for any pair of disjoint closed sets A, B of X, there is a cut C between them with $DgC \leq n - 1$. Here a closed set C disjoint from A and B is called a cut between them, if C is closed and meets every continuum, i.e. every compact and connected subspace, of X that meets both A and B.

In the last few years, a number of papers have been published on Dg. In one of them by V.A. Chatyrko and V.V. Fedorchuk the question was raised whether $DgX = 1$ for every snake-like (= chainable) compact space X. For each $n > 1$, we construct a compact, first countable, separable, snake-like space X_n such that $dimX_n = 1 \leq DgX_n < indX_n = IndX_n = n$. While $DX_2 = 1$, the precise value of DgX_n for $n > 2$ is an open problem.

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A note on the dimension of cosmic spaces.

Abstract. We construct in ZFC a cosmic space that, despite being the union of countably many metrizable subspaces, has covering dimension equal to 1 and inductive dimensions equal to 2. This answers questions raised by A.V. Arhangel'skiĭ and S. Oka.

A cosmic space that is the union of countably many metrizable subspaces and has $dim = 1$ and $Ind > 1$ was first announced, assuming the continuum hypothesis, by Delistathis and Watson in 2000. Their proof, however, contains gaps.

A. Dow and K.P. Hart have recently announced the construction under the assumption of Martin's Axiom of a cosmic space X with $dimX = 1 < 2 \leq IndX$.

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Additive and product theorems for ind

Abstract. All spaces considered are assumed to be regular T_1. Let d be a dimension function which is monotone with respect to closed subsets. Recall that *the finite sum theorem for d* holds in a space X (in dimension $k \geq 0$), in brief, FST(d) (respectively, FST(d, k)), if $\mathrm{d}(A \cup B) = \max\{\mathrm{d}A, \mathrm{d}B\}$ for any closed in X sets A and B (such that $\mathrm{d}A, \mathrm{d}B \leq k$). For any space X let us define

$$\mathrm{FST}(\mathrm{d}, X) = \begin{cases} \infty, & \text{if FST(d) holds in the space } X; \\ \min\{k \geq 0 : \mathrm{FST}(\mathrm{d}, k) & \text{does not hold in } X\}, \text{otherwise.} \end{cases}$$

We improve some known statements from [1], [2], [3] to the following effects.

Theorem 1. Let $X = X_1 \cup X_2$ be a space and $\operatorname{Ind} X_1 = m \geq 0$, $\operatorname{Ind} X_2 = n \geq 0$. Then

(i) $\operatorname{ind} X \leq 2(m + n + 1)$;

(ii) moreover, $\operatorname{ind} X \leq m + n + 1$, if FST(ind) holds in the space X.

Theorem 2. Let X and Y be spaces with $\operatorname{ind} X = n \geq 0$ and $\operatorname{ind} Y = m \geq 0$. Assume also that $\mathrm{FST}(\operatorname{ind}, X)$, $\mathrm{FST}(\operatorname{ind}, Y) \geq k$ for some $k \geq 0$. Then

$$\operatorname{ind}(X \times Y) \leq \begin{cases} n + m, & \text{if } n = 0, \text{or } m = 0, \text{ or } n, m \leq k, \\ 2(n + m) - k - 1, & \text{otherwise.} \end{cases}$$

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There is no upper bound of small transfinite compactness degree in metrizable spaces

Abstract. In [1], Aarts and Nishiura investigated several types of dimensions modulo a class $\mathcal{P}$, $\mathcal{P}$-ind, of spaces. The small inductive dimension modulo $\mathcal{P}$, $\mathcal{P}$-ind has a natural transfinite extension $\mathcal{P}$-trind. When we consider as $\mathcal{P}$ the class of compact spaces $\mathcal{K}$, $\mathcal{K}$-trind X is called the *small transfinite compactness degree* and denoted by trcmp X. It is clear that for a regular T_1-space X with weight $w(X) \leq \aleph_\alpha$, $\operatorname{trcmp} X \leq \operatorname{trind} X \leq \omega_{\alpha+1}$ holds. It is well known that for every countable ordinal α there is a separable metrizable space Z_α such that $\operatorname{trcmp} Z_\alpha = \alpha$. On the other hand, there are metrizable spaces having arbitrarily high small transfinite dimension. In this note, we shall strength this fact as follows.

Theorem 1. For each ordinal number α, there exists a metrizable space X_α such that $\operatorname{trcmp} X_\alpha = \alpha$ and $\operatorname{trind} X_\alpha$ is an ordinal number.

To prove Theorem 1, we use a completely metrizable spaces M^α, where α is an ordinal, constructed in [2]. For the space M^α it is known that $\operatorname{trind} M^\alpha$ is an ordinal number, and if $\alpha = \omega \cdot \beta$ then $\operatorname{trind} M^\alpha \geq \beta$. Let X be a metrizable space and $\mathcal{C}$ the class of completely metrizable spaces. Then the *small transfinite completeness degree* tricd X is defined as $\operatorname{tricd} X = \mathcal{C}\text{-trind}\, X$. We have the following theorem which implies Theorem 1.

Theorem 2. For each ordinal number α we have

$$\operatorname{trind} M^\alpha = \operatorname{trcmp}(\mathbb{Q} \times M^\alpha) = \operatorname{tricd}(\mathbb{Q} \times M^\alpha),$$

where $\mathbb{Q}$ denotes the space of rational numbers.

From the theorem we also get that there is no upper bound of small transfinite completeness degree in metrizable spaces.

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Lattices of equivariant open mappings, d-spaces and Dugundji compacta

Abstract. L.Ivanovskii and V.Kuzminov, answering the question of P.S.Alexandroff, proved that any compact topological group is a dyadic compactum. M.Choban showed that any compact G_δ-subspace in a factor space G/H of some topological groups G is a dyadic compactum and B.Pasynkov strengthened this result showing that such compacta is a Dugundji compacta. Of course any topological group and its any factor space are G-spaces with a natural action of the same topological group. These results allowed A. Archangelśkiĭ to formulate the following problem. Let X be a compact G-space and the action is transitive. For actions of what groups is compactum X a Dugundji compactum? V.Uspenskij has given a positive answer on the question of A. Archangelśkiĭ for $\aleph_0$-bounded groups. He also introduced the class of (d-) od-spaces which correspond to the class of Dugundji compacta in noncompact case.

Other results obtained by V.Uspenskij [1] are based on his further development of the spectral method of V.Ščepin [2].

Let X be a G-space. The action $\alpha : G \times X \to X$ is (d-) open if for each $x \in X$ the mapping $\alpha_x(., x) : G \to X$ is (d-) open (V.Uspenskij called the mapping $f : X \to Y$ d-open if for any open subset O in X one has $f(O) \subset int(cl(f(O))))$. It turned out that actions in all the cases when the G-space is Dugundji compactum or d-space has the property of (d-) opennes.

In our joint work G-spaces with such actions are investigated. Two natural lattices of equivariant mappings on a G-space will be presented, and it will be shown that all known cases a G-space to be Dugundji compactum or d-space are realized by one of them. Among the new results presented are the following.

Theorem 1. Let X be a compact G-space, the action $\alpha : G \times X \to X$ is d-open, and G is $\aleph_0$-balanced group. Then X is Dugundji compactum.

It is worth noting, first of all, that the class of $\aleph_0$-balanced groups (subgroups of the products of metrizable groups) is much wider than its subclass of $\aleph_0$-bounded groups (subgroups of the products of separable metrizable groups). Secondly, the action is not transitive, it is only dense. In the noncompact case we obtained.

Theorem 2. Let X be a (psedocompact) G-space, the action $\alpha : G \times X \to X$ is (d-) open, and G is $\aleph_0$-balanced group (balanced group). Then X is *od*- (d-) space.

In connection with results of V.Uspenskij it is worth noting that the class of $\aleph_0$-bounded groups and balanced groups doesnt́ contain one another.

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Σ-classification of real functions

Abstract. The standard classification of real functions given by René-Louis Baire in 1899 is well-known by mathematicians. It has been developed with choosing various initial classes or changing the kind of convergence.

In my researches I construct a classification starting with an arbitrary class of real functions and defining each subsequent class as the family of all functions which are sums of some pointwise convergent series of functions from the previous classes. I call the above procedure Σ-*classification*.

Clearly if the class we start with is a group with respect to addition (for instance, if it is the family of all continuous functions with respect to some topology), then we obtain the standard Baire classes. Therefore only the other case is interesting.

The goal of the talk is to present the classifications obtained when starting with several classes of functions, in particular, by the families of quasi-continuous functions and Darboux quasi-continuous functions.

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Π-classification of real functions

Abstract. In 1899 in his thesis René-Louis Baire introduced his standard classification of real functions. He started with the class of all continuous functions, and defined each subsequent class as the family of all limits of pointwise convergent sequences of functions from the previous classes. This procedure has been repeated by many mathematicians, who changed either the class we start with or the kind of convergence we consider.

In our talk we want to present a new classification patterned on that of Baire. Namely we start with any class of real functions, and we define each subsequent class as the family of all pointwise limits of products of functions from the previous classes. We call the above procedure Π-*classification*.

First we consider the class of all continuous real functions defined on $\mathbb{R}$. It is easy to show that in this case each Π-class is a proper subset of the standard Baire class with the same index. The goal of the talk is to present this Π-classification.

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The A. H. Stone Conjecture Revisited

Abstract. Nearly sixty years ago, A. H. Stone formulated the following conjecture: a normal locally connected, connected space X is multicoherent iff, for every $n > 2$, X is

the union of a circular chain of n continua. In 1978, H. Bell and R. F. Dickman constructed a counterexample—a hereditarily locally connected multicoherent plane curve which cannot be covered by a circular chain of more than 6 subcontinua (a 6-element circular chain always exists for a compact X).

We prove that *each nondegenerate Peano continuum X can be covered either by a circular chain or by a linear chain of n subcontinua, for arbitrary n*
in$\mathbb{N}$*.* This is equivalent to the following statement: *either I is X-like or $S1$ is X-like*, where I and $S1$ denote the unit interval and the unit circle, respectively. It follows that every indecomposable or 2-indecomposable arc-like continuum is X-like.

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Exponentiality for the construct of affine sets

Abstract. The topological construct **SSET** of affine sets over the two-point set S contains many interesting topological subconstructs such as **TOP**, the construct of topological spaces, and **CL**, the construct of closure spaces. For this category and its subconstructs cartesian closedness is studied. We first give a classification of the subconstructs of **SSET** according to their behaviour with respect to exponentiality. It is well known that **TOP** is not cartesian closed because the functor $X \times -$ does not always preserve quotients, except for corecompact X. For **CL** it is just the other way around, $X \times -$ generally does not preserve coproducts, except for X being indiscrete. We formulate sufficient conditions implying that a subconstruct

behaves similar to **CL**. On the other hand, we characterize a conglomerate of subconstructs with behaviour similar to **TOP**. Finally, we construct the cartesian closed topological hull of **SSET**.

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Metrically generated constructs

Abstract. Metrics (or any of their various generalizations) lie at the basis of many topological and analytical theories. Natural functors describe the transition from metric spaces to objects in a given category **X**. With a metric d one can associate e.g.a topology $\mathcal{T}_d$, a uniformity $\mathcal{U}_d$ or an approach structure $\mathcal{A}_d$, and restricting to suitable metrics one associates their zerodimensional or totally bounded counterparts. In each of these examples, a natural functor K from a category of metric spaces to the category **X** is given. Some of these examples give rise to intuitive notions of what one could call a metrically generated construct. For instance (1) every uniform space, every completely regular space as well as every uniform approach space, is a subspace of a product of metrizable spaces. It is also known that (2) for these categories the objects can be isomorphically described by means of sets structured by collections of metrics. In this talk we investigate the relationship between the two points of view (1) and (2), in a general setting of generalized metric spaces that allowes application to many examples besides the ones mentioned above.

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On finitistic fuzzy subsets, finitistic and completely finitistic spaces in L-topology

Abstract. In this paper, we have introduced the concepts of finitistic L-fuzzy subsets, completely finitistic spaces in L-topology and proved some of their properties. We have proved some results which give some relationship between finitistic spaces and finitistic L-fuzzy subsets. This paper also contains some more properties, examples and counter examples of finitistic spaces in L-topology. At the end, we have proved that sum space of two L-topological spaces is finitistic if and only if each L-toplogical space is finitistic. We have also given an example to show that $(X, \delta(D))$ need not be finitistic where $\delta(D)$ is an L-topology on X generated by an L-fuzzy quasi uniformity D on X.

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Group action RxQ

Abstract. Let X be a Tychonoff space, $H(X)$ the group of all self-homeomorphisms of X and $e : (f, x) \in H(X) \times X \to f(x) \in X$ the evaluation function. Call an *admissible group topology* on $H(X)$ any topological group topology on $H(X)$ that makes the evaluation function a group action.

Denote by $\mathcal{L}_H(X)$ the upper-semilattice of all admissible group topologies on $H(X)$ ordered by the usual inclusion. The existence of the minimum in $\mathcal{L}_H(X)$ for non-compact spaces X goes back to R. Arens. He proved that if X is locally compact T_2, then the g-topology, which is generated by the collection of all sets of the form:

$$[C, W] = \{f \in H(X) : f(C) \subset W\}$$

where C is closed, W is open in X, and C or $X - W$ is compact, results in that minimum. Beyond the class of locally compact spaces the author proved the same result for rim-compact, T_2 locally connected spaces and for zero-dimensional spaces satisfying the property:*any two non-empty clopen subspaces are homeomorphic,* and their products. We show that rim-compactness is not a necessary condition for the existence of the minimum in $\mathcal{L}_H(X)$. Notoriously, $R \times Q$ is not rim-compact when it carries the euclidean topology, but $\mathcal{L}_H(R \times Q)$ admits the minimum. This is because $H(R \times Q)$ embeds as subgroup in $H(S^1 \times \beta(Q))$ and the minimum is achieved from the relativization to $H(R \times Q)$ of the compact-open topology on $H(S^1 \times \beta(Q))$.

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Topological characterization of linearly ordered residuated lattices

Abstract. Residuated lattices are essential mathematical objects in many-valued logics and fuzzy set theory. The topological aspects of the particular linearly ordered residuated lattice $([0,1], \leq, T)$, where T stands for a left continuous t-norm, has been studied by Klement, Pap and Mesiar. In this study, we give a topological characterization of a general linearly ordered residuated lattice $M = (L, \leq, \otimes)$ (i.e. $(L, \leq, \bigwedge, \bigvee, \bot, \top)$ is a linearly ordered complete lattice where $\bigwedge$, $\bigvee$, $\bot$ and $\top$ are, respectively, the meet and join operations on L and the bottom and the top elements of L, and $(L, \otimes, \top)$ is a commutative monoid provided that $\otimes$ is distributive over arbitrary joins, i.e. $\alpha \otimes (\bigvee\limits_{i \in J} \beta_i) = \bigvee\limits_{i \in J} (\alpha \otimes \beta_i)$ for all $\alpha \in L$ and $\{\beta_i \mid i \in J\} \subseteq L$), and then their results become starightforward conclusions of the present results. In particular we show that a linearly ordered residuated lattice $M = (L, \leq, \otimes)$ is divisible, or equivalently is a GL-monoid if and only if the monoid operation $\otimes$ is a continuous map from $L \times L$ to L with respect to the topology induced by the underlying linear ordering $\leq$ on L. Furthermore, we are interested in the formulation of positive rational powers of elements of $M = (L, \leq, \otimes)$ and the problem of their existence. With the help of topological characterization of linearly ordered residuated lattices, we give a satisfactory answer to this problem.

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Relative minimality of subgroups in topological groups

Abstract. M. Megrelishvili introduced the notion of a *relatively minimal* subgroup X of a topological group G (namely, when every coarser Hausdorff group topology on G induces on X the original topology of X) and found some interesting applications of this notion [2]. We discuss joint recent results with M. Megrelishvili on relative minimality of subgroups of topological groups, with particular emphasis on the generalized Heisenberg groups (in the sense of [1]).

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Irregularity spectra

Abstract. Regular convergences form a concretely reflective subcategory of the category of convergences. The corresponding reflector can be attained by a (transfinite) iteration of a functor that we call the partial regularizer. An ordinal β belongs to the irregularity spectrum of a point x if there is a filter that converges to x in the β-th iteration of the partial regularizer, but does not for a sequence of iterations tending to β. Irregularity spectra are studied with the aid of embedded special convergences on sequential cascades.

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Asymptotic analogs of classical theorems of dimension theory.

Abstract. We discuss asymptotic analogs of the following classical theorems from dimension theory: The union theorem of Menger-Urysohn, the Hurewicz mapping theorem, the Morita theorem, and the Alexandroff theorem.

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On Unilateral Weighted Shifts in Noncommutative Operator Theory

Abstract. The condition for the functional model of unilateral weighted shift operator system is given. With the help of this model we also present the necessary and sufficient conditions for the algebra generated by this system to be isometrically isomorphic to the ball algebra.

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$a-$minimal sets and their properties

Abstract. $a-$minimal setsápproach introduced some closed right ideals of Ellis semigroup of a transformation semigroup which behave like minimal right ideals of Ellis semigroup in some senses. From 1997 till now they cause some new ideas in distality, proximal relation, transformed dimension, ...
Here we will compare the above mentioned ideas and will improve them.

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On f-density topologies

Abstract. There are investigated properties of f-density topologies introduced using a notion of f-density points: x is a *right-hand f-density point* of a Lebesgue measurable subset A of the real line if $\lim_{h\to 0^+} \frac{|(x;x+h)\setminus A|}{f(h)} = 0$, where a function $f : (0;\infty) \to (0;\infty)$ is nondecreasing, tends to zero at zero and $\liminf_{x\to 0^+} \frac{f(x)}{x} < \infty$. We show that

if $\liminf_{x\to 0^+} \frac{f(x)}{x} > 0$ then f-density topology is similar to density topology, but if $\liminf_{x\to 0^+} \frac{f(x)}{x} = 0$, f-density topology has quite different properties.

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Between weakly infinite-dimensional spaces and C-spaces.

Abstract. Let X be a topological space and m be an interger ≥ 2. Set

$$\mathrm{cov}_m(X) = \{u \in \mathrm{cov}(X) : m \geq 2\}$$

and

$$\mathrm{cov}_\infty(X) = \cup\{\mathrm{cov}_m(X) : m \geq 2\}.$$

Recall that a family $\mathcal{U} = \{u_\alpha : \alpha \in A\} \subset \mathrm{cov}(X)$ is said to be *essential* if for any disjoint open families v_α, $\alpha \in A$, such that v_α refines u_α for each α, the family $\cup\{v_\alpha : \alpha \in A\}$ does not cover X. If $\mathcal{U}$ is not essential, it is called *inessential*.

A normal space X is called an m-C-space ($m \geq 2$ or $m = \infty$) if every countable family $\mathcal{U} \subset \mathrm{cov}_m(X)$ is inessential. The class of all m-C-spaces is denoted by m-C. We have the following increasing sequence

$$C \subset \infty\text{-}C \subset \ldots m\text{-}C \subset \ldots \subset 2\text{-}C = wid,$$

where C is the class of all C-spaces and wid is the class of all weakly infinite-dimensional spaces in the sense of P.S. Alexandroff.

Intrinsic properties of m-C-spaces coincide with those of wid-spaces. In particular, compact m-C-spaces admit

transfinite classification by ind_m of R. Pol, and transfinite dimension $\dim_m$ of P. Borst. It is unknown, whether the equality m-$C = (m+1)$-C holds in the class of compact spaces.

There is another approach to a generalization of C-spaces. Let $\varphi_m(X)$ be the set of all disjoint collections Φ of closed subsets of X with $|\Phi| \leq m$ and $\varphi_\infty(X) = \cup\{\varphi_m(X) : m \geq 2\}$. A *neighbourhood* $O\Phi$ of a family $\Phi = \{F_1, \ldots, F_k\} \in \varphi_\infty(X)$ is a collection $\{OF_1, \ldots, OF_k\}$ of disjoint neighbourhoods of members of Φ. A set $\varphi = \{\Phi_\alpha : \alpha \in A\} \subset \varphi_\infty(X)$ is said to be *inessential* if there exist neighbourhoods $O\Phi_\alpha$ such that the family $\cup\{O\Phi_\alpha : \alpha \in A\}$ is a cover of X. A normal space X is said to be a w-m-C-space ($m \geq 2$ or $m = \infty$) if every countable family $\varphi \subset \varphi_m(X)$ is inessential. The class of all w-m-C-spaces is denoted by w-m-C. As in the class of m-C-spaces, there is an increasing sequence

$$w\text{-}\infty\text{-}C \subset \ldots \subset w\text{-}m\text{-}C \subset \ldots \subset w\text{-}2\text{-}C = wid.$$

Every m-C-space is a w-m-C-space and it is unknown, whether the converse is true even for compact spaces.

The theory of m-C-spaces and the theory of w-m-C-spaces resemble one another. The only exeption concerns mappings. It is unknown, for example, whether a compact space X is a w-∞-C-space if X admits a w-∞-C-mapping onto a w-∞-C-space, in particular, if $X = Y \times Z$, where Y and Z are w-∞-C-spaces.

There are a lot of other questions. For example:

Is it true that Borsts compactum E_{ω_0} from [1] is a w-3-space?

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On fully closed absolutes.

Abstract. We shall discuss several results concerning the properties of fully closed absolutes [1]. For a compact space X the fully closed absolute faX is obtained by "inserting" into each point x of X the remainder of the Stone-Cech compactification of $X \setminus \{x\}$; the resulting space has the topology of a fiber product. A fully closed absolute of an arbitrary regular space X can be described in a similar way. Ulýanov [2] gave a sufficient condition for a space X to satisfy an equality $fa(faX) = faX$. We generalize Ulýanovś result; we show, in particular, that the mentioned equality holds for all normal first countable spaces. We present an example of a compact space X such that the canonical mapping $fa^{(\alpha+1)}X \to fa^{(\alpha)}X$ (where α is a given ordinal) is not a homeomorphism. Also we construct an example of a compact space X such that faX is homeomorphic to X but for which a canonical mapping $faX \to X$ is not a homeomorphism.

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Nielsen type invariants and the location of coincidence sets for pair of mappings in positive codimension.

Abstract. Let two maps of compact smooth manifolds $f, g : M^{n+m} \to N^n$ with codimension $m > 0$ and $n > 2$ be given. We consider some aspects of the Minimum problem for the coincidence set in the homotopy class of f, g. This problem for the case $m = 0$ is considered in the classical Nielsen theory. In the classical definition the Nielsen equivalence of coincidence points is based on paths. And in case of positive codimension it is replaced with an equivalence of (singular) submanifolds of the coincidence set C based on bordisms. More exactly, two (singular) submanifolds A and B of the coincidence set C are called Nielsen equivalent if they are bordant in M via some bordism W, and the restrictions $f\mid_W, g\mid_W$ are homotopic (rel $A \cup B$). There are some recent papers concerning the mentioned problem (D.Gonsalves, J.Jeziersky, U.Koschorke, P.Saveliev, P.Wong and others). Essential results have been obtained for the cases $m = 1, m < 2n - 2$, and in general case $m > 0$ - only for particular cases of maps and/or spaces.

We investigate here the possibility of relocations for coincidences under local homotopies in case of an arbitrary codimension $m > 0$. We suppose $A \subset C$ and B are bordant $m-$ submanifolds (via some bordism W) in M, such that the images $f(A) = g(A) = a, f(B) = b$, $g(B) = c$ are points in N, and the restrictions $f\mid_W, g\mid_W$ are homotopic paths in N. Let the homotopy F joining these paths

be constant on A and factor some Morse function φ on W. More exactly, $F : W \times [0;1] \xrightarrow{\varphi \times id} [0;1] \times [0;1] \xrightarrow{\gamma_s} N$. We suppose also that the normal bundle on W is trivial, and f, g preserve the product structure on a small tubular neighbourhood of W. Let also $U = U(W) \subset M$ be a neighbourhood such that $Coin(f, g) \cap U = A$. Under the described conditions the following statement is proved:

THEOREM. There exists a pair of maps f', g' homotopic to f, g such that $Coin(f', g') = (Coinf, g \setminus A) \cup B$, and the homotopy is constant out of U. ■

In particular, if $B \subset C$, this result allows to reduce the Nielsen coinsidence class. And in general, it can be viewed as a way to relocate so called weakly common Nielsen classes analogous to ones in the relative version of classical Nielsen Theory in the spirit of X.Zhao's works.

The implication of this result for the Positive Codimension Nielsen Theory will be discussed.

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Topology and Geometry of Integrable Hamiltonian Systems on Lie algebras New Development.

Abstract. New results in the theory of topological classification of integrable Hamiltonian systems where obtained in the last years. Each system of differential equations, which is integrable in Liouville sense on symplectic manifold M, generates the so called Liouville foliation, i.e. the foliation of M into the union of tori and some singular fibers which are obtained as some gluings and degenerations of several Liouville tori. The analysis of topology of Liouville folia-

tions gives a lot of information about the behaviour of the solutions of initial system of differential equations. Such systems appear in many concrete problems of mathematical physics, algebra and topology.

Some time ago Mischenko and Fomenko proved fundamental theorem, that each semisimple Lie algebra there exists at least one integrable Hamiltonian system with polinomail integrals on any orbit of general position in Lie algebra. Then they formulated the conjecture that this is true for any finite-dimensional Lie algebra. Many mathematicians in the series of works proved this conjecture for different types of Lie algebras. Recently the general Mischenko-Fomenko conjecture was finally proved for any Lie algebra by S.T.Sadetov. He discovered some new important mechanism of polinomial integrability. Basing on this result, Fomenko and his pupils obtained new theorems clarifying the geometry of Lie algebras. In particular, for all Lie algebras of low dimensions (up to 6) are discovered the direct and explicit formulas for the polinomial integrals.

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Generalized Pontryaginś duality theorem.

Abstract. We characterize the existence of a nonnegative, sublinear and continuous order-preserving function for a not necessarily complete preorder on a real convex cone in an arbitrary topological real vector space. As a corollary of the main result, we present necessary and sufficient conditions for the existence of such an order-preserving function for a complete preorder.

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A definition of topological invariants for wild knots and links by means of construction of non standard internal S.Albeverio integral. Wild Knots and Quantum Gravity.

Abstract. Recent work on the loop representation of quantum gravity has revealed previously unsuspected connections between knot theory and quantum gravity,or more generally, 3-dimensional topology and 4-dimensional generally covariant physics. We review how some of these relationships arise from a 'ladder of field theories' including quantum gravity and BF theory in 4 dimensions,Chern-Simons theory in 3 dimensions, and gauged WZW model in 3 dimensions. We also describe the relation between wild link (or wild multiloop) invariants and generalized measures on the space of connections. In addition,we pose some research problems and describe some new results, including a proof that the Chern-Simons path integral is given by a non standard internal generalized measure.

A definition of topological invariants for wild knots and links by means of construction of internal S.Albeverio integral. During last 15 years the theory of topological invariants for tame knots and catches was meaningfully developed. At the same time the theory of wild knots and catches is still in initial condition, without looking the fact that non-trivial examples of corresponding constructions were known in the first half of XX century yet. In the present work we shall show that by means of internal integrating technique proposed by S. Albeverio [2] the theory of topological invariants can be developed for the case of wild knots and links too.

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Sharpness of the Nielsen preimage number

Abstract. Let $f : X \to Y$ be a continuous map of topological spaces and $B \subset Y$ a proper subset. A point $x \in X$ such that $f(x) \in B$ is called a preimage point. In the paper of R. Dobreńko and Z. Kucharski (1990) preimage points are divided into classes, the notion of (topological) essentiality of a class is introduced, and the Nielsen preimage number $N(f, B)$ is defined as the number of essential classes. The Nielsen number is a homotopy invariant, and hence it is a lower bound for the number of preimage classes of $g^{-1}(B)$ for each map g homotopic to f. We state conditions under which this estimate is exact, i.e., there exists a map $g \sim f$ such that its preimage set $g^{-1}(B)$ has exactly $N(f, B)$ preimage classes. As corollaries, we derive results for roots and coincidences which were obtained by Brooks in 1973.

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Recent results concerning generalized open sets

Abstract. In this talk we shall present some recent results concerning preopen sets, semi-open sets, b-open sets and semi-preopen sets.

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Čech function

Abstract. A closure operator on a set X is a mapping $\varphi : \mathcal{P}X \longrightarrow \mathcal{P}(X)$ satisfying (a) $\varphi(\emptyset) = \emptyset$, (b) $M \subseteq \varphi(M)$, (c) $\varphi(M \cup N) = \varphi(M) \cup \varphi(N)$ whenever $M, N \subseteq X$. A Čech function is a closure operator different from the identity, which maps $\mathcal{P}(X)$ onto $\mathcal{P}(X)$. We shall present a ZFC construction of a Čech function, solving thus Eduard Čechś problem from 1947.

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Games on classes of spaces

Abstract. Using the construction of Containing Spaces given in [1] we define some kind of games considered on topological classes of spaces.

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Dimension-like functions and universality

Abstract. In this paper, we define two dimension-like functions denoted by $\mathrm{dm}_{\mathrm{E}}^{\mathrm{K,B}}$ and $\mathrm{Dm}_{\mathrm{E}}^{\mathrm{K,B}}$. For a positive integer n we prove that in the families $\mathrm{P}(\mathrm{dm}_{\mathrm{E}}^{\mathrm{K,B}} \leq \mathrm{n})$ and $\mathrm{P}(\mathrm{Dm}_{\mathrm{E}}^{\mathrm{K,B}} \leq \mathrm{n})$ of all spaces X of weight τ, where τ is an infinite cardinal, such that $\mathrm{dm}_{\mathrm{E}}^{\mathrm{K,B}}(\mathrm{X}) \leq \mathrm{n}$ and $\mathrm{Dm}_{\mathrm{E}}^{\mathrm{K,B}}(\mathrm{X}) \leq \mathrm{n}$, respectively, there exist universal elements.

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A study of a physical problem using fuzzy linear regression models and metric spaces

Abstract. In this paper we study a physical problem using fuzzy linear regression models and metric spaces.

The research is funded by the European Social Fund (ESF), Operational Program for Educational and Vocational Training II (EPEAEK II), and particularly the Program PYTHAGORAS II.

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Movable categories

Abstract. The notion of movability is one of the fundamental concepts of the theory of shape [1]. The movability of topological spaces was defined by means of neighborhoods of the given space (embedded as closed set in a certain AR-space) or by means of inverse systems, depending on the approach to shape theory used.

Here we define the notion of a movable category and prove that the movability of a topological space X coincides with the movability of the comma category $\mathcal{W}^X$, defined by S.Mardesic [2].

Let K be an arbitrary category and K' any subcategory of the category K.

Definition 1. We say that a subcategory K' is *movable* in a category K, if for any object $X \in \mathrm{Ob}(K')$ there exists an object $Y \in \mathrm{Ob}(K')$ and a morphism $f \in K'(Y, X)$ such that for any object $Z \in \mathrm{Ob}(K')$ and any morphism $g \in K'(Z, X)$ there is a morphism $h \in K(Y, Z)$ such that $g \circ h = f$.

We say that a category is *movable* if it is movable in itself.

It is not difficult to prove that any category with zero-morphisms or initial objects is movable.

On the categorical approach to shape theory of S. Mardevsic [2] for each topological space X it is introduced a new comma category $\mathcal{W}^X$ and then it is defined a shape map $f : X \to Y$ as a some covariant functor $f : \mathcal{W}^Y \to \mathcal{W}^X$.

Theorem 1. The topological space X is movable if and only if the comma category $\mathcal{W}^X$ is movable.

Recently I. Pop [3] extended a notion of movable category by defining the notion of uniformly movable category and proving that if the comma category $\mathcal{W}^X$ of a topological space X is uniformly movable then X is uniformly movable. A converse assertion is also proved.

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Approach Theory in a category: Compactness

Abstract. A finitely complete category $\mathcal{X}$ with a proper $(\mathcal{E}, \mathcal{M})$-factorization structure is endowed with a pre-approach operator by defining a concrete functor $\mathbf{\Lambda} : \mathcal{X} \to \mathbf{Prap} : \underline{X} \mapsto (X, \lambda^{\underline{X}})$ to the construct **Prap** of pre-approach spaces and contractions. The following class of "closed morphisms" arises naturally: $\mathcal{F}_{\mathbf{\Lambda}} := \{f |\ \mathbf{\Lambda} f$ is a closed expansive contraction in $\mathbf{Prap}\}$. For $\mathbf{\Lambda}$ preserving subobjects, all the axioms put forward in [2] in order to develop a "functional approach to general topology" with respect to the class $\mathcal{F}_{\mathbf{\Lambda}}$ are fulfilled. Here, we concentrate on $\mathcal{F}_{\mathbf{\Lambda}}$-compactness of an object $\underline{X}$ in $\mathcal{X}$ which expresses the fact that the projection $p_Y : \underline{X} \times \underline{Y} \to \underline{Y}$ is in $\mathcal{F}_{\mathbf{\Lambda}}$ for every $\mathcal{X}$-object $\underline{Y}$. Under certain conditions on $\mathbf{\Lambda}$, we prove that $\mathcal{F}_{\mathbf{\Lambda}}$-compactness of $\underline{X}$ is equivalent to 0-compactness of $\mathbf{\Lambda}(\underline{X})$ in **Prap**.
We also concentrate on some examples. In **Top**, we capture the classical compactness and b-compactness if we consider for $\mathbf{\Lambda}$ the functor describing the Kuratowski or b-closure, respectively. On the construct **Ap** of approach spaces natural functors $\mathbf{\Lambda}$ arise, which can not be described by means of a closure operator. Besides 0-compactness, we also obtain several notions of compactness that were not considered before in the literature.

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Semitopological homomorphisms

Abstract. Arnautov defined a continuous isomorphism $f : (G, \tau) \to (H, \sigma)$ of topological groups to be *semitopological* if there exist a topological group $(\widetilde{G}, \widetilde{\tau})$ containing G as a topological normal subgroup and an open continuous homomorphism $\widetilde{f} : \widetilde{G} \to H$ extending f. He characterized the semitopological isomorphisms $f : (G, \tau) \to (H, \sigma)$ in (internal) terms of the topological groups (G, τ) and (H, σ).

We study the counterpart of this notion for continuous surjective homomorphisms and analyze further the properties of semitopological isomorphisms and semitopological surjective homomorphisms. In particular we extend the internal characterization of Arnautov to the case of surjective homomorphisms and we discuss the open problems left by Arnautov.

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Covering mappings on solenoids and their dynamics

Abstract. Let $P = (p_1, p_2, \ldots)$ be an arbitrary sequence of primes. We say that a prime $q \in \mathbb{N}$ *occurs infinitely often* in P if q is equal to infinitely many terms of P. The *P-adic solenoid* Σ_P is the inverse limit of the inverse sequence $\{X_n, f_n^{n+1}, \mathbb{N}\}$, where each X_n is the unit circle in the complex plane, and every bonding mapping f_n^{n+1} is given by $f_n^{n+1}(z) = z^{p_n}$, $z \in X_{n+1}$. The solenoid is a compact connected abelian group under the coordinatewise multiplication with the identity $e = (1, 1, \ldots)$. It is not locally connected.

For $k \in \mathbb{N}$, we consider the k-th potency mapping $h_P^k : \Sigma_P \to \Sigma_P$, that is, $h_P^k(g) = g^k, g \in \Sigma_P$, which is a covering mapping [2]. Covering mappings on solenoids were studied in papers by Charatonik J.J., Covarrubias P.P., Fox R.H., Mardešić S., Matijević V., Zhou Yousheng and others (see, e.g., [2],[3] and references cited therein). One has the following dichotomy.

Theorem 1. *If k is a multiple of a prime which occurs infinitely often in P, then there is no a k-fold connected covering space of the P-adic solenoid Σ_P. Otherwise, h_P^k is a k-fold covering mapping and, moreover, each k-fold covering mapping from connected space onto Σ_P is equivalent to h_P^k.*

In what follows we put $k \geq 2$ and use the terminology of [1,§1.8].

Proposition 2. *The covering mapping h_P^k is topologically transitive.*

We denote by $S(P)$ a subset in the set of positive inte-

gers $\mathbb{N}$ consisting of all primes which do not occur infinitely often in the sequence P.

Proposition 3. *Let $S(P) = \emptyset$. Then the identity e is the only periodic point of the covering mapping h_P^k.*

Proposition 4. *Let $S(P)$ be an infinite set. Then the covering mapping h_P^k is chaotic on the solenoid Σ_P.*

Proposition 5. *Let $S(P)$ be a nonempty finite set. If k is a multiple of the product of all primes from $S(P)$, then the identity e of the P-adic solenoid is the only periodic point of the covering mapping h_P^k. If there exists a prime from $S(P)$ which is not a divisor of k, then the covering mapping h_P^k is chaotic on the solenoid Σ_P.*

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C-embedded G_δ-dense subsets of products

Abstract. All spaces here are assumed Tychonoff. A subspace Y of X is *G_δ-dense* if Y meets every nonempty G_δ subset of X.

We recall two theorems from the literature.

1. M. Ulmer [1970/1973]. In a product of first countable spaces, each Σ-product is C-embedded.

2. N. Noble [1972]. In a product of separable metric spaces, each G_δ-dense subset is C-embedded.

We generalize those theorems simultaneously, as follows.

Theorem. In a product of first countable spaces, each G_δ-dense subset is C-embedded.

Remark. It is known that a Σ-product (and therefore a G_δ-dense subset) in a product space $X = \Pi_{i\in I} X_i$ need not be C-embedded, even if each point in each X_i is a G_δ-point.

In the book C̈hain Conditions in Topology ̧ W. W. Comfort and S. Negrepontis noted that if Y is a Σ-product in X, then each continuous $f : Y \to \mathbb{R}$ remains continuous when the topology of Y is strengthened to the topology inherited from X^d, the product of the spaces X_i each with the discrete topology; by Ulmerś Theorem, f extends to continuous $\overline{f} : X^d \to \mathbb{R}$, and $\overline{f}$ is the unique candidate for a continuous extension of f from X to $\mathbb{R}$. It follows from the above theorem that the function $\overline{f}$ is also well-defined when Y is any G_δ-dense subset of X for which $\pi_J[Y] = \Pi_{i\in J} X_i$ for every countable $J \subseteq I$. In that context we show the following.

Theorem. Let Y be G_δ-dense in X such that $\pi_J[Y] = \Pi_{i\in J} X_i$ for each countable $J \subseteq I$. Then Y is C-embedded in X if and only if for every continuous $f : Y \to \mathbb{R}$ and for each $r \in \mathbb{R}$ the set $f^{-1}(r)$ has identical closures in the two spaces X and X^d.

(1) This is joint work with W. W. Comfort.

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On the cohomology with inner symmetry of operator algebra

Abstract. We are concerned with the dihedral cohomology of Banach algebra with an involution, in short, B* algebra. We split the Helemskii sequence [2] into two exact sequences using the fact, that the cyclic cohomology of Banach algebra isomorphic to the direct sum of dihedral cohomologies [1], [3]. We get the Connes-Tsygan long exact sequences for the dihedral cohomology under the conditions Helemskii in [2]. Finally we calculate the dihedral cohomology of some classes of C*-algebras.

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Fuzzy metric spaces

Abstract. The problem of constructing a satisfactory theory of fuzzy metric spaces has been investigated by several authors from different points of view. In particular George and Veeramani have introduced and studied a notion of a fuzzy metric space. In this talk we make a little survey and give some new results about completion in this class of fuzzy metric spaces.

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On uniform classification of the spaces $C_p(N \cup \{\xi\})$ for $\xi \in \beta N \setminus N$

Abstract. It is proved that the spaces $C_p(N_\xi)$ and $C_p(N_\eta)$ are uniformly homeomorphic if and only if there exists a bijection of N on itself which transfers ξ into η. It improves our result from [1] where a linear homeomorphism was considered.

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Completeness on Hutton uniform spaces

Abstract. This paper deals with $[0, 1]$-Hutton (quasi)-uniform spaces.Completeness of a Hutton $[0, 1]$-uniform space is investigated. The main result states the equivalence between completeness of any fuzzy-(quasi)-metric space $(X, M, *)$

and completeness of the induced $[0,1]$-Hutton quasi-uniform space X. Also it is prooved that a $[0,1]$-Hutton (quasi)-uniform space induced by a fuzzy-(quasi)-metric space has a bicompletion unique up to quasi-uniform isomorphisms. The obtained results come from an appropriate definition of Cauchy L-filter (where L stands for a complete lattice, with aditional properties).

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Extension of Families of Pairwise Disjoint Real-valued Functions and Hedgehog-valued Functions

Abstract. After Frantzś seminal paper [M. Frantz, Controlling Tietze-Urysohn extensions, Pacific J. Math. 169 (1995), 53-73] there has recently been an interest in extending families of continuous pairwise disjoint real-valued functions. Our first observation is that a family (of cardinality κ) of bounded continuous pairwise disjoint functions and a continuous function with values in a hedgehog (with κ spines) with its Lawson topology are equivalent concepts. Also, extending a family of pairwise disjoint functions reduces to extending the corresponding hedgehog-valued function and vice versa. However, we do not only simplify formulations of known results related to extending families of pairwise disjoint functions. Main result provides a complete characterization of spaces having the extension property with respect to closed subspaces and functions taking valued in a hedgehog with its Lawson topology.

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Continuity in computer topology

Abstract. In this talk we introduce the notion of weak (k_0, k_1)-continuity and investigate its various properties to study computer topological properties of spaces in $\mathbf{Z}^n$.

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Lawson Topology of the space of formal balls and the hyperbolic subbase

Abstract. The set $B(X)$ of formal balls in a metric space X has two natural topologies; the Lawson topology and the product topology. We investigate the difference of these two topologies, in particular for the case X is a normed vector space. Then, we consider another topology induced by the metric of X, generated by insides and outsides hyperbolic curves in X, and study its relation to the Lawson topology of $B(X)$.

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Some notes about biseparating group isomorphims defined

Abstract. We will report here on some results obtained in order to find out to what extent the unitary group $\mathcal{U}(A)$ of a

C^*-algebra $\mathcal{A}$ determines the structure of the C^*-algebra. The main tool we have considered here is the notion of *separating map*. We prove that when two C^*-algebras $\mathcal{A}$ and $\mathcal{B}$ have lgebraically isomorphic unitary groups and this isomorphism is given by a *biseparating* map with certain other conditions, then the algebras are isomorphic.

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Symmetric neighbourhood assignments and metrization

Abstract. Given a topological space X. If for every point $x \in X$, we assign a neighbourhood $U(x)$ to it, we have $\mathcal{U} \equiv \{U(x) : x \in X\}$, an assignment of neighbourhoods. We emphasize that those neighbourhoods need not be *open* neighbourhoods. If $y \in U(x) \Leftrightarrow x \in U(y)$ for all $x, y \in X$, we say the assignment is *symmetric*. If $\{T(U) \equiv \{x \in X : U(x) = U\} : U \in \mathcal{U}\}$ is *closure-preserving*, we say the assignment is *tufted*, and speak of the collection $\{T(U) : U \in \mathcal{U}\}$ as the *tufting*.

Theorem. A regular T_0-space X is metrizable if, and only if, there is, for every $n \in \mathbb{N}$, an assignment $\mathcal{U}_n \equiv \{U_n(x) : x \in X\}$ of neighbourhoods, *symmetric* and *tufted*, such that given $\xi \in X$ and any neighbourhood U there is such a $\nu \in \mathbb{N}$ that $U_\nu(\xi) \subset U$.

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Products of topological spaces and submeasurable cardinals

Abstract. It was shown earlier how productivity of certain classes of topological spaces is related to submeasurable cardinals. We can now add a generalization of a result of Dow and Watson to show that, in case no submeasurable cardinal exist, there is no proper class of topological spaces closed under products, quotients and disjoint sums. What about if submeasurable cardinals exist?

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Borel actions of groups and universality

Abstract. All considered spaces are assumed to be separable metrizable.
A mapping f of space X into a space Y is said to be *Borel* if the set $f^{-1}(U)$ is a Borel set for every open subset U of Y. If moreover the set $f^{-1}(U)$ is a set of the additive class $\alpha \in \omega^{+}$, then f is called a *Borel mapping of the class* α.
An action $F : G \times X \to X$ of a topological group G on a space X is said to be *Borel* (*of the class* α) if F is a Borel mapping (of the class α). In this case, the pair (X, F) is called a *Borel G-space (of the class* α*).*

Let $\mathbb{R}$ be a class of G-spaces. An element (T, F^T) of $\mathbb{R}$ is said to be *universal* in $\mathbb{R}$ if for every element (X, F^X) of $\mathbb{R}$ there exists an embedding i of X into T such that

$i \circ F^X = F^T \circ (e_G \times i)$, where e_G is the identical mapping of G and $e_G \times i$ is the mapping of $G \times X$ into $G \times T$ for which $(e_G \times i)(g, x) = (g, i(x))$ for every $g \in G$ and $x \in X$.

In [1] the notion of a *saturated class of spaces* is given. In particular, the following classes are saturated:

(1) The class of all spaces.

(2) The class of all countable-dimensional spaces.

(3) The class of all strongly countable-dimensional spaces.

(4) The class of all locally finite-dimensional spaces.

(5) The class of all spaces of dimension less than or equal to a fixed non-negative integer.

(6) The class of all spaces of dimension ind less than or equal to a fixed (non-finite) countable ordinal.

Proposition. *Let* $\mathbb{P}$ *be a saturated class of spaces and* α *a countable ordinal. Then, in the class of all Borel* G*-spaces* (X, F^X) *of the class* α*, where* $X \in \mathbb{P}$*, there exist universal elements.*

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Mapping Class groups and Curve Complexes

Abstract. Let R be a connected orientable surface, $\mathcal{C}(R)$ be the complex of curves on R and Mod_R^* be the extended mapping class group of R. Let K be a finite index subgroup of Mod_R^* and f be an injective homomorphism $f : K \to Mod_R^*$. We prove that if $g \geq 2$ then a simplicial map $\lambda : \mathcal{C}(R) \to \mathcal{C}(R)$ is superinjective if and only if it is induced by

a homeomorphism of R. As a corollary, we prove that if R is not a closed surface of genus 2 and $g \geq 2$, then f has the form $k \to hkh^{-1}$ for some $h \in Mod_R^*$, and f has a unique extension to an automorphism of Mod_R^*. If R is a closed surface of genus 2, then f has the form $k \to hkh^{-1}i^{m(k)}$ for some $h \in Mod_R^*$ where m is a homomorphism $K \to \mathbb{Z}_2$ and i is the hyperelliptic involution on R. We also prove that $Aut(\mathcal{N}(R)) \cong Mod_R^*/\mathcal{C}(Mod_R^*)$, where $\mathcal{N}(R)$ is the complex of nonseparating curves on R.

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Local characterization of absolute co-extensors

Abstract. Suppose that A is a subspace of a topological space X and $f : A \to K$ is a map (i.e., continuous function). Then the problem of determining whether f has a continuous extension to X, i.e. whether there is a map $F : X \to K$ such that $F|A = f$, is called the extension problem. The most investigated case occurs when A is a closed subset of X, X belongs to a certain class of spaces and K is a CW-complex or a polyhedron. The subject is then referred to as *extension theory*. Just to illustrate, among the most widely used theorems which are the solutions of appropriate extension problems, are the Tietze Extension Theorem and the Homotopy Extension Theorem.

In extension theory we use the following notation and terminology. One says that K is an *absolute extensor* for X, $K \in AE(X)$, or that X is an *absolute co-extensor* for K, $X\tau K$, if for each closed subset A of X and map $f : A \to K$, there exists a map $F : X \to K$ such that F is an

extension of f. If A is a closed subset of X and $X\tau K$ holds, then $A\tau K$ holds too. So the property $X\tau K$ is inherited by closed subsets but not necessarily by open subsets. We have previously proved that if X is stratifiable, K a CW-complex, and $X\tau K$, then for *any* subspace A of X, $A\tau K$.

A space X is a *local absolute co-extensor* for K, means that every point of X has an open neighborhood U with $U\tau K$.

Suppose that X is a paracompact space and K a space. Now we show that X is an absolute co-extensor for K if and only if it is a local absolute co-extensor for K. We also provide a similar characterization using a weaker local extension property.

We also present several extension results for open subsets of stratifiable spaces if K is a CW-complex.

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Space Structures:Theory and Applications

Abstract. The lecture contains a short description of the book S̈pace Structures: Theory and Applications". This book consists of three parts of the same name, which are published separately. The first part was published in Zap.Nauchn.Semin. POMI, v. 287 (2002). It contains authoŕs results on metric spaces and their generalizations, in particular fixed point theorems and metric axioms of euclidean space, authoŕs theory of continuity structure, of extension structures, of general bitopological spaces as pairs (X, β) where β is a topological structure on $X \times X$. English translation of this part was published in Journal of

Mathematical Sciences, v.125 No.1 (2005), its electronic version is in SpringerLink. The second part was published in Zap.Nauchn.Semin. POMI, v. 313 (2004). It contains authorś results on convergence structures, on P-topological spaces, i.e. pairs (X, ζ), where ζ is a topological structure on $PX = \{A | A \subset X\}$, on connections between P-topological stuctures and proximity, contiguity, nearness, uniform structures, differential and piecewise linear structures, on connections between topological structures on products and function spaces. This part contains also some applications of space structures: bitopological manifolds, presentation of piecewise linear structures by bitopological structures, bitopological and P-topological manifolds, bitopological and P-topological groups. The third part will be published this year also in Zap.Nauchn.Semin. POMI. It contains authorś results on $P2$-topological spaces, i.e. pairs (X, θ) where θ is a topological structure on P^2X, on P-bitopological spaces, on $P3$-topological spaces, on some interesting applications of space structures. The significances of these structures lies in the fact that they are defined on sets of such objects as topological structures on X (elements of P^2X), uniform structures of entourages (elements of $P(X \times X)$), uniform structures of coverings (elements of P^3X), mappings X into Y (elements of $P(X \times Y)$) and others.

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d-Closure iterations in compact spaces

Abstract. Given a space X and $A \subset X$, the set $d - Cl(A) = \cup\{Cl(D) : D \subset A$ and D is discrete$\}$ is called

the d-closure of A in X. Iterations of the d-closure of A is defined as follows: let $A_0 = A$; if we have A_α, set $A_{\alpha+1} = d - Cl(A_\alpha)$. If we have sets $\{A_\alpha : \alpha < \beta\}$ where β is a limit ordinal, let $A_\beta = \cup\{A_\alpha : \alpha < \beta\}$. If X is compact then for every $A \subset X$ there is an ordinal α such that $Cl(A) = A_\alpha$ [1].

Definition. Let X be a compact space. The d-closure iteration number $idc(X)$ is the least ordinal α such that for every $A \subset X$ $Cl(A) = A_\alpha$.

Theorem 1. If $\chi(X) \leq \omega_n$, then $idc(X) \leq n + 1$.

A space X is discretely generated [1] iff $idc(X) = 1$. Every first countable space is discretely generated.

Theorem 2 (CH). There exists a compact (zero-dimensional) space X such that $w(X) = \omega_1$, $\chi(x, X) = \omega_0$ for every $x \neq x_0$ (x_0 is a fixed point in X) and $idc(X) = 2$.

Theorem 3 (CH). There exists a compact space X such that $w(X) = \omega_1$, $\chi(x, X) = \omega_0$ for every $x \neq x_0$, $idc(X) = 1$ and $idc(X2) = 2$ Theorem 3 givs a negative answer to [1] Problem 5.1.

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On the computational complexity of basic decision problems in 3-dimensional topology

Abstract. We study the computational complexity of basic decision problems of 3-dimensional topology, such as

to determine whether a triangulated 3-manifold is irreducible, prime, ∂-irreducible, or homeomorphic to a given 3-manifold M. For example, we prove that the problem to recognize whether a triangulated 3-manifold is homeomorphic to a 3-sphere, or to a 2-sphere bundle over a circle, or to a real projective 3-space, or to a handlebody of genus g, is decidable in nondeterministic polynomial time (NP) of size of the triangulation. We also show that the problem to determine whether a triangulated orientable 3-manifold is irreducible (or prime) is in PSPACE and whether it is ∂-irreducible is in coNP.

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sg-Compact spaces and multifunctions

Abstract. This paper deals with obtaining properties of sg-compact spaces by using nets, filterbases, sg-complete accumulation points and so on. By introducing the notion of 1-lower (resp. 1-upper) sg-continuous functions and considering the known notion of 1-lower (resp. 1-upper) compatible partial orders we investigate some more properties of sg-compactness. We also investigate sg-compact spaces in the context of multifunctions by introducing 1-lower(resp. 1-upper) sg-continuous multifunctions. Lastly we also obtain some characterizations of sg-compact spaces by using lower (resp. upper) sg-continuous multifunctions

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Countably compact hyperspaces and countably compact products

Abstract. Let $H(X)$ denote the set of all non-empty closed subsets of X with the Vietoris topology. A subbase for the Vietoris topology consists of sets of the form U^+, U^- where U is open in X and $U^+ = \{C \in H(X) : C \subseteq U\}$ and $U^- = \{C \in H(X) : C \cap U \neq \emptyset\}$. We introduce a property called $R(\kappa)$ and prove two theorems.

Theorem 1: Let X be a T_2-space with $H(X)$ countably compact, then X satisfies $R(\kappa)$ for all κ. A space has $R(1)$ iff all its finite powers are countably compact, so this theorem generalizes a theorem of J. Ginsburg (if X is T_2 and $H(X)$ is countably compact, then X^n is countably compact for $n < \omega$).

Theorem 2: If X is T_3, X satisfies $R(\kappa)$, $\kappa < \mathfrak{t}$, the orbit of every point is dense, and X has a family of κ many pairwise disjoint nonempty open sets, then X^κ is countably compact. An infinite regular space has an infinite collection of pairwise disjoint sets; so Theorem 2 generalizes a theorem of J. Cao, T. Nogura, and A. Tomita (if X is T_3, homogeneous and $H(X)$ countably compact, then X^ω is countably compact).

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Covering compacta by discrete subspaces

Abstract. Let $dis(X)$, called the *discrete covering number* of the space X, denote the minimum number of discrete subspaces of X that cover X. Clearly, $dis(X) \leq |X|$.

Our main result is the following strengthening of the celebrated Čech - Pospi šil theorem.

THEOREM. Let X be a compact Hausdorff space in which all points have character at least κ. Then $dis(X) \geq 2^{\kappa}$.

This result follows from the following, whose proof makes essential use of a recent lemma of G. Gruenhage.

THEOREM. Let X be a compact space in which there is a λ-branching family of closed sets. Then $dis(X) \geq \lambda$.

A family of sets $\mathcal{F}$ is called λ-branching if $|\mathcal{F}| < \lambda$ but one can form λ many pairwise disjoint intersections of subfamilies of $\mathcal{F}$.

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The infimum operation in the lattice of quasi-uniformities

Abstract. Let X be a set and let $q(X)$ be the set of all quasi-uniformities on the set X, partially ordered under set-theoretic inclusion $\subseteq$. It is well known that $(q(X), \subseteq)$ is a complete lattice.

The supremum of a family $(\mathcal{U}_i)_{i\in I}$ of quasi-uniformities on a set X is the filter on $X \times X$ generated by the subbase $\bigcup_{i\in I} \mathcal{U}_i$.

This description of an explicit subbase allows one to see quite easily that several properties of quasi-uniformities are preserved under suprema.

No similarly useful description of a subbase for the infimum of a family of quasi-uniformities is known. Therefore corresponding results regarding infima of families of quasi-uniformities are in general much more difficult to obtain. As a consequence the infimum operation has often been neglected in studies about quasi-uniformities.

In our talk we hope to show that with some additional work nevertheless numerous interesting results concerning the infimum operation in $(q(X), \subseteq)$ can be obtained.

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The Application of Inverse Limits to Models in Economics

Abstract. We discuss the application of the theory of inverse limits to solve problems coming from models in economics that have the property of not being well-defined forward in time. The work involves putting an appropriate measure on the resulting inverse limit space, proving that integration then makes sense, and computing relevant integrals.

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n b-Ro and b-R1 spaces via b-open sets

Abstract. We introduce b-Ro and b-R1 spaces and investigate their properties via b-open sets and b-closure operators.

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A note on a theorem of V.V.Fedorchuk

Abstract. Let $\mathcal{F}$ be a normal functor of degree ≥ 3 acting in the category of compact spaces. In 1989, V.V.Fedorchuk proved that if a compact space $\mathcal{F}(X)$ is hereditarily normal, then X is metrizable. The present abstract contains a theorem, Theorem 1, which is a generalization of the Fedorchuk theorem. In 1970, J.Mack defined δ-normal spaces. A space is said to be δ-normal if every regular G_δ-set has arbitrarily small closed neighborhoods. Recall that a subset G of a topological space is a regular G_δ-set if it is the intersection of the closures of a countable collection of open sets each of which contains G. **Theorem 1** *Let $\mathcal{F}$ be a normal functor of degree ≥ 3 acting in the category of compact spaces. If a space $\mathcal{F}(X) \setminus X$ is hereditarily δ-normal, then X is metrizable.*

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Covering properties of $\mathbb{K}(X)$

Abstract. We discuss relationships between covering properties of a space X and covering properties of the space $\mathbb{K}(X)$ of nonempty compact subsets of X. The properties which we consider are defined in terms of selection principles.

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Homotopy Decompositions of Polyhedra

Abstract. n 1968, at the Topological Conference in Herceg-Novi, K. Borsuk asked (in shape-theoretical language), if every polyhedron homotopy dominates only finitely many different homotopy types. By a polyhedron we mean here a finite one.

We showed earlier that the answer to the above question is negative and counter examples exist even with nilpotent fundamental groups.

Here we prove that for every polyhedron P with nilpotent fundamental group there is only finitely many different homotopy types of X_i such that $P \simeq X_i \times S1$.

At the same time we obtain a negative answer to the question, if there exists $X \in FANR$ such that Sh(X) = Sh(X_i)$\times$ Sh($S1$), for infinitely many different shapes Sh(X_i), for $FANRs$ with nilpotent first shape groups.

It should be noted that there exist polyhedra with nilpotent fundamental groups which can be decomposed in the homotopy category in two different ways into a product with the second factor $S1$.

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A direct way from reality to topology

Abstract. The connection between topology and the reality (the nature, the Universe, etc.) usually is not immediate or direct. In most cases, some – more or less – thick “filteröf other mathematical structures is also an important part of the game. And the topologists may sometimes hear an argument that almost everything that can be done in topological terms also can be done, possibly in a less elegant way, but still without using topology. Although this is not completely true, the filter of other mathematical structures may prevent topology to reflect some important information from the reality anyway.

In this contribution we will discuss an elementary simple construction of a pointless topological structure which directly reflects relationship between various locations which are glued together by possible presence of a physical object or a virtual “observer.· No other mathematical structures “in the middleäre necessary. The way how the information, usually received from finite observations or measurements is handled has some common background with FCA, formal concept analysis founded by B. Ganter and R. Wille. But as author believes, the theory promises to connect three relatively independent scientific disciplines – topol-

ogy, computer science and theoretical physics, lying beyond the standard model.

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Separating sets by quasi-continuous and Darboux quasi-continuous functions

Abstract. The classical Urysohn Lemma states that if X is normal space and the sets $A_0, A_1 \subset X$ are disjoint and closed, then there is a continuous function $f\colon X \to \mathbb{R}$ such that $f = 0$ on A_0 and $f = 1$ on A_1, and if moreover A_0 and A_1 are G_δ-sets, then $0 < f < 1$ on $X \setminus (A_0 \cup A_1)$.

In 2002 Aleksander Maliszewski (Fund. Math. 175 (2002), 271–283) considered two separation properties for a fixed class $\mathcal{F}$ of functions from $\mathbb{R}$ into $\mathbb{R}$: 1) when, given two sets $A_0, A_1 \subset \mathbb{R}$, we can find a function $f \in \mathcal{F}$ such that $f = 0$ on A_0 and $f = 1$ on A_1; 2) when, given two sets $A^-, A^+ \subset \mathbb{R}$, we can find a function $f \in \mathcal{F}$ such that $f < 0$ on A^- and $f > 0$ on A^+.

The goal of the talk is to present the solutions of both above problems for the families of quasi-continuous functions and Darboux quasi-continuous functions.

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Space-time-topologies with and without neighbourhood-bases

Abstract. The nonregularity of "fine" topologies in space-time-manifolds leeds to further modelling questions concerning the lightcone.

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On the Metrically Chebyshev Nets in a Riemannian Space with Semi-Symmetric Metric Connection

Abstract. In this paper we introduce metrically Chebyshev nets in a Riemannian manifold with semi-symmetric metric connection. We also studied the conformal transformations of these nets and obtained some geometric conditions.

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Darboux Function of a Hypersurface in a Riemannian Manifold with Semi-Symmetric Metric Connection

Abstract. In this paper, using the special type of a semi-symmetric connection, we define the Darboux function in a Riemannian hypersurface admitting semi-symmetric connection.

The semi-symmetric metric connection on a Riemannian manifold was introduced and studied by Yano before.

An n-dimensional differentiable manifold having a symmetric connection ∇ and metric tensor g is called a Riemannian manifold. Let ∇ be a Riemannian connection of M_n, in this case, this manifold is denoted by $M_n(\nabla, g)$. A linear connection ∇^* on M_n is said to be semi-symmetric if the torsion tensor of the connection ∇^* satisfies the expression $T(X,Y) = w(Y)X - w(X)Y$ where w 1-form.

A semi-symmetric connection ∇^* is called semi-symmetric metric connection if it further satisfies the equation

$$\nabla^* g = 0 \tag{1}$$

By using the equation (1), we find that $\Gamma^i_{jk} = \left\{ {i \atop jk} \right\} + \delta^i_k w_j - g_{jk} w^i$ where $\left\{ {i \atop jk} \right\}$ and Γ^i_{jk} are the Christoffel symbols of ∇ and ∇^*, respectively.

We choose an orthogonal ennuple of the unit vector field $\vec{v}_{r|}$ $(r = 1, 2, ..., n)$ on a hypersurface $M_n(\nabla^*, g)$.

We find the Darboux function of the direction $\vec{v}_{p|}$ with respect to the direction $\vec{v}_{r|}$ $(r$
$neqp)$ in a hypersurface with semi-symmetric metric connection.

The purpose of this paper is that the relations between the Darboux function with respect to the Riemannian connection and the linear connection are found. Here, we study on the conditions for the equality of the Darboux functions D and D^*. Some special cases are examined and some theorems are proved.
endabstract

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Realcompactness of locally precompact groups

Abstract. A subset B of a (Hausdorff) topological group G is said to be *precompact* (or *bounded*) if for every neighborhood U of the identity in G, there is a finite subset $F \subseteq G$ such that $B \subseteq FU$. Every topological group G admits a completion $\bar{G}$ with respect to its two-sided uniformity, and the completion is unique up to a topological isomorphism. Thus, precompact groups are precisely subgroups of compact groups.

Precompact groups have been in the focus of interest since Comfort and Ross' seminal paper of 1964 on duality theory of abelian precompact groups (cf. [2]). It was followed shortly by a second study, where the same authors proved that a group G is pseudocompact if and only if it is precompact and G_δ-dense in $\bar{G}$ (cf. [3]).

A group G is *locally precompact* if it contains a precompact neighborhood of the identity. The completion of a locally precompact group is locally compact, and thus

such groups are precisely the subgroups of locally compact groups. Comfort and Trigos-Arrieta extended the Comfort-Ross criterion, and proved that a locally precompact group G is locally pseudocompact if and only if it its G_δ-dense in $\bar{G}$ (cf. [4]). Locally pseudocompact groups were also studied by Sanchis (cf. [5]).

In this talk, we present a characterization of realcompactness in the class of (not necessarily abelian) locally precompact groups.

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Lax algebras and topological theories

Abstract. In this talk we give an overview of the various ways in which certain topological categories such as the categories of topological, approach and metric spaces can be viewed as categories of lax algebras [1], [2], [3]. In particular, in [5] the authors gave a characterization of approach spaces which involved the introduction of a new auxiliary category of numerical relations. We will present another lax algebraic characterization which does not require this but which, as in topology, considers lax algebras over the category of relations.

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Inf of Hausdorff-Bourbaki Uniform Hypertopologies

Abstract. Let X be a Tychonoff space and let $CL(X)$ be the hyperspace of all non-empty closed subsets of X. We characterize the infimum of all Hausdorff-Bourbaki uniform topologies on $CL(X)$ which are induced by compatible uniformities on X.

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Hit-And-Far-Miss Topology

Abstract. The Wijsman topology has been extensively studied in the context of a metric space (X, d). Its notation emphasized its dependence on the metric d. Recently, it was shown that the Wijsman topology is a hit-and-far-miss (or HAFM-) topology which depends on the family of

closed balls and the metric proximity. This alternate way of looking at the Wijsman topology helps in giving simple and short conceptual proofs of results involving comparisons of the Wijsman topology with others. Moreover, this viewpoint helps to study the generalized Wijsman topology as the HAFM-topology on the hyperspace of a topological T_1 space X equipped with a compatible LO-proximity.

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Every topological group is a group retract of a minimal group

Abstract. A Hausdorff topological group G is *minimal* if G does not admit a strictly coarser Hausdorff group topology. We show [5] that every Hausdorff topological group is a group retract of a minimal topological group. This first was conjectured by Pestov in 1983. Our main result leads to a solution of four problems of Arhangelśkii [1]. One of them is the problem about representability of a group as a quotient of a minimal group (Problem 519 in the book Open Problems in Topology [6]. Our approach is based on generalized Heisenberg groups and on groups arising from group representations in Banach spaces [4], [3], [5].

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Stabilizers and orbits of smooth functions

Abstract. Let $f : R^n \to R^1$ be the germ of smooth (C^∞) function such that $f(0) = 0$ and let $\phi : R1 \to R1$ be the germ of preserving orientation diffeomorphisms at 0, i.e. $\phi(0) = 0$ and $\phi'(0) > 0$.

Suppose that f satisfies the following condition (*): there are smooth functions $\alpha_1, \dots, \alpha_n$ such that

$$(*) \qquad f(x) = \alpha_1 f'_{x_1} + \cdots + \alpha_n f'_{x_n}.$$

This condition is very general, and holds for many singularities, e.g. if $f(x) = \pm x2 \pm y2$, then $f = \frac{x}{2} f'_x + \frac{y}{2} f'_y$.

Theorem 1.[1] If f satisfies (*), then for arbitrary ϕ there exists a solution h_ϕ of (*). Moreover, the correspondence $\phi \mapsto h_\phi$ is a homomorphism of the corresponding groups of germs of diffeomorphisms $Diff(R^1) \to Diff(R^n)$.

Let M be a closed manifod, $I = [0, 1]$ and $\hat{C}^\infty(M, I)$ be the subset of $C^\infty(M, I)$ consisting of surjective smooth

functions f, i.e. $f(M) = I$. There is a natural action of the group $Diff(I) \times Diff(M)$ on $\hat{C}^\infty(M, I)$ called left-right and defined by:

$$(\phi, h) \cdot f = \phi \circ f \circ h,$$

for $(\phi, h) \in Diff(R) \times Diff(M)$ and $f \in \hat{C}^\infty(M, I)$. This action also called left-right. Identify the group $Diff(M)$ with the subgroup $id_I \times Diff(M) \subset Diff(R) \times Diff(M)$. Then the induced action of $Diff(M)$ on $\hat{C}^\infty(M, I)$, called right, is given by:

$$h \cdot f = f \circ h.$$

Thus for each $f \in \hat{C}^\infty(M, I)$ we have two stabilizers St_{IM} and St_M and two orbits O_{IM} and O_M. Evidently, $St_M \equiv id_I \times St_M \subset St_{IM}$ and $O_M \subset O_{IM}$. The following theorem is a global variant of Theorem 1:

Theorem 2.[1] Let $f : M \to I$ be a surjective smooth function having only finitely many, say n, critical values and satisfying condition (*) at each of its critical points. Then St_M is a strong deformation retract of S_{IM}. Moreover, the inclusion $O_M \subset O_{IM}$ extends to a homeomorphism $O_M \times R^{n-2} \subset O_{IM}$.

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On embeddings of proper and equicontinuous actions in zero-dimensional compactifications

Abstract. We provide a tool for studying properly discontinuous actions of non-compact groups on locally compact, connected and paracompact spaces, by embedding such an action in a suitable zero-dimensional compactification of the underlying space with pleasant properties. Precisely, given such an action (G, X) we construct a zero-dimensional compactification μX of X with the properties: (a) there exists an extension of the action on μX, (b) if $\mu L \subseteq \mu X \setminus X$ is the set of the limit points of the orbits of the initial action, then the restricted action $(G, \mu X \setminus \mu L)$ remains properly discontinuous, is indivisible and equicontinuous with respect to the uniformity induced on $\mu X \setminus \mu L$ by that of μX, and (c) μX is the maximal among the zero-dimensional compactifications of X with these properties. Proper actions are usually embedded in the end point compactification εX of X, in order to abtain topological invariants concerning the cardinality of the space of the ends of X, provided that X has an additional "nice̋property of rather local character ("property Z̋, i.e., every compact subset of X is contained in a compact and connected one). If the considered space has this property, our new compactification coincides with the end point one. On the other hand, we give an example of a space not having the "property Z̋for which our compactification is different from the end point compactification. As an application, we show

that the invariant concerning the cardinality of the ends of X holds also for a class of actions strictly containing the properly discontinuous ones and for spaces not necessarily having "property Z".

The proof of the results stated above relies on a *new construction*: The action $(G, \mu X)$ is obtained as an equivariant inverse limit of properly discontinuous G-actions on polyhedra, which are constructed via G-invariant locally finite open coverings of X, generated by locally finite coverings of suitable fundamental sets of the initial action.

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Monotone insertion of lattice-valued functions

Abstract. Insertion of lattice-valued functions in a monotone manner is investigated. For L a $\lhd$-separable completely distributive lattice (i.e. L admits a countable base which is free of supercompact elements), a monotone version of the Katětov-Tong insertion theorem for L-valued functions is established. We also provide a monotone lattice-valued version of Urysohnś lemma. Both results yield new characterizations of monotonically normal spaces. Moreover, extension of lattice-valued functions under additional assumptions is shown to characterize also monotone normality.

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Maps preserving compactness and path connectedness

Abstract. In the recent paper C̈haracterizing continuity by preserving compactness and connectedness, the autors J. Gerlitis, I. Juhasz, L. Soukup, Z. Szentmiklossy investigate the old problem about continous maps and maps preserving compactness and connectedness. We investigate similar kind of problem replacing connectedness by path connectedness.

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Alexandroff bispaces Lower and Upper cozero sets of bitopological spaces

Abstract. Alexandroff bispaces are defined as extensions of Alexandroff spaces [1]. Urysohnś lemma for Alexandroff bispaces is used to show that lower and upper cozero sets of bitopological spaces are Alexandroff bispaces. An adjunction between Alexandroff bispaces and completely regular bitopological spaces is established.

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Roots of geometric objects

Abstract. Let C be a class of geometric objects of some kind. Given a set of simplifying moves on C, we apply them to a given object M as long as possible. What we get is a root of M. Our main result is that under certain conditions the root of any object exists and is unique. We apply this result to different situations and get several new results and new proofs of known results.

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Homogeneous spaces and transitive actions by topological groups

Abstract. We construct an example of a homogeneous Polish space which on which no $\aleph_0$-bounded topological group acts transitively. This implies that this space admits no metrizable compactification in which it is "homogeneously" embedded. But it has a metrizable compactification in which it has only two types of points.

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Retral spaces and continua with the fixed point property.

Abstract. A space X is called a retral space if it is a retract of a topological group. We show that every retral continuum with the fixed point property is locally connected. It follows that an indecomposable continuum with the fixed point property is not a retract of a topological group.

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Selection principles, hyperspace topologies and $\gamma-$sets in closure spaces

Abstract. Relations between closure-type properties of hyperspaces over a Čech closure space (X, u) and covering properties of (X, u) are investigated.

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On Homotopy Invariant of Fixed Points

Abstract. Fixed point and homotopic invariance results are presented for set-valued generalized -contractive maps defined on complete metric spaces.

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Uniformly Ergodic Theorem for Semigroups With k-Decomposable Kato Infinitesimal Generators

Abstract. In this paper we shall extend the technical assumption (E-k) to semigroups. We prove that if $T = (T(t), t \geq 0)$ is C_0-semigroup of operators in $L(X)$ with k-decomposable Kato infinitesimal generator A satisfying the condition $(E - k)$, then T is uniformly ergodic. These results are of interest in view of recent activity in the ergodic theory and its applications.

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On embedding of metrizable spaces into uniform Eberlein compacts

Abstract. According to the Archangelśkii theorem, every metrizable space has an Eberlein compactification. Our purpose is to investigate which metrizable spaces can be embedded into uniform Eberlein compacts, i.e. weakly compact subspaces of the Hilbert space.

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Pontryagin's works

Abstract. The authors of the talk were fortunate to lern and work under the direct guidance of academician Lev Pontryagin. We remind his main topological results and describe its influence on other branches of mathematics, in particular, on the Theory of Differential Equations (in a broad sens) and on the Theory of Optimal Control.

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Poincare duality and signature for topological manifolds

Abstract. The signature of the Poincare duality of compact topological manifolds with local system of coefficients can be described as a natural invariant of nondegenerate symmetric quadratic forms defined on a category of infinite dimensional linear spaces.

The objects of this category are linear spaces of the form $W = V \oplus V^*$ where V is abstract linear space with countable base. The space W is considered with minimal natural topology.

The symmetric quadratic form on the space W is generated by the Poincare duality homomorphism on the abstract cochain group induced by nerves of the system of atlases of charts on the topological manifold.

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On weakly Whyburn spaces

Abstract. A topological space X is called *Whyburn* if for every $A \subset X$ and every $x \in \overline{A}$ there is a set $B \subset A$ such that $\overline{B} \setminus A = \{x\}$.

A space X is called *weakly Whyburn*, if for every non-closed $A \subset X$ there is $B \subset A$ such that $|\overline{B} \setminus A| = 1$.

We extract a simple principle from an example of Gerlits and Nagy (also used by some authors in context of weakly Whyburn spaces), which allows us to bring some new examples to the topic.

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On H-closedness via a typical extension

Abstract. The notion of θ-closure operator in a topological space, first introduced by Veličko [Amer.Math.Soc.Transl. 78(1968)], is known to be an extremely useful appliance for studying numerous concepts of topology, especially in dealing with the H-closed spaces. According to Alexandroff and Urysohn [Math.Ann.92(1924)], a Hausdorff space X is H-closed if it is closed in every Hausdorff space in which it is embedded as a subspace. There have been extensive investigations on H-closed spaces and allied topics in course of the last 70 years or so. The present talk is aimed at a description of H-closed spaces in terms of a type of extensions, called θ-extensions of topological spaces. The concept of θ-extensions, and θ-equivalence of two such exten-

sions by means of θ-closure operator are introduced. Like trace systems in a closure space, we define the θ-trace systems of θ-extensions of a space. These concepts ultimately lead us to characterize the θ-equivalence of H-closed θ-extensions of a Hausdorff space. A particular type of θ-extension is formulated, which is termed as θ-principal extension. We then construct a typical θ-extension X^* of a Hausdorff space X, where X^* is a certain collection of grills on X, and show that any θ-principal extension of a Hausdorff space is θ-equivalent to X^*. A condition for X^* to be H-closed is obtained. All these results finally yield some formulations, in terms of the space X^*, of the H-closedness of a Hausdorff space.

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Strong nearness sigma-frames and completion

Abstract. We consider the notion of a nearness structure on a σ-frame as a generalization of uniformities and metrics on σ-frames. Particularly, we focus on the category **SN**σ**Frm** of strong nearness σ-frames. We exhibit a completion of a strong nearness σ-frame and show that completion is a coreflection in this category. This in essence generalizes the uniform (and metric) σ-frame case.

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On D-Recurrent Spaces with Semi-Symmetric Recurrent-Metric Connection

Abstract. The properties of Riemannian spaces admitting a semi-symmetric metric connection and semi-symmetric recurrent-metric connection were studied in [1-3]. On the other hand, the notion of the recurrent Riemannian space was introduced and studied in [4]. Furthermore, D-recurrent spaces with semi-symmetric connection were investigated in [5].

In this work, we introduce D- recurrent spaces with semi-symmetric recurrent-metric connection denoted by (M_n, g, D) and we obtain some properties of the curvature tensor L^m_{ijk} of (M_n, g, D). For such a space, it is shown that $D_l L_{ijkh} = \rho_l \, L_{ijkh}$, $(\rho_l \neq 0)$, where L_{ijkh} is the curvature tensor corresponding to the D connection and ρ_l is a non-zero covariant vector field. Also, an example of these spaces is given.

Key words. Recurrent spaces, semi-symmetric spaces, metric connection, semi-symmetric recurrent-metric connection.

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Categorical view of di-uniformities

Abstract. Textures and ditopological spaces were introduced as a means of representing fuzzy sets and topologies in a point based setting. By a texturing of a set S we mean a subset $\mathcal{S}$ of the power set $P(S)$ which is a point separating, complete, completely distrubutive lattice with respect to inclusion which contains S and $\emptyset$ and for which arbitrary meets coincide with intersections and finite joins with unions. The pair $(S, \mathcal{S})$ is called a texture space.

In [1] the authors laid the foundations of a theory of uniformities on textures. The effect of complementation on the texture and the relation with classical quasi-uniformities and uniformities were discussed in [2] and in [3] the notions of completeness and total boundness were given for di-uniformities on a texture.

In this talk we discuss various fundamental aspects of di-uniform texture spaces in a categorical setting. We will introduce the category $drUnif$ whose objects are direlational uniformities, whose morphisms are uniformly bicontinuos difunctions and also the category $\mathcal{DC}Unif$ and several related categories and some properties will be discussed including the existence of products.

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Spaces of monotone continuous functions $CM_p(X)$: compactness and extension properties

Abstract. Real-valued function is called *monotone* if preimages of all points are connected. We consider the space of all monotone continuous function on the connected topological space X with the topology of pointwise convergence – $CM_p(X)$. The space X is called *monotonically degenerate* if all continuous function on X are constant.

Theorem. These conditions for Tychonoff space X are equivalent:
(1) $CM_p(X)$ is locally compact;
(2) $CM_p(X)$ is σ-countably compact;
(3) $CM_p(X)$ has Hurewicz property;
(4) X is monotonically degenerate.

Let $X \subseteq Y$ and f is a continuous function on X. Does exist a monotone extension of f from X to Y ?

Theorem. Let X be a Tychonoff space and $p \in (0; 1)$. Let f is a bounded function on a space $X \times \{p\}$, where

$X \times \{p\} \subseteq X \times [0;1]$. Then there is a monotone extension of f to $X \times [0;1]$.

Example. There exists the continuous function on a space $[0;1] \times \{0\}$ which has no monotone extension to $[0;1] \times [0;1]$.

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Approach frames: the pointfree counterpart of approach spaces

Abstract. Approach spaces form a topological construct in which the categories of topological spaces and metric spaces are embedded. It has various characterisations, one of which is the so-called regular function frame, a collection of functions from the space to $[0, \infty]$ stable for suprema, finite infima and translations. This can be abstracted to a pointfree notion, which we call approach frames.

To be precise, it is a frame equipped with two extra unary operations: for each positive real constant there is an up shift and a ‘truncateddown shift, where the involved operations have to satisfy all algebraic laws that hold on $[0, \infty]$. On the latter interval, the standard operations are considered, that is, the up shift is given by addition and the down shift given by subtraction, upwardly truncating to 0 when the outcome is a negative number [1].

We will introduce the basic functors between approach spaces and approach frames and demonstrate that approach frames form a category which has the same interaction with frames as approach spaces have with topological spaces, namely the latter are embedded in the former and this embedding has both a left and right adjoint.

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On Conformally Flat Pseudo-symmetric Kahler Spaces

Abstract. In a recent paper [1], M.C.Chaki introduced and studied a type of non-flat Riemannian space (M_n, g) $(n \leq 2)$ whose curvature tensor satisfies the condition

$$R^h_{ijk,l} = 2\lambda_l R^h_{ijk} + \lambda^h R_{lijk} + \lambda_i R^h_{ljk} + \lambda_j R^h_{ilk} + \lambda_k R^h_{ijl}$$

where λ_i is a non-zero vector and comma denotes covariant differentiation with respect to the metric g_{ij}. Such a space was called by him a pseudo-symmetric space, the vector λ_i was called its associated vector and n-dimensional space of this kind has been denoted by $(PS)_n$.

In this paper, firstly, we proved a theorem that the scalar curvature of the conformally flat pseudo symmetric Kahler spaces is constant and we proved that this space is also Ricci symmetric. By using this theorem, we obtain the Ricci tensor as

$$R_{ij} = \alpha g_{ij} + \beta v_i v_j$$

where α and β are scalars. Without assuming any restriction on Ricci tensor, by using some theorems, it is proved that this space is being translated to special spaces.

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On Weakly and Pseudo Concircular Symmetric Structures On a Riemannian Manifold

Abstract. A non-flat Riemannian manifold (M_n, g), $n > 2$ is called weakly symmetric if the curvature tensor R_{hijk} satisfies the condition $\nabla_l R_{hijk} = A_l R_{hijk} + B_h R_{lijk} + D_i R_{hljk} + E_j R_{hilk} + F_k R_{hijl}$ where A,B,D,E,F are 1-forms (non-zero simultaneously) and the comma "," denotes covariant differentiation with respect to the metric tensor of the manifold. The 1-forms are called the associated 1-forms of the manifold and an n-dimensional manifold of this kind is denoted by $(WS)_n$.

For a conformal transformation $\bar{g}_{ij} = \rho^2 g_{ij}$ of the metric tensor of a Riemannian manifold, the function ρ satisfies the relation $\rho_{ij} = \phi g_{ij}$, $\rho_{ij} = \rho_{i,j} - \rho_i \rho_j + \frac{1}{2} g^{\alpha\beta} \rho_\alpha \rho_\beta g_{ij}$, $\rho_j = \frac{\partial}{\partial u^j} \ln \rho$ this transformation is called concircular transformation where the comma "," denotes the covariant differentiation and ϕ is a function of u^i.

The present paper deals with non-concircular flat Riemannian manifold (M_n, g) whose concircular curvature tensor Z_{hijk} satisfies the condition

$$\nabla_l Z_{hijk} = A_l Z_{hijk} + B_h Z_{lijk} + D_i Z_{hljk} + E_j Z_{hilk} + F_k Z_{hijl}$$

where $Z_{hijk} = R_{hijk} - \frac{R}{n(n-1)}(g_{hk}g_{ij} - g_{hj}g_{ik})$, R_{hijk} is the curvature tensor and R is the scalar curvature. Such a manifold will be called a weakly concircular symmetric manifold and denoted by $(WZS)_n$. It was shown that, Z^h_{ijk} is invariant under a concircular transformation.

After these definitions, in the second part of this paper, totally umbilical hypersurface of a weakly concircular symmetric space is examined and some theorems are proved:

Theorem. Let us suppose that the totally umbilical hypersurface of $(WZS)_{n+1}$ is $(WZS)_n$. If this hypersurface holds the condition $R = (\frac{n-1}{n+1})\bar{R}$ ($R \neq const.$) then, the hypersurface is totally geodesic $(n > 2)$.

Theorem. If the totally geodesic hypersurface of $(WZS)_{n+1}$ satisfies the condition $R = (\frac{n-1}{n+1})\bar{R}$ then (M_n, g) is $(WZS)_n$, $(n > 2)$.

Theorem. If the totally hypersurface of $(PZS)_{n+1}$ is $(PZS)_n$ and the condition $\nabla_s M^2 = \lambda_s M^2$ holds then this hypersurface is totally geodesic $(n > 2)$.

In the last part of this paper, an example is given for the existence of the weakly concircular recurrent Riemannian manifold.

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Different Kinds of Closedness in S(n)-Spaces.

Abstract. In 1966, Velichko [11] introduced the notion of θ-closedness. For any subset M of a topological space, the θ-closure is defined to be the set $\mathrm{cl}_\theta M$ consisting of points $x \in X$ such that any closed nb of x intersects M. Many topologists actively applied this notion when studying nonregular Hausdorff spaces. For example, Hamlett [4], Jankovic [5], Porter and Votaw [9], and Gryzlov [6] use the θ-closure operator in investigating properties inherent in H-closed spaces and spaces close to them (functionally compact, C-compact, and others). In 1986 Dikranjan and Giuli [3] introduced a more general notion of the θ^n-closure operator and developed a theory of $S(n)$-spaces, $S(n)$-closed and $S(n)$-θ-closed spaces. In 1997, Jiang, Reilly, and Wang [12] used the θ^n-closure in studying properties of $S(n)$-noncondensable spaces.

The present work continues the study of properties inherent in $S(n)$-closed and $S(n)$-θ-closed spaces, using the θ^n-closure operator; in addition, wider classes of spaces—weakly $S(n)$-closed and weakly $S(n)$-θ-closed spaces— are introduced.

Let X be a topological space, $M \subseteq X$, and $x \in X$. For any $n \in \mathbb{N}$, we consider the θ^n-*closure* operator: $x \notin \mathrm{cl}_{\theta^n} M$ if there exists a set of open nbs $U_1 \subseteq U_2 \subseteq ... \subseteq U_n$ of the point x such that $\mathrm{Cl}U_i \subseteq U_{i+1}$ for $i = 1, 2, ..., n-1$ and $\mathrm{cl}U_n \cap M = \varnothing$ if $n > 1$; $\mathrm{cl}_{\theta 0} M = \mathrm{cl}M$ if $n = 0$; and, for $n = 1$, we get the θ-closure operator, i.e., $\mathrm{cl}_{\theta 1} M = \mathrm{cl}_\theta M$.

A set M is θ^n-closed if $M = \mathrm{cl}_{\theta^n} M$. For any $n \in mathbbN$, a point $x \in X$ is $S(n)$-*separated* from a subset M if $x \notin \mathrm{cl}_{\theta^n} M$. For example, x is $S(0)$-separated from M if $x \notin \overline{M}$. For $n > 0$, the relation of $S(n)$-separability of points is symmetric.

X is called an $S(n)$-*space* if any two distinct points of X are $S(n)$-separated.

It is obvious that $S(0)$-spaces are T_0-spaces, $S(1)$-spaces are Hausdorff spaces, and $S(2)$-spaces are Urysohn spaces.

In the Aleksandrov and Urysohn memoir on compact spaces [1], the notion of a θ-accumulation point was introduced. A point x is called a θ-*accumulation point* of a set F if $|F$
$cap\overline{U}| = |F|$ for any nb U of the point x. It was noted that any H-closed space has the following property:

$(*)$ any infinite set of regular power has a θ-accumulation point.

However, the converse is not true. The first example of a space possessing property $(*)$ and not being H-closed was constructed by Kirtadze [7].

Definition 1. *A nb U of a point x is called an n-hull of the point x if there exists a set of nbs $U_1, U_2, \ldots, U_n = U$ of the point x such that* $\mathrm{cl} U_i \subseteq U_{i+1}$ *for $i = 1, \ldots, n-1$.*

Definition 2. *A point x from X is called a $\theta 0(n)$-accumulation ($\theta(n)$-accumulation) point of an infinite set F if $|F \cap U| = |F|$ ($|F \cap \overline{U}| = |F|$) for any U, where U is an n-hull of the point x.*

Note that, for $n = 1$, a $\theta 0(1)$-accumulation point is a point of full accumulation, and a $\theta(1)$-accumulation point is a θ-accumulation point.

Definition 3. *A topological space X is called weakly $S(n)$-θ-closed (weakly $S(n)$-closed) if any infinite set of regular power of the space X has a $\theta 0(n)$-accumulation ($\theta(n)$-*

accumulation) point.

Note that any $\theta 0(n)$-accumulation point is a $\theta(n)$-accumulation point; hence, any weakly $S(n)$-θ-closed space is weakly $S(n)$-closed. Moreover, since a $\theta(n)$-accumulation point is a $\theta 0(n+1)$-accumulation point, it follows that a weakly $S(n)$-closed space will be weakly $S(n+1)$-θ-closed.

For $n = 1$, weakly $S(1)$-θ-closed and weakly $S(1)$-closed spaces are compact Hausdorff spaces and spaces with property $(*)$, respectively.

Theorem 1. *Let X be an $S(n)$-θ-closed $S(n)$-space. Then X is weakly $S(n)$-θ-closed.*

Theorem 2. *Let X be an $S(n)$-closed $S(n)$-space. Then X is weakly $S(n)$-closed.*

It was proved in [3] that $S(n)$-closedness implies $S(n+1)$-θ-closedness (see also [8]).

The existence of a finite subcovering for any Urysohn covering of a space X is a characteristic of the U-θ-closedness of this space. A regular space X is called regular-closed if it is a closed subspace in every regular space in which it is embedded. A topological space X is feebly compact if any open locally finite family of its subsets is finite.

In 1982, Pettey [10] proved that the product of regular-closed spaces is regular-closed if it is feebly compact. The validity of a similar theorem in the class of U-θ-closed spaces was discussed in the work of Dikranjan and Giuli [3] in 1988, where the problem on the product of U-θ-closed spaces (Problem 5) was formulated. Namely, it is required to prove or to disprove that the product of U-θ-closed spaces is feebly compact. In the present work, two Urysohn U-θ-closed spaces whose product is not feebly compact are constructed. Thus, the question is negatively solved.

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A new characterization of the plane

Abstract. Characterizations of the plane is a classical topic in topology starting from the work of Moore. In this talk we give another such characterization improving an earlier result of Papasoglu:
Theorem. Let X be a locally compact, locally connected, non degenerate metric space. Then X is homeomorphic to the plane iff the following hold:

(i) X has no cut points

(ii) X is simply connected

(iii) For any two points of X there is a line separating them.

We define a line to be the image of a proper 1-1 map $f : \mathbb{R} \to X$. We say that a line L separates a, b if a, b lie in distinct components of $X - L$.

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On the classification of homogeneous continua

Abstract. Locally connected 1-dimensional homogeneous continua have been classified by Anderson. In this talk we

focus on the 2-dimensional case and we prove the following inspired by geometric group theory:

Theorem. Let X be a non-degenerate, homogeneous, locally connected and simply connected continuum. Then no simple arc separates X.

It is an interesting question whether this holds for 2-dimensional homogeneous l.c. continua in general. We remark that Krupski has shown that homogeneous continua are Cantor manifolds, so our result is interesting only for 2-dimensional continua. One could hope for a classification of homogeneous simply connected continua in dimension 2 similar to Andersonś classification. To our knowledge there are no examples of homogeneous, locally connected and simply connected continua of dimension 2 other than the sphere and the 2-dimensional universal Menger space.

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Continuous maps between finite degrees of Sorgenfrey line.

Abstract. Recall that the Sorgenfrey line $\mathbb{S}$ is the real line topologized by a basis of half-open intervals closed on the left. It is well known that there is no continuous map of $\mathbb{S}$ onto $\mathbb{S}^n$ for $n \geq 2$ and, clearly, for any $n > m > 0$ there exists open map of $\mathbb{S}^n$ onto $\mathbb{S}^m$.

In 1987 D.K. Burke and D.J. Lutzer [1] proved that for any $n > m > 0$ there exists continuous one-to-one map of $\mathbb{S}^n$ onto $\mathbb{S}^m$. We obtain the following theorems.

Theorem 1. For any $2 \leq n < m$ there exists continuous one-to-one map of $\mathbb{S}^n$ onto $\mathbb{S}^m$

Theorem 2. There is no factor map of $\mathbb{S}^2$ onto $\mathbb{S}^m$ for $m > 2$.

Theorem 3. There is no closed map of $\mathbb{S}^n$ onto $\mathbb{S}$ for $n > 1$.

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On dimensional properties of Corson and (strong) Eberlein compacta

Abstract. Let: τ be an infinite cardinal number; $\mathcal{A}$ a set of cardinality τ;

$$I^\tau = \prod\{I_\alpha = [0,1] : \alpha \in \mathcal{A}\};$$

$$\sigma^\tau = \{x = \{x_\alpha\}_{\alpha\in\mathcal{A}} \in I^\tau : |\{\alpha : x_\alpha \neq 0\}| < \omega\}$$

and

$$\Sigma^\tau = \{x = \{x_\alpha\}_{\alpha\in\mathcal{A}} \in I^\tau : |\{\alpha : x_\alpha \neq 0\}| \leq \omega\}.$$

Recall that a compactum is called *strong Eberlein* if it is homeomorphic to a subspace of σ^τ for some τ. Recall also that a countably compact space X can be embedded topologically in Σ^τ iff there exists in X a point-countable, of cardinality $\leq \tau$ and T_0-separating points of X family of functionally open subsets. All such spaces are normal and sequentially compact Frechet-Urysohn spaces.

I. **Theorem 1.** For any ordinal number α, there exists a strong Eberlein compactum E_α with $dim\ E_\alpha = 1$ and $ind\ E_\alpha = \alpha$.

Theorem 2. There exists a strong Eberlein compactum F with $dim\ F = 1$, $ind\ F = 2$ and $Ind\ F = 3$.

II. **Theorem 3.** For any metrizable space M with $dim\ M = n$, $n = 0, 1, \ldots$, there exists an Eberlein compactification eM (i.e., eM is an Eberlein compactum) such that $w(eM) = w(M)$, $dim\ eM = ind\ eM = Ind\ eM = dim\ M$, eM has a 0-dimensional map onto a metrizable compactum, there exist a 0-dimensional Eberlein compactum $E0$ and an onto map $f : E0 \to eM$ with $|f^{-}1y| \leq n+1$ for any $y \in eM$, M is d-posed in eM (see B.A.Pasynkov [London Math. Soc. Lect. Note Ser., **93** (1985), 227-250].

Remark. In 2002 T.Kimura and K.Morishita [Fund. Math., **171**(2002), 223-234] proved that every metrizable space M has an Eberlein compactification KM with $dim\ KM = dim\ KM$. In 2004 M.G.Charalambous [Fund. Math., **182**(2004), 41-52] proved that, additionally, $Ind\ KM = dim\ KM$ (and so $ind\ KM = dim\ KM$).

The following generalization of Theorem 3 is obtained by me jointly with Ant.Belyaeva (for a compactum X, $\Delta\ X \leq n$ means that there exist a 0-dimensional compactum $X0$ and an onto map $f : X0 \to X$ such that all fibers of f consist of not greater than $n+1$ points).

Theorem 4. For any metrizable space M and any σ-locally finite family λ of closed sets in M, there exists an Eberlein compactification eM of M having 0-dimensional map onto a metrizable compactum such that M is d-posed in eM and, for any $F \in \lambda$,

$$
\begin{aligned}
1.\ dim\ cl_{eM}F &= dim\ F = ind\ cl_{eM}F = \\
&= Ind\ cl_{eM}F = \Delta\ cl_{eM}F;
\end{aligned}
$$

2. $w(cl_{eM}F) = w(F)$.

III. **Theorem 5.** [The factorization theorem for maps to Valdivia, Corson and (strong) Eberlein compacta] If f is a map of a space X of dimension $dim\ X = n$, $n = 0, 1, \ldots$, to a Corson (respectively, to a (strong) Eberlein or Valdivia) compactum Z then there exists a Corson (respectively, (strong) Eberlein or Valdivia) compactum Y and maps $g : X \to Y$, $h : Y \to Z$ such that $f = h \circ g$, $dim\ Y \leq n$ and $w(Y) \leq w(Z)$.

Corollary 6. [The theorem on Corson, (strong) Eberlein and Valdivia compactifications] Let cX be a Corson (respectively, (strong) Eberlein or Valdivia) compactification of a Tychonoff space X. Then there exists a Corson (respectively, (strong) Eberlein or Valdivia) compactification bX of X following cX with $w(bX) = w(cX)$ and $dim\ bX \leq dim\ X$.

Theorem 7. [The theorem on a universal n-dimensional countably compact space contained in Σ^τ] For any $\tau \geq \omega$ and $n = 0, 1, \ldots$, there exists an n-dimensional (in the sense of dim) countably compact space $U^n_{\Sigma^\tau} \subset \Sigma^\tau$ (of weight τ) such that any n-dimensional subspace of Σ^τ can be embedded topologically in it.

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Higher-dimensional hereditarily indecomposable continua with exactly n continuous surjections

Abstract. We show that for every $n \in N$ and $m \in N \cup \{\infty\}$ there exists a hereditarily indecomposable m-

dimensional continuum which has exactly n continuous surjections onto itself (each one being a homeomorphism). Moreover, we construct an uncountable family of continua of this type such that no two different continua from this family are comparable by continuous mappings. In the special cases when $m = 1$ or $n = 1$, these results were obtained by the author in 2005.

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On strong K-homology

Abstract. In [5] I introduced strong homology with the coefficients in a spectrum E by the following:

Definition 1. Let X be a topological space, and let $X \to \mathbf{P}$, where $\mathbf{P}$ is a pro-object in the category of ANRs, be a strong ANR-expansion in the sense of S. Mardesic [4]. We define

$$\overline{H}_n(X,E) := \pi_n \mathbf{holim}_{i \in I} \left(\omega\left(\mathbf{P}(i)^+ \wedge E\right)\right)$$

where $i \longmapsto \mathbf{P}(i)$ is a functor $I \longrightarrow TOP$ corresponding to the pro-space $\mathbf{P}$, $\omega(D)$ is an Ω-spectrum corresponding to the spectrum D, Y^+ is a one-point extension of Y, and **holim** is the inverse homotopy limit of spectra in the sense of R. W. Thomason [6].

In the case when E is the K-theory spectrum bu it is quite natural to call the corresponding strong homology $\overline{H}_n(X, bu)$ **strong K-homology**, and we denote

$$\overline{K}_n(X) := \overline{H}_n(X, bu), n \in \mathbb{Z}.$$

This theory satisfies many useful properties [4]: exactness (in two senses), suspension axiom, triad sequence, excision, strong wedge axiom, and
Atiyah-Hirzebruch type spectral sequence (when X has a finite shape dimension). Nothing more special can be added in the case of **general** spaces. However, when X is compact (= compact Hausdorff), then the following property can be proved. The K-homology below is the usual homology with respect to the spectrum bu:
$K_n(Y) := H_n(Y, bu)$.

Theorem 1. Let a compact space X be represented as the inverse limit of finite polyhedra: $X = \mathbf{lim}_\alpha X_\alpha$. Then there exists a natural exact sequence

$$0 \longrightarrow \mathbf{lim}^1_\alpha K_{n+1}(X_\alpha) \longrightarrow \overline{K}_n(X) \longrightarrow \mathbf{lim}_\alpha K_n(X_\alpha) \longrightarrow .$$

In [1] Brown, Douglas, and Fillmore introduced the Abelian group functor $Ext(X)$ for compact metric spaces X, and a homology theory which they called Steenrod K-homology: ${}^sK_n(X) = Ext(X)$ if n is even and ${}^sK_n(X) = Ext(SX)$ if n is odd.

In [2] and [1] Kahn, Kaminker and Schochet introduced another variant of Steenrod K-homology, and proved that their variant is isomorphic to ${}^sK_n(X)$ in the case when X is a **finitely dimensional** compact metric space. Our theory, however, is isomorphic to ${}^sK_n(X)$ in a more general context:

Theorem 2. The functors $\overline{K}_n$ and ${}^sK_n$ are isomorphic on the category of compact metric spaces.

A natural question arises: does there exist an equivalent **analytic** definition of $\overline{K}_n$ on the category of
textbfall compact spaces?

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Coexistence of ω-limit sets

Abstract. In topological dynamics Sharkovskiis theorem about the interrelationships of periodic points (cycles) of continuous self-mappings of the unit interval is widely known. It was generalized in several different directions. In 1990 Ye Xiangdong has introduced a new topological invariant: D-function of a minimal set, with structure similar to period of a periodic point. Ye Xiangdong has included D-functions in Sharkovskii cycle coexistence ordering and extended Sharkovskiis theorem to minimal sets. The notion of D-function is extended to ω-limit set of an arbitrary point. We show which properties are induced and which new phenomena arise. We prove that the Sharkovskii-Ye Xiangdong D-functions ordering holds true for this case.

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A covering characterization of strong quasi-metric spaces

Abstract. A quasi-metric space is called strong if its topology is smaller than its conjugate topology. Raghavan and Reilly (J. London Math. Soc.15 (1977), 169-172) showed that any quasi-metric space whose topology is countably paracompact with respect to its conjugate topology is strong. Marin and Romaguera (Publ. Mat. Debrecen (2006) to appear) have proved that the converse of this result is true, thus providing a characterization of strong quasi-metric spaces in terms of bitopological countable paracompactness.

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Embeddability of cones and suspensions

Abstract. This will be a survey talk on how the methods of geometric topology have proven to be quite successful in answering questions of embeddability. Basic results of embeddability theory were laid in 1930's by Claytor. In 1990's

Repovš, Skopenkov and Ščpin investigated the conditions for embedding products of higher dimensional polyhedra and an interval into Euclidean space. One of the results they proved was that for each n-dimensional polyhedron X, the product $X \times I$ embeds into R^{2n+1}. More recently, Malešič, Repovš, Rosicki and Zastrow showed that such products with the unit interval often have a
nonunique structure as a Cartesian product. We are interested to find out for which dimension the cone CX of an n-dimensional polyhedron X embeds into R^k. Related low-dimensional versions of this question have already been answered - it was shown by Rosicki that for a locally connected continuum, provided it embeds into $R3$ and is a nontrivial Cartesian product, that either one of the factors is an arc, or that it is a simple closed curve and the other factor is also embeddable into $R2$. He also proved that if X is a locally connected continuum whose cone CX embeds into R^n with $n < 4$, then X embeds into S^{n-1}. Cauty showed that if X is a locally connected continuum such that $X \times I^{n-2}$ embeds into an n-! manifold, then X must be locally planar. We shall also present some new results and conjectures.

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Stratifiable function spaces

Abstract. Stratifiable spaces $C_k(X)$ of continuous real-valued functions on separable metrizable spaces X with the compact-open topology are studied. Suppose that X is a metrizable compact space and λ is a family of compact subsets of X. Let $C_\lambda(X)$ denote $C(X)$ with the topology

of uniform convergence on the elements of λ. Conditions on families λ under which $C_\lambda(X)$ is stratifiable are studied. It is proved that $C_\lambda(X)$ is stratifiable if λ is (i) the family of countable compact sets or (ii) the family of countable compact sets of finite scattered height.

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On More Properties of a- Covering Dimension

Abstract. Zougdani(H. Zougdani, Covering Dimension of Fuzzy Spaces,Math-Vesnik, 36(1984),104-118) introduced the concept of α-Covering Dimension by using α-shading and proved its several properties. In this paper, we have studied some more properties of α-Covering Dimension. While studying the relationship of α-Covering Dimension of a fuzzy topological space with Covering Dimension of its background space, we have proved that α- dim(X, δ) = dim(X, $[\delta]$). By using this theorem, we proved some other interested properties of α- Covering Dimension. We have also shown that α- Covering Dimension is a topological property where as this concept need not be preserved by a continuous map. While studying the relationship of Finite Covering Dimensional fuzzy topological space with some other fuzzy topological space which are also defined with help of α-shading,We checked that an a-compact (T.E.Gantner,R.C.Steinlage and R.H.Warren, Compactness in fuzzy topological spaces,J.Math.Anal.Appl. 62(1978),547-562) space need not be finite dimensional and a finite dimensional space need not be α-compact(Note here finite dimensional space means a fuzzy topological

space of finite α- Covering Dimension). We also given examples to show that a finite dimensional space need not be α-finitistic(Shakeel Ahmed, On α-finitistic Spaces, Tamsui Oxford Journal of Mathematical Sciences Vol.22,No2(to appear)) and an α-finitistic space need not be finite dimensional. At the end we have proved that the sum space of any finite number of finite dimensional space is finite dimensional if and only if each factor is finite dimensional.

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Splitting problem for selections

Abstract. Let $F_1 : X \to Y_1$ and $F_2 : X \to Y_2$ be any convexvalued lower semicontinuous mappings and let $L : Y_1 \oplus Y_2 \to Y$ be any linear surjection. The *splitting problem* is then problem of representation of any continuous selection f of the composite mapping $L(F_1; F_2)$ in the form $f = L(f_1; f_2)$, where f_1 and f_2 are some continuous selections of F_1 and F_2, respectively. The typical example gives the the following question. When an arbitrary continuous selection of $F_1 + F_2$ is sum of continuous selections of F_1 and F_2, respectively?

We prove that the splitting problem always admits an approximative solution with f_i being an ε-selections, theorem 2.1. We also propose two partial cases for finding of exact splittings, which occurs are stable with respect to continuous variations of the data, theorems 3.1 and 3.3 and show that, in general, exact splittings does not exist even for finite-dimensional range spaces.

Some examples, counterexamples and applications for measurable, Lipshitz, etc. selections are presented.

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Cardinal functions concerning Katětov extensions

Abstract. In the present paper, an attempt has been made to investigate the cardinal functions corresponding to certain concepts associated with the Katětov H-closed extension of an infinite discrete space. The concepts of density, spread, cellularity, weight and tightness of a topological space, as given by Juhász, are recalled here for their determination in respect of the said Katětov extension of an infinite discrete space.

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Some properties of functors M_τ and M_R

Abstract. For a Tychonoff space X by $M(X)$ we denote the set of all σ-additive non-negative Borel measures on X. This set is equipped by $\star$-weak topology. For a continuous mapping $f : X \to Y$ one can define a mapping $M(f) : M(X) \to M(Y)$ as follows:

$$M(f)(\mu)(B) = \mu(f^{-1}(B))$$

for any $\mu \in M(X)$ and Borel set $B \subset Y$. This mapping is continuous and we get a covariant functor $M : \mathcal{T}ych \to \mathcal{T}ych$. By $M_\tau(X)$ (respectively $M_R(X)$) we denote the set of all τ-additive (respectively Radon) measures $\mu \in M(X)$. For a continuous mapping $f : X \to Y$ we have
$M(f)(M_\tau(X)) \subset M_\tau(Y)$ and $M(f)(M_R(X)) \subset M_R(Y)$.

This gives us functors M_τ and M_R which are subfunctors of M.

These functors doń t preserve empty set, a point and compact spaces. All other properties of normality of covariant functors $F : \mathcal{T}ych \to \mathcal{T}ych$ are fulfilled for the functors M_τ and M_R. Besides, these functors preserve metrizable spaces, p-paracompact spaces, and Čech-complete spaces.

For a bounded metric space (X, ρ) we construct a metric $M_\tau(\rho)$ on the space $M_\tau(X)$. This construction is functorial. Thus the functor M_τ is lifted to a functor $M_\tau^{metr} : \mathcal{BM}etr \to \mathcal{M}etr$, where $\mathcal{M}etr$ is the category of all metric spaces and their continuous mappings and $\mathcal{BM}etr$ is a full subcategory of $\mathcal{M}etr$ consisting of all bounded metric spaces.

The functor M_τ^{metr} preserves complete metric spaces, but it doesń t preserve a uniform continuity of mappings even in a realm of compact metric spaces. Nevertheless, the functors M_τ and M_R can be lifted to the category $\mathcal{U}nif$ of uniform spaces.

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On Peano Curves

Abstract. Squre-to-linear ratio of a mapping $p : I \to I2$ of an interval onto a square is defined as maximum of $\frac{|p(x)-p(y)|^2}{|x-y|}$, $x, y \in I$. The problem considered is to construct mapping with minimal possible square-to-linear ratio.

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Some Applications of infinitesimal pseudo homothetic transformations

Abstract. A non-flat Riemannian space is called generalized pseudo Ricci symmetric and denoted by $(GPRS)_n$ if the Ricci tensor is non-zero and satisfies the condition:

$$R_{ij,k} = \lambda_k R_{ij} + \lambda_i R_{kj} + \lambda_j R_{ik} \tag{1}$$

where λ is 1-form (non-zero simultaneously) and comma denotes the operator of the covariant
derivative with respect to the connection. Generalized pseudo symmetric space was introduced by Chaki and Koley, [1].

Firstly, we shall consider an infinitesimal conformal transformation in $(GPRS)_n$.

We call an infinitesimal transformation pseudo-homothetic if it is an infinitesimal conformal transformation under which the curvature tensor in invariant.

The object of this paper is to study pseudo-homothetic motions in a $(GPRS)_n$. It is proved that if a $(GPRS)_n$ of dimension n admits either homothetic motion or this space is Ricci-flat.

In $(GPRS)_n$ which is not Ricci-flat, a pseudo-homothetic motion is also affine motion.

Last part of this paper, we give an example for this motion.

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Zero-Dimensional Metrizable Topological Groups

Abstract. The celebrated theorem of Katětov says that $\dim X = IndX$ for any metric space X; however, there exist examples of metrizable spaces with noncoinciding dimensions *ind* and dim. The first (very complicated) example of such a space was constructed by Roy in 1968. Since then, much simpler examples with various additional properties have been constructed, but the question about the coincidence of dimensions for metrizable topological groups has remained open (apparently, for the first time, it was stated by Mishchenko in 1964).

The spaces embeddable in zero-dimensional topological groups occupy an intermediate position between zero-dimensional and strongly zero-dimensional metrizable spaces (any strongly zero-dimensional metrizable space X can be metrized by a non-Archimedean metric, and this metric can be assumed to take only rational values. The Graev extension of such a metric to the free group $F(X)$ takes only rational values as well; therefore, the group $F(X)$ with the

Graev metric has dimension *ind* zero, and it contains X as a subspace). We suggest the following criterion for the embeddability of zero-dimensional metrizable topological spaces in zero-dimensional metrizable topological groups.

Theorem. A topological space X can be embedded in a zero-dimensional metrizable topological group if and only if the topology of X is generated by a uniformity which has a countable base consisting of open-and-closed sets.

Using this criterion, we prove that a special case of Mrowkaś metrizable space with noncoinciding dimensions can be embedded as a closed subspace in a zero-dimensional metrizable group (and it is not strongly zero-dimensional); thereby, an example of a metrizable group with noncoinciding dimensions *ind* and dim is obtained. On the other hand, applying the same criterion, we show that one of Kuleszaś zero-dimensional metrizable spaces cannot be embedded in a metrizable zero-dimensional group; thus, not all zero-dimensional metrizable spaces can be embedded in zero-dimensional metrizable groups. Some open questions and conjectures are also discussed.

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Integer-valued fixed point index for acyclic maps on ANRś

Abstract. Integer-valued fixed point index for compositions of $(\mathbf{m}, GZ)-$acyclic multivalued maps on compact ANR spaces is constructed. This integer-valued fixed point index has the properties: additivity, homotopy invariance, normalization,commutativity, multiplicativity and mod-p.

The $GZ-$acyclicity is with respect to the Cech cohomology theory. The technique of chain approximation is used.

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Dimension-raising maps theorems

Abstract. Versions of the Hurewicz and Freudenthalś theorems on dimension-raising maps are established for cohomological and extension dimensions.

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Categorical convergence structures

Abstract. We introduce and study a concept of a convergence structure on a concrete category. The concept is based on using certain generalized filters for expressing the convergence. Some basic properties of the convergence structures are discussed. In particular, we study convergence separation and convergence compactness and investigate relationships betweenthe convergence structures and the usual closure operators on categories.

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Universal dendrites for some families of dendrites with a countable set of end points.

Abstract. We prove that for any integer $\kappa \geq 3$ and for any countable ordinal α there exists a universal dendrite in the family of all dendrites X with orders of ramification points $\leq \kappa$ and with a closed countable set of end points $E(X)$ such that the α-derivative of $E(X)$ contains at most one point.

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Linear Coverings Of Graps

Abstract. We define a concept of finite coverings of connected graphs by connected subgraphs with a linear ordering and such that there is a supremum for the number of such coverings. An algorithmic procedure is given that counts this supremum. Also it is proved that if this number is finite then it is even. (Mathematical Subject Classification (1991), Primary 54D05, Secondary 05C10).

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On weakly CLP-compactspaces

Abstract. Recall that a space is called CLP-compact if every its cover by clopen (closed and open) sets has a finite subcover. CLP-compact spaces generalize compact spaces as well as connected spaces and inherit many of their nice properties. Unforunately, as it was shown in [1] and in [2], CLP-compactness is not preserved by (even finite) products. (Note that multiplicativity is one of the most crucial features of compactness and connectedness.) Even worse, there exist three CLP-compact spaces such that the produc of every pair of them is CLP-compact while the product of all three spaces fails to be CLP-compact. In order, to remedy this defect we introduce here a more general concept of a *weakly CLP-compact space*, which has many nice properties of CLP-compact spaces, and b esides is multiplicative.

Given a topological space (X, τ), let CLP stand for the family of all clopen subsets of (X, τ). Further, let τ^{clp} de-

note the topology generated by the family CLP as a base.

Definition 1. A family $\mathcal{U} \subseteq \tau^{CLP}$ closed under finite unions and such that $\bigcup\{U \mid U \in \mathcal{U}\} = X$ is called a saturated-CLP-cover, or sCLP-cover for short, of (X, τ) if for each $U \in \mathcal{U}$ there exists $V \in CLP$ such that $U \subseteq V \neq X$.

It is easy to see that a cover $\mathcal{U} \subseteq \tau^{CLP}$ of X closed under finite unions is an sCLP-cover of X if and only if for each $U \in \mathcal{U}$ there exists $\emptyset \neq W \in CLP$ such that $U \cap W = \emptyset$.

Definition 2. A space (X, τ) is called weakly-CLP-compact, or wCLP-compact for short if every its sCLP-cover has a finite subcover.

Remark. Note that the analogious definition of weak compactness as a generalization of compactness makes no sense since it is equivalent to ordinary compactness.

Main results:

Theorem 1. The product $\prod_{i \in I}(X_i, T_i) = (X, T)$ of non-empty spaces is a weakly-CLP-compact space if and only if every factor (X_i, T_i) is weakly-CLP-compact.

Theorem 2. An image of a weakly-CLP-compact space under a continuous mapping is weakly-CLP-compact.

Theorem 3. If $f : (X, T_X) \to (Y, T_Y)$ is a continuous clopen mapping (that is the image of every clopen set is clopen), and besides the pre-image of every point $y \in Y$ is a weakly CLP-compact subset of (X, T), then (X, T) is weakly CLP-compact whenever (Y, T_Y) is weakly CLP-compact.

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Some remarks on weakly Whyburn spaces

Abstract. A topological space X is called Whyburn if for any non-closed set $P \subseteq X$ and any $x \in \overline{P} \setminus P$ there is a subset $B \subseteq P$ such that $\{x\} = \overline{B} \setminus B$. It is a weakly Whyburn space if for every non-closed subset P there is a point $x \in \overline{P} \setminus P$ and a set $B \subseteq P$ such that $\{x\} = \overline{B} \setminus B$. In this talk, in particular, relations between almost radial spaces and weakly Whyburn spaces are examined.

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On the commutivity of interior and closure operators in topological spaces

Abstract. Levine, in the year 1961, defined a set A in a topological space has the property Q if $A^{-o} = A^{o-}$ and characterized the sets having the property Q where A^{-} and A^{o} respectively denote the closure and interior operators in a topological space. In this paper we characterize regular open sets, regular closed sets, semi-open sets, semi-closed sets, α-open sets, α-closed sets, pre-open sets, pre-closed sets, semi-pre-open sets and semi-pre-closed sets using the property Q. Using this property we investigate the commutivity of the operators namely α-closure, semi-closure, pre-closure, semi-pre-closure, a-interior, semi-interior, pre-interior and semi-pre-interior

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Graphs and surfaces

Abstract. It is well-known that the property of being locally connected simplifies the structure of a metric space considerably. Nevertheless, a complete description of the locally connected, compact metric spaces seems hopeless. However, a complete description becomes possible if we add the condition that the space does not contain an infinite complete graph and if we also strengthen the local connectivity condition to local 2-connectedness, that is, for every element x in the space, and every neighborhood U of x, there exists a neighborhood V of x contained in U such that both V and $V - x$ are connected. Surprisingly, such a space must be locally 2-dimensional, that is, it is contained in a 2-dimensional surface. Some applications will be given.

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On Suslinian Continua

Abstract. A continuum is a compact connected Hausdorff space and a continuum is said to be Suslinian, if it does not contain an uncountable collection of disjoint subcontinua. Recently, various properties of Suslinian continua have been studied in relation to problems concerning generalizations of the Hahn-Mazurkiewicz Theorem, rim-metrizability, perfect normality and homogeneity. In this talk, we will survey these results.

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On an L-valued Category of L-valued Topological Spaces

Abstract. Let X be a set and $L = (L, \wedge, \vee, *)$ be a GL-monoid. Following U.Hohle [2], by a (global) L-valued equality on X we call a mapping $E : X \times X$
toL such that

(i) $E(x, x) = 1 \ \forall x \in X$ – reflexivity ;

(ii) $E(x, y) * E(y, z) \leq E(x, z)$ – transitivity ;

(iii) $E(x, y) = E(y, x)$ –symmetry .

The pair (X, E) is called an L-valued set.

We extend the L-valued equality E on X to an L-valued equality $\mathcal{E}$ on the L-powerset L^X as follows. Given $A, B \in$

L let

$$\mathcal{R}(A,B) = \bigwedge_{x,y} (\mathcal{R}(x,y) * A(x) \longrightarrow B(y)).$$

Proposition 1. $\mathcal{R} : L^X \times L^X \to L$ is a reflexive transitive relation on L^X, that is

(i) $\mathcal{R}(A,A) = 1 \forall A \in L^X$;

(ii) $\mathcal{R}(A,B) * \mathcal{R}(B,C) \leq \mathcal{R}(A,C) \forall A,B,C \in L^X$.

Further, by setting $\mathcal{E}(A,B) = \mathcal{R}(A,B) \wedge \mathcal{R}(B,A)$ we obtain an L-valued equality on L^X.

Proposition 2. Let SET(L) denote the category of L-valued sets and extensional mappings between them (that is such mappings $f : (X, E_X) \to (Y, E_Y)$ that $E_X(x,x') \leq E_Y(f(x), f(x'))$ Then

$$\mathcal{R}_X(f^{-1}(C), f^{-1}(D)) \geq \mathcal{R}_Y(C,D) \ \forall C, D \in L^Y,$$

and hence

$$\mathcal{E}_X(f^{-1}(C), f^{-1}(D)) \geq \mathcal{E}_Y(C,D) \ \forall C, D \in L^Y.$$

Thus, by assigning to an L-valued set (X,E) its L-valued powerset $(L^X, \mathcal{E})$ and to an extensional mapping $f : (X, E_X) \to (Y, E_Y)$ the mapping

$$f^{\leftarrow} : (L^Y, \mathcal{E}_Y) \to (L^X, \mathcal{E}_X)$$

we define a contravariant functor from SET(L) into itself.

Some properties of this functor, in particular the behaviour of initial and final structures with respect to it, will be discussed.

Further, let (X, E, τ) be an L-topological space, that is (X,E) is an L-valued set and τ is an L-topology on X [1], that is a family of its L-subsets, satisfying

(i) $0_X, 1_X \in \tau$;

(ii) $U, V \in \tau \Longrightarrow U \wedge V \in \tau$;

(iii) $U_i \in \tau \quad \forall i \in \mathcal{I} \Longrightarrow \bigvee_{i \in I} U_i \in \tau$.

We define an L-valued (or L-fuzzy) category LF-TOP(L) with L-topological spaces as objects and mappings (not necessarily continuous) as potential morphisms. Namely, given a mapping $f : (X, E_X \tau_X) \to (Y, E_Y, \tau_Y)$ we define its *measure of continuity* by setting

$$\mu(f) = \inf_{V \in \tau_Y} \mathcal{R}_X(f^{-1}(V), int f^{-1}(V)).$$

Proposition 3. If $f : (X, E_X, \tau_X) \to (Y, E_Y, \tau_Y)$ and $g : (Y, E_Y, \tau_Y) \to (Z, E_Z, \tau_Z)$ are mappings then $\mu(g \circ f) \geq \mu(g) * \mu(f)$. Besides, $\mu(id_X) = 1$ for the identical mapping $id_X : (X, E_X, \tau_X) \to (X, E_X, \tau_X)$. Thus LF-TOP(L) indeed is an L-valued category in the sense of [3]

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Unitary representations of free abelian topological groups

Abstract. For every Tikhonov space X the free abelian topological group $A(X)$ and the free locally convex vector space $L(X)$ admit a topologically faithful unitary representation. The case of a compact space X is due to Jorge Galindo.

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Completeness for metrically generated constructs

Abstract. The context we will be working in is the one of $\mathcal{C}$-metrically generated constructs as discussed in the talk by Eva Colebunders. A $\mathcal{C}$-metrically generated construct $\mathbf{X}$ is very suitable for studying categorical completeness in the sense of [1], which consists in considering the class of complete objects in the subconstruct $\mathbf{X}_0$ of T_0-objects, as a firm $\mathcal{U}$-reflective subcategory of $\mathbf{X}_0$, for $\mathcal{U}$ some given class of morphisms.

We will discuss the symmetric situation and assume that $\mathcal{C}$ stands for the class of all metrics, all totally bounded metrics or all ultrametrics.

A first attempt to investigate completeness in $\mathcal{C}$-metrically generated constructs is inspired by the well-known completion theories known for the categories **Unif** and **UAp** [2]. A second approach consists in describing a

functor from $\mathcal{C}$-metrically generated constructs to the category of regular nearness spaces, a category which is known to be very suitable for studying completeness. For each of these views on completeness we develop a firm completion theory and recover several well-known and some new examples.

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Quantified functional analysis: an overview

Abstract. An arbitrary uncountable product of metrizable topological spaces is hardly ever metrizable, and even for a countably infinite product there is no preferred, canonical, metric that overlies the product topology. Approach theory has emerged from the study of suitably axiomatized point-set distances, which generally cannot be retrieved from the metric point-point distance. These approach structures capture exactly that part of the metric information that can be retained in concordance with topological products, thus providing a remedy for the lack of stability under products of the concept of metrizability in topology.

An interesting phenomenon occurs when they are mixed with the algebraic structure of a vector space. It turns out

that any locally convex approach space—an approach vector space that has a local base of convex functions—can be described by an ideal of seminorms. Moreover, the category of locally convex approach spaces contains the category of topological spaces (with linear continuous maps as morphisms) and the category of seminormed spaces (with linear contractions) as full subcategories. Locally convex approach spaces moreover solve again the productivity problem from above, now for seminorms in relation to locally convex topological spaces. Classically, for non-normable topological vector spaces, only an isomorphic theory exists, whereas now, when we start from a well-chosen defining set of seminorms, working on the approach level allows an isometric theory, where canonical numerical concepts exists. In this talk we give an overview of the known results in the relatively new research area of quantified functional analysis.

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An application of Michaelś selection theorems to maps of splittable spaces

Abstract. All topological spaces are assumed to be Tychonoff (completely regular T_1). Let $\mathcal{P}$ be a class of spaces (a topological property). A space X is called *splittable* over $\mathcal{P}$ if for any two disjoint subsets $A, B \subseteq X$ there exists a continuous mapping $f : X \to Z$ onto a space $Z \in \mathcal{P}$ (depending on the pair A, B) such that $f(A) \cap f(B) = \varnothing$. For a space Y, a space X is called splittable over Y if it is splittable over the class $\mathcal{P} = 2^Y$ of all subspaces of the space Y.

The concept of splittability (or cleavability), introduced by Arhangelśkii in mid 1980s, is a natural generalization of one-to-one mappings. It has been studied in detail for various particular cases. A number of works is devoted to splittability over $\mathbb{R}^\lambda$ (for $\lambda = n$ finite, and especially for $\lambda = \omega$ countable — see, in particular, [*J. Soviet Math.*, 1990, **50**(2), 1497–1512]), over LOTS/POTS (linearly/partially ordered topological spaces), etc.

The author proved [*Topology Appl.*, 1995, **61**(2), 101–114; *Moscow Univ. Math. Bull.*, 1992, **47**(5), 14–16] that open perfect mappings preserve splittability over $\mathbb{R}^\lambda$ for infinite λ, over the class of all metrizable spaces (of weight $\leq \lambda$), and over any LOTS, whereas wider classes of continuous mappings such as clopen, or perfect (even two-to-one), or open compact ones, may destroy splittability over $\mathbb{R}^\omega$. However, for the last clause the following holds.

Theorem 1. *Let $f : X \to Y$ be an open pseudocompact mapping of a space X onto a paracompact space Y with $dimY = 0$, and X be splittable over a metric space M. Then Y is also splittable over M.*

The proof is based on Michaelś zero-dimensional selection theorem [*Ann. Math.*, 1956, **64**(3), 562–580] and a general technical statement (M being not necessarily complete). A similar, though not so straightforward, application of Michaelś convex-valued selection theorem (from the same 1956 paper) yields

Theorem 2. *Let a space X be splittable over $\mathbb{R}$ and $f : X \to Y$ be an open monotone* (that is, all point inverses are connected) *pseudocompact mapping onto a paracompact space Y. Then Y is also splittable over $\mathbb{R}$.*

Note that the latter statement fails for splittability over $\mathbb{R}3$ (because the cube $I3$ admits an open monotone mapping onto I^n for any finite $n > 3$) and for non-monotone mappings f (because the circle $S1$ is not splittable over $\mathbb{R}$).

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On invariant skew symmetric tensors on the group ring of finite groups

Abstract. The cohomology groups of a finite group G are defined using G-invariant functions on the group ring $\mathbb{Z}[G]$ of the group G. This definition may be translated on the language of G-invariant tensors and henceforth allows to use the standard tensor theory to study the homology (co-homology) groups. Bearing in mind a special role of skew symmetric tensors in the classical calculus and differential geometry and topology it is natural to look at the skew symmetric part of these considerations separately and, in particular, to define groups of exterior homology (cohomology) of a finite group based on invariant skew symmetric tensors (see my paper *Exterior Homology and Cohomology of finite groups*, Proc. Steklov Institute of Mathematics, Vol. 225, 1999, pp. 190-213). In the cited paper was shown that many geometrical properties of homology (cohomology) of groups are tied just with their skew symmetric counterparts. My report presents further results on the structure of invariant (with respect to the left action) skew symmetric tensors on the group ring $L[G]$ or on the group algebra $L(G)$, where L is usually a subring of the complex number field $\mathbb{C}$.

Methods. 1) The decomposition of the group ring $L[G]$ into two-sided minimal ideals W_r related to irreducible characters χ_r (or central idempotents π_r) gives rise to the associated decomposition of tensors.

2) To describe the structure of invariant skew symmetric tensors we use mostly geometrical arguments given by the hermitian scalar product in $\mathbb{C}[G]$ and realized by a decomposition of the central minimal idempotents into the sum of pair-wise orthogonal idempotents. In particular, that gives in this case an explicit form of Wedderburn–Artin theorem.

3) Invariant 2-forms are, as usual, of special interest. They serve both concrete representatives of some important forms and tools for organization of the further investigations due to Lefschetz type decompositions.

4) Non degenerate skew symmetric integer-valued forms (if they exist) supply us with additional perspectives by invoking methods of symplectic geometry, geometric and algebraic number theory.

Results. 1) Denote by σ, σ' symmetrizations by the action of the group G on polyvectors by the left and by the right. Then $\sigma(1 \wedge g)_{g \in G^+}$ form a basis of invariant bivectors, while the right action is expressed by a relation $\sigma(1 \wedge g)h = \sigma(1 \wedge h^{-1}gh)$.

2) An invariant bivector $\sum_{g \in G} a(g)\sigma(1 \wedge g)$ is biinvariant iff $a(g)$ is a central function which satisfies a relation $a(g^{-1}) = -a(g)$. In particular,

$$\left\{\Omega_r = \tfrac{d_r}{|G|} \textstyle\sum_{g \in G} \chi_r(g)\sigma(1 \wedge g)\right\}; \text{ (where } d_r = \dim_{\mathbb{C}} W_r),$$

is a basis of biinvariant bivectors.

3) The rank of the group of biinvariant bivectors with integer coefficients is equal to one-half of the number of complex characters, hence if all irreducible characters are real there exist no non-trivial biinvariant bivectors (2-forms).

4) The bivector Ω_r is concentrated on a two-side ideal

$W_r \oplus W_r$ and coincides with a bivector

$$\sum_{i,j=1}^{d_r} e^r_{ij} \wedge e^r_{ij},$$

where e^r_{ij} is an orthonormal basis of matrixes constructed starting from a maximal system of pair-wise orthogonal idempotents. It follows that any bivector $\Omega = \sum_r c_r \Omega_r$ is decomposable.

5) The bivector Ω has integral coefficients iff all c_r are imaginary and conjugate under the action of some cyclotomic field.

6) A non-degenerated bivector on the augmentation ideal $J = [\mathbb{Z}[G] \to \mathbb{Z}]$ (i.e. integral symplicial structure) exists iff the order $n = |G|$ of G is odd, $n = 2m + 1$.

7) We call an orientation of G an ordered sequence $g_1, g_2, \ldots, g_n$ of all elements of G, as well as an element $\alpha = g_1 \wedge g_2 \wedge \ldots \wedge g_n$. The inner differentiation d applied to α gives an orientation $\beta = d(\alpha)$ in $\Lambda^{2m} J$, henceforth we may evaluate the volume of the form Ω as a coefficient of Ω^m and obtain the following formula

$$\Omega^m = m! \prod_r c_r^{d2_r} \beta.$$

8) If Ω is a boundary of an invariant 3-vector, $\Omega = d\sigma(b)$, then

$$\begin{aligned} \frac{\Omega^{m-1}}{(m-1)!} \wedge \Omega = m \prod_r c_r^{d_r 2} \beta &= d\Big(\frac{\Omega^{m-1}}{(m-1)!} \wedge \sigma(b)\Big) \\ &= \kappa |G| \beta \end{aligned}$$

for some integer κ. Thus the order $|G|$ of the group G

should divide the number

$$\prod_r c_r^{d_r 2}$$

that implies several interesting conclusions about the structure of finite groups of odd order.

9) In a case of a group of odd order the symplectic structure given on the augmentation ideal J by the invariant integral decomposable bivector Ω provide the Lefschetz decomposition and in this way reduces structural problems to primitive polyvectors.

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Minimal Markov partitions for pseudo-Anosov homeomorphisms

Abstract. It is well known that for Anosov diffeomorphism of 2-torus there exist Markov partitions, consisting of two rectangles which obviously is minimal. The main question considered in the talk is what is the minimum of the number n of rectangles of Markov partitions for generalized (i.e. possibly with 1-prong singularities of contracting and expanding invariant foliations) pseudo-Anosov homeomorphisms f of closed surface M.

The answer depends on properties of invariant foliations. Let b_d be the number of d-prong singularities of invariant foliations for f. In the case of Anosov diffeomorphism we assume that there is unique 2-prong singularity. In other cases we assume that there are no 2-prong singularities. The sequence $\{b_d : d \in \mathbb{N}\}$ is called the *singular*

type of f. Then $n_{\min} = m_{\min} + \frac{1}{2}\sum_d db_d$, where $m_{\min}$ is minimal m such than there exists the lief W of one of invariant foliations such that $f^m(W) = W$. It is easy to see that $m_{\min} \leqslant \min db_d$.

The proof is based on the consideration of some special Markov partitions called *band partitions*. It is proved the existence of such partitions for every generalized pseudo-Anosov homeomorphism and the fact that the minimum of the number of rectangles is realized on partitions in this class. The band Markov partitions are essential by themselves because it is possible using them to apply combinatorial methods developed in [1] to the problem of classification of generalized pseudo-Anosov homeomorphism up to topological conjugacy.

In this connection it is natural to classify Markov partitions with respect to the following relation. Two Markov partitions of generalized pseudo-Anosov homeomorphism f are called equivalent if rectangles of one of them are images of those of another under some iteration of f. It is proved that there exist only finite number of non-equivalent band Markov partitions with fixed number of rectangles.

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Some classes of sequences

Abstract. We consider sequences of positive real numbers which satisfy certain asymptotic conditions (slow variability, rapid variability and so on). Several kinds of convergence related to such sequences are defined and investigated.

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Partially automorphic spaces

Abstract. We work in a category whose objects we call spaces are sets endowed with algebraic and topological structures: topological spaces can be considered at one extreme and Banach spaces at the opposite.

A space will be called automorphic if every isomorphism between two subspaces can be extended to an automorphism of the space. This can be understood as if every subspace could be placed in a unique position.

We will also consider spaces which have a partially automorphic character, which still might have a few different meanings: either that the space behaves in an automorphic way with respect to some prefixed class of its subspaces, or else that it behaves ñearlyäutomorphic. Of course, the notion is more interesting as more complex is the structure, wider is the class of subspaces with respect to which the space behaves automorphically, or tighter is the

ñearlyäutomorphic character. The techniques will range from basic combinatorics to new homological algebra principles.

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Extending automorphisms in spaces of continuous functions

Abstract. We show that there are no automorphic spaces of the form $C(K)$ with K continuous image of Valdivia compact except the spaces $c_0(\Gamma)$. Nevertheless, when K is an Eberlein compact of finite height not isomorphic to $c_0(\Gamma)$, all isomorphism between subspaces of $C(K)$ of size less than $\aleph_\omega$ extend to automorphisms of $C(K)$.

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Category liftings for product spaces

Abstract. A category lifting for a triple $(X, \mathfrak{B}_c(X), \mathcal{M}(X))$, where X is a Baire topological space, $\mathfrak{B}_c(X)$ is the σ-algebra of all subsets of X having the Baire property, $\mathcal{M}(X)$ is the ideal of all first category subsets of X, is a Boolean homomorphism $\rho : \mathfrak{B}_c(X) \longrightarrow \mathfrak{B}_c(X)$ which selects elements modulo $\mathcal{M}(X)$. One problem we discuss is the existence of liftings π for a product Baire space $X \times Y$ compatible with the product structure in the sense that

for given liftings ρ for X and σ for Y, π is a product of ρ and σ and

$$[\pi(E)]_x = \sigma([\pi(E)]_x) \quad \text{for all} \quad E \in \mathfrak{B}_c(X\times Y) \quad \text{and all} \quad x \in X.$$

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Intutionistic fuzzy strongly sets

Abstract. A new class of intuitionistic generalized fuzzy open sets is defined. Intuitionistic fuzzy strong precontinuous mappings and intuitionistic fuzzy strongly preopen (presclosed) mappings between intuitionistic fuzzy topological spaces are introduced. Some of their properties and relationship are studied.

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On the plane fixed point problem

Abstract. A continuum is a compact connected metric space. Does every non-separating plane continuum have the fixed point property? This very accessible and simply stated question has gone unanswered for nearly a century, and it has been the main focus of topologists around the globe. We will present major partial results of attempts towards a solution and the current state of the problem.

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Interior operator in a simple density topology

Abstract. Let $\Phi^1(A)$ be a set of all simple density points of a measurable set $A \subset R$ and let $\Phi^{\alpha+1}(A) = \Phi^1(\Phi^\alpha(A))$ for a countable ordinal α and $\Phi^\alpha(A) = \cap_{\beta<\alpha}\Phi^\beta(A)$ if α is a limit countable ordinal. It is proved that if $B \subset R$ is an arbitrary set (not necessarily measurable), then there exists $\beta <$
$omega_1$ such that $Int\,(B) = B \cap \Phi^\beta(A)$, where A is a measurable kernel of B.

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Continuous selections on spaces of continuous functions

Abstract. For a space Z, we denote by $\mathcal{F}(Z)$, $\mathcal{K}(Z)$ and $\mathcal{F}_2(Z)$ the hyperspaces of non-empty closed, compact, and subsets of cardinality ≤ 2 of Z, respectively, with their Vietoris topology. For spaces X and E, $C_p(X,E)$ is the space of continuous functions from X to E with its pointwise convergence topology.

We analyze in this article when $\mathcal{F}(Z)$, $\mathcal{K}(Z)$ and $\mathcal{F}_2(Z)$ have continuous selections for a space Z of the form $C_p(X,E)$ where X and E are zero-dimensional and E is completely metrizable. We prove that if X is separable, then $\mathcal{K}(C_p(X,E))$ has a continuous selection, and that $C_p(X,E)$ is weakly orderable if and only if X is separable. Moreover, we obtain that the separability of X, the existence of a continuous selection for $\mathcal{K}(C_p(X,E))$, the existence of a continuous

selection for $\mathcal{F}_2(C_p(X,E))$ and the weak orderability of $C_p(X,E)$, are equivalent when X is N-compact.

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Some relative versions of star selection principles

Abstract. We introduced some relative version of star selection principles first considered in [2], [3]. Some of the work estends results from [1] and gives some examples.

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Quasicontinuous Functions and Domains

Abstract. A subset of a topological space is said to be quasiopen if has a dense interior and a function is quasicontinuous if the inverse image of every open set is quasiopen. We survey a number of recent results that have arisen in the theory of quasicontinuous functions involving dense points

of continuity and sections of closed relations and USCO maps. We then restrict our attention to the case that the codomain is an n-fold product of the extended reals (or more generally a bicontinuous lattice) in order to bring to bear the tools of domain theory in our investigation, in particular to introduce an interesting and useful function space topology on the quasicontinuous function space. We apply this machinery to define an extended differential calculus in the quasicontinuous function space.

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The existace of continuous, cancellative, semigroup operations on R with the density topology