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A SIMULATION OF SELF-ORGANIZED PLASTIC ACTORS IN AN ELASTIC NETWORK

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Abstract: Here we intend to re-examine Axelrod's 'adaptive culture model,' i.e., a simulation of the dissemination of culture through social interaction proceeding by local convergence and resulting an emergence of global polarization at some degree (Axelrod, 1997). Our purpose is twofold: (i) to increase the produced relatively low heterogeneity in Axelrod's model and (ii) to explore how the social network might change when actors are permitted to modify their links through their interactions. Thus, by increasing the range of actors' plasticity and allowing them to perform both convergent and divergent interactions, the level of attained global diversity also increases. Moreover, by allowing actors to follow their intrinsic biases in linking with others, implied from their predispositions to convergent or divergent interactions, the elastic dynamic network turns out to be self-organizing. Finally, diffusion patterns produced by the rigidity or persistence of certain actors on invariant interactions are examined too.

Introduction

That individual actors and the network, in which they are embedded, constitute a co-evolving system is a commonplace today: Network ties are affecting actors' behavior, by providing structural niches to their interactions, and actors are shaping the web of their connections, by actively constructing their own social worlds through their interactional contacts and choices (e.g., Lazer, 2001). In this paper, we intend to manifest this co-evolution of actors and network (or agency and structure) through a simulation, which is self-organizing both actors' attributes and network positions in certain stabilized (equilibrium) patterns, exhibiting various degrees of diversity of attributes and network structuration.

In order to give a general description of this simulation, let us first assume that actors are embedded in a certain known social network and that each actor possesses some known attributes. In fact, these assumptions would be produced by certain sociometric and attitudinal data from which the actors' network and attributes could be derived. However, as it usually happens in reality, data change in time: attitudinal data are commonly longitudinal and time dependent sociometric data result dynamic networks. Therefore, a meaningful simulation of the actors-network interplay has to permit actors' attributes and network to be updated iteratively (from step to step). Indeed, our simulation does so by defining two sets of rules: (i) rules of how the network makes actors change their attributes and (ii) rules of how the field of attributes over actors changes the network. In David Lazer's (2001, p. 70) terminology, these two sets of rules are just setting up the following two mechanisms of co-evolving actors-network systems: (i) the network-dependent *plasticity* of actors and (ii) the actors-dependent *elasticity* of the network.

Although details will be given later, let us now sketch broadly how these rules are understood in our simulation. First, we are assuming that any actor possesses an inherent predisposition towards either convergent or divergent (local) interactions. Then, on the one side, actors' plasticity (i.e., the ways actors' attributes might change because of network contacts) is molded by local interactions between adjacent actors; these local interactions might result that the interacting actors' attributes become either more 'similar' or more 'dissimilar,' depending on actors' predispositions. On the other side, network elasticity (i.e., the ways network links might change according to whether they are enabled or disabled by actors' attributes) is produced by actors' tendency to cluster with other actors, who are possessing the most possible 'similar' or the most possible 'dissimilar' attributes with them, again depending on actors' predispositions towards convergent or divergent interactions.

It is clear even at this general level of description that our simulation constitutes a generalization of Axelrod's (1997) 'adaptive culture model,' which is a model of the dissemination of culture through social interaction proceeding by local convergence and resulting an emergence of global polarization at a limited degree. In fact, Robert Axelrod was dealing with the dissemination of culture over a very simple form of a social network: a rectangular grid (regular lattice) the nodes (or cells) of which were supposed to be the interacting actors. In this model, each actor possessed a list of numerical attributes, which Axelrod was interpreting as the actor's cultural characteristics. What Axelrod's simulation was resulting was that over the long run the assumed processes of local convergent interactions were globally homogenizing culture: almost all actors were adopting the same culture or in the best case only a very small number of cultures were surviving throughout the simulation. In other words, the outcome of Axelrod's simulation was either a complete cultural homogenization or a fragmentation into a small number of heterogeneous cultural zones.

As we are going to see later with more details, the present simulation constitutes a generalization of Axelrod's model along the following directions: (i) actors' geometry becomes an arbitrary network (a nondirected graph), (ii) actors possess inherent predispositions towards either convergent or divergent (local) interactions and (iii) the network is dynamic and resilient: network links may change before the simulation ends up.

Therefore, our simulation is self-organizing both actors and network, which are, thus, co-evolving as the simulation runs. By this we mean that, starting initially from an arbitrary or random field of attributes and from any given initial network, after running the simulation for a long number of iterations, equilibrium between actors' plasticity and network elasticity is attained. In other words, at this emergent 'plastic-elastic equilibrium,' both actors' attributes and network configuration are eventually stabilized into certain invariant patterns. We call these emergent patterns 'field of equilibrium attributes' and 'equilibrium network.' Notice that this simulation is not deterministic but stochastic (its rules are probabilistic). Furthermore, the emergent patterns strongly depend on the history of the 'moves' performed on previous iterations before reaching equilibrium. Said in another way, this simulation exhibits the property of 'path dependence' or 'nonergodicity' (Arthur, 1989).

Consequently, it is only stochastically that we may investigate the equilibrium patterns of such a simulation. But there are a number of interesting topics and questions on the properties of the emergent equilibrium patterns that we intend to tackle in the analysis of our simulation. For instance, in what concerns the field of equilibrium attributes, it is very interesting to know the degree of diversity (or the entropy) of these patterns. Of course, the case of all actors turning out to have the same attributes (homogeneity) is a possible outcome that, as we are going to discuss later, in certain settings is less welcomed than the case of actors being stabilized with a high degree of diversity in their attributes (heterogeneity). Furthermore, in what concerns the equilibrium network configuration, we are interested in its structural forms: Does the equilibrium network break into disconnected components (subnetworks)? How are actors and their attributes distributed over these components? Finally, if we assume that certain actors are rigid or permanently persistent in their initial attributes and, thus, these actors are not influenced by the simulation, the questions to investigate would be: Are the attributes of rigid actors diffusing over the network? Do they dominate over the field of equilibrium attributes? Does such diffusion depend on the structural (network) positions of the rigid actors? These are some of the topics and questions that we will try to discuss and answer by experimentation through our simulation in the sequel.

Actors' Attributes

As we have said, actors possess attributes; in here, the attributes of an actor will be considered to constitute a vector of m components – called 'features' – such that each feature may take its values – called 'traits' – on the set of nonnegative integers $\{0, 1, \ldots, q\}$. Typically, in most of our simulation experiments, $m = 5$ and $q = 9$, i.e., attributes will be composed of 5 features and each feature's traits will be chosen among the 10 numbers from 0 to 9.

In Axelrod's (1997) model, actors' attributes are interpreted as cultural characteristics the number of which is the same and equal to m for each actor and each characteristic being quantified to take a numerical value (a trait) from 0 to q. However, the cultural interpretation of attributes is not the only one: According to Johnson and Huckfeldt (2001) actors' attributes might be interpreted as opinions, issue stances, political allegiances or any other judgments actors may have on the political sphere through which they might orient their voting choices or their political participation in one way or another.

However, as it has been already mentioned, actors' attributes do change throughout this type of simulations. In fact, actors' attributes might change whenever actors interact. Therefore, it is important to understand what sort of interactions these simulations allow before going to examine in details what possible changes in attributes are implied by these interactions. In Axelrod's (1997) adaptive culture model, actors' interactions were taken to be local and convergent. Being local means that an interaction is performed only between connected (or adjacent) actors. Being convergent means that an interaction would tend to produce more similarity between the attributes of the interacting actors.

But, if local convergence were the only mechanism of interactions, then even at this level of generality it would not be too strange to expect that the outcome of such a simulation would be an almost total global homogeneity in the field of the attributes over the stabilized equilibrium patterns. Of course, selecting certain appropriate attributes on connected actors and even considering certain peculiar graph topologies might produce a certain degree of heterogeneity in the field of the equilibrium attributes. However, the fact remains that local convergence generically tends towards low global diversity at equilibrium (always in competition with the structure of communication channels provided by the social network).

Johnson and Huckfeldt (2001) have argued that this type of simulations should break away from the above 'homogenization predictions' partly because of empirical reasons and partly because of normative reasons. On the one side, they claim that recent empirical studies have shown that high diversity and considerable disagreement can be sustained by interpersonal networks (Huckfeldt, Johnson & Sprague, 2002). On the normative side, Johnson and Huckfeldt (2001) have argued that "this model has the implication that interaction does erase all differences" and, thus, "one might argue in favor of segregation, or cultural apartheid, as the only way to preserve diversity on the aggregate social level," which of course is a rather discouraging contention.

For these reasons, in our model, we are going to assume that each actor possesses an inherent predisposition towards either convergent or divergent local interactions with other actors (depending on the latter's predispositions). Then, as we are going to see in the sequel when we perform experiments of our simulation, by putting just a little room for divergent interactions the levels of diversity in the equilibrium patterns arise significantly and, thus, the model becomes more realistic. This is why, in our simulation, we are going to consider that any actor can be either 'homophilic' or 'heterophilic':

- A *homophilic* actor is one determined to sustain local convergent (or amicable) interactions with other actors (but in different ways depending on whether the other actor is homophilic or heterophilic). In other words, a homophilic actor a priori tends to agreement or adoption of the otherness, something which might produce more similarities among the actors' attributes.
- A *heterophilic* actor is one determined to sustain local divergent (or contentious) interactions with other actors (again in different ways depending on whether the

other actor is homophilic or heterophilic). In other words, a heterophilic actor is inclined a priori towards disagreement or rejection of the otherness, something which might proliferate dissimilarities among the actors' attributes.

Sometimes in what follows, when dealing with attributes of homophilic or heterophilic actors, we will call them *signed attributes* and we will put in front of them a sign + or – in order to indicate the actors' predispositions.

Rules of Interaction of Plastic Actors

Now, let's define what are the outcomes of interactions among homophilic and heterophilic actors. They have been both named 'plastic,' because their attributes are changing by (local) interactions as they are prescribed by the network (local) topology.

First, we have to say that we are following Axelrod's (1997) 'protocol of interactions,' by which we mean the following: When actor *i* interacts with actor *j*, then actor *j* influences actor *i*. In other words, when actor *i* interacts with actor *j*, it is *i*'s attributes that change, not *j*'s! How this happens depends, in our simulation, on the homophilia or heterophilia of actors:

- If both actors *i* and *j* are homophilic, then select randomly a feature on which *i* and *j* have different traits and with probability equal to the proportion of similar features (i.e., features for which their traits are equal in the two actors) change the trait on this feature of *i* to take the value of the trait of the corresponding feature on *j* (this is exactly Axelrod's rule of convergent local interactions).
- If actor *i* is homophilic but actor *j* is heterophilic, then select randomly a feature on which *i* and *j* have different traits and with probability equal to the half of the proportion of similar features change the trait on this feature of *i* to take the value of the trait of the corresponding feature on *j*.
- If both actors *i* and *j* are heterophilic, then select randomly a feature on which *i* and *j* have the same trait and with probability equal to the proportion of dissimilar features (i.e., features for which their traits are different in the two actors) change the trait on this feature of *i* to take randomly any value which is different from the common value of *i* and *j*.
- If actor *i* is heterophilic but actor *j* is homophilic, then select randomly a feature on which *i* and *j* have the same trait and with probability equal to the half of the proportion of dissimilar features change the trait on this feature of *i* to take randomly any value which is different from the common value of *i* and *j*.

Rules of Rewiring Links in an Elastic Network

As we have already said in the introduction, in our simulation, the underlying social network (over which actors are positioned and, thus, the structural niches of their interactions are prescribed) is assumed to be 'elastic' in the sense that it might change according to the field of attributes on network nodes (actors' loci). Since networks are commonly represented by graphs, the way to see how such a network elasticity is effected is by allowing actors to 'rewire' their links: We say that an actor *rewires* one of its links if an existing link of this actor with one adjacent actor is disabled (deleted) and a new link with another actor (nonadjacent till then) is enabled (added).

In our simulation, the updating of links should satisfy the following ‘protocol of rewiring’: A homophilic actor might rewire only a link with an adjacent actor having at least one of its features dissimilar with the former actor; similarly, a heterophilic actor might rewire only a link with an adjacent actor having at least one of its features similar with the former actor. In particular, no rewiring of links can occur in the following cases: a homophilic actor surrounded only by actors with completely similar features and a heterophilic actor surrounded only by actors with completely dissimilar features.

Having said when rewiring of links occurs, it remains to add how rewiring is implemented, i.e., to specify the rules for the rewiring of links in our simulation. These are the following:

- Let i be a homophilic actor (not surrounded only by actors with completely similar features with it) and let j be an adjacent (to i) actor such that j has the larger number of features dissimilar with the corresponding features of i (j is randomly chosen in case there exist more than one such actors). Moreover, let k be a nonadjacent (to i) actor such that k has fewer features dissimilar with i than j and there exists no other nonadjacent (to i) having fewer features dissimilar with i (k is randomly chosen in case there exist more than one such actors). Then the link (i,j) is replaced by the link (i,k).
- Let i be a heterophilic actor (not surrounded only by actors with completely dissimilar features with it) and let j be an adjacent (to i) actor such that j has the smaller number of features dissimilar with the corresponding features of i (j is randomly chosen in case there exist more than one such actors). Moreover, let k be a nonadjacent (to i) actor such that k has more features dissimilar with i than j and there exists no other nonadjacent (to i) having more features dissimilar with i (k is randomly chosen in case there exist more than one such actors). Then the link (i,j) is replaced by the link (i,k).

What the above rules of rewiring of links say is that actors might distort the underlying elastic network (by changing links), as they pursue to cluster with more similar or dissimilar actors (depending on their inherent predispositions): The bias of homophilic actors is to cluster with other actors which are more similar to them by abandoning links with actors which are not so similar to them; similarly, heterophilic actors tend to cluster with other more dissimilar actors than the ones they are already connected.

Thus, as the simulation runs (to be described formally in next section), on the one side, the network is shaping the field of attributes over actors and, on the other side, the field of actors’ attributes is rewiring the network (and so on). Typically, cases of such cyclical feedback mechanisms are self-organized and they are self-organizing emergent equilibrium patterns as our simulation experiments show.

The Formal Simulation

We assume that the actors’ network is represented by a nondirected graph G of N nodes, on which the actors $i = 1, \dots, N$ are positioned.

Let a_i denote the attributes of actor i. For any actor $i = 1, \ldots, N$, a_i is a vector in Q^m, where $Q = \{0, 1, \ldots, q\}$ and q, m are nonnegative integers. Thus, $a_i = (a_{1i}, a_{2i}, \ldots, a_{mi})$ and the values (called 'traits') that the m components (called 'features') of actor i, a_{1i}, $a_{2i}, \ldots, a_{mi}$, take are numbers from 0 to q.

The set of attributes Q^m becomes a metric space, when equipped with the following distance between two attributes a_i and a_j:

$$d(a_i, a_j) = \text{number of features with different traits.}$$

For instance, all the corresponding features of the attributes a_i and a_j have the same traits (i.e., the attributes are the same) if and only if $d(a_i, a_j) = 0$; they differ in r features ($0 < r < m$) if and only if $d(a_i, a_j) = r$; they are completely different if and only if $d(a_i, a_j) = m$.

Our simulation starts with:

- A graph G of N nodes over which the actors are positioned.
- A randomly (or arbitrarily) chosen field of signed attributes a_i of the actors $i = 1, \ldots, N$.
- A proportion n_+ (0 § n_+ § 1) of homophilic actors and a proportion $n_- = 1 - n_+$ of heterophilic actors.

<u>Step 1</u>: At random, pick an actor i and then pick an actor j adjacent to i such that $0 < d(a_i, a_j) < m$. If there exists no such adjacent actor, then go to Step 2. Otherwise, compare a_i with a_j and then modify a_i as follows:

- If both actors i and j are homophilic, then select randomly a feature such that $a_{pi} \neq a_{pj}$ and, with probability equal to $1 - d(a_i, a_j)/m$, make a_{pi} equal to a_{pj}.
- If actor i is homophilic but actor j is heterophilic, then select randomly a feature such that $a_{pi} \neq a_{pj}$ and, with probability equal to $[1 - d(a_i, a_j)/m]/2$, make a_{pi} equal to a_{pj}.
- If both actors i and j are heterophilic, then select randomly a feature such that $a_{pi} = a_{pj}$ and, with probability equal to $d(a_i, a_j)$, make a_{pi} to take randomly any value different from a_{pj}.
- If actor i is heterophilic but actor j is homophilic, then select randomly a feature such that $a_{pi} = a_{pj}$ and, with probability equal to $d(a_i, a_j)/2$, make a_{pi} to take randomly any value different from a_{pj}.

<u>Step 2</u>: At random, pick an actor i and then examine whether all actors j adjacent to i have $d(a_i, a_j) > 0$, if i is homophilic, or $d(a_i, a_j) < m$, if i is heterophilic. If there exist no such adjacent actors, then go to Step 1. Otherwise, rewire a link of i as follows:

- Let i be a homophilic actor and let j be adjacent to i such that $0 \neq d(a_i, a_j) = \max\{d(a_i, a_r)$: r adjacent to $i\}$ (j is randomly chosen in case there exist more than one such actors). Moreover, let k be nonadjacent to i such that $d(a_i, a_j) > d(a_i, a_k) = \min\{d(a_i, a_p)$: p nonadjacent to $i\}$ (k is randomly chosen in case there exist more than one such actors). Then the link (i,j) is replaced by the link (i,k). If there exists no such actor k, then go to Step 1.

- Let i be a heterophilic actor and let j be adjacent to i such that $m > d(a_i, a_j) = \min\{d(a_i, a_r): r \text{ adjacent to } i\}$ (j is randomly chosen in case there exist more than one such actors). Moreover, let k be nonadjacent to i such that $d(a_i, a_j) < d(a_i, a_k) = \max\{d(a_i, a_p): p \text{ nonadjacent to } i\}$ (k is randomly chosen in case there exist more than one such actors). Then the link (i,j) is replaced by the link (i,k). If there exists no such actor k, then go to Step 1.

Apparently, the simulation ends whenever there are no actors that may change either their attributes or their links. Thus, we say that an actor i is *invariant* under the simulation (or actor i is '*at equilibrium*') either when $d(a_i, a_j) = 0$ or m, for all actors j adjacent to i, or when i is isolated in the network. Clearly, when all actors are or become invariant, then the following invariant *equilibrium patterns* are emerging: a *field of equilibrium attributes* a_{ei} of the invariant N actors and an *equilibrium network* (graph) G_e over which the N actors are stabilized. In the following sections, we are going to examine some aspects of certain characteristics of these equilibrium patterns.

Diversity of Plastic Actors' Equilibrium Attributes

The first characteristic of the equilibrium patterns that we are going to examine is the *diversity* of the field of equilibrium actors' attributes. To measure this property, we will use the following *diversity index* D:

$$D = D_+ + D_-,$$

D_+ = number of clusters of homophilic actors with completely similar attributes,

D_- = number of clusters of heterophilic actors with completely dissimilar attributes.

Above, by 'cluster' we mean a connected component of the network (graph) including at least one node (actor); trivially, any isolated (and, thus, invariant) actor constitutes a cluster. Another remark that we should make is that apparently this diversity index depends on the number of actors N; thus, comparisons of diversity through the above index make sense only if they refer to the same number of actors.

Obviously, $D \geq 1$ and $D = 1$ either whenever all actors are homophilic, nonisolated and possess the same attributes or whenever all actors are heterophilic, nonisolated and possess completely different attributes. Moreover, $D \leq N$ and $D = N$ whenever all actors are isolated (absolute heterogeneity).

If there exist no heterophilic (nor isolated) actors, then, apparently, $D = 1$ can be interpreted as absolute homogeneity. Of course, if there exist both homophilic and heterophilic actors, then $D \geq 2$ and the lowest D is, the more homogeneous the field of actors' attributes could be characterized. On the other side, in any case, as D increases, the field of actors' attributes turns out to be more heterogeneous. This is why D is considered here as an index of diversity.

In this section, for the sake of comparison with Axelrod's (1997) computations, let us consider in our simulation just the case of plastic actors, omitting network elasticity. Indeed, under these conditions, our simulation (restricted to Step 2) coincides with Axelrod's simulation if there were no heterophilic actors. Again for the sake of comparison with Axelrod, let us consider network (graph) topologies of one-dimensional lattices of N nodes (actors) with degree 4, which are topologically equivalent to rectangular grids of N_1 x $N_2 = N$ actors with periodic boundaries (that Axelrod has been considering). Then, our aim is to add just a few heterophilic actors and to compare the resulting diversity index D with the corresponding index without any heterophilia (that index was called 'number of stable regions' by Axelrod but here it is still denoted by D here).

In general, Axelrod in his simulation (1997) has found that the average number of stable regions takes relatively low values, almost always equal to 1 (unless $m = 5$ and $q = 9$ or 14, when D increases somehow – see Table 7-2, p. 160, in Axelrod [1997]). To see what happens in our simulation under similar conditions, let us perform the following experiment:

Experiment 1 (Non-elastic regular graph, $N = 100$, $n_- = 0.02$): Consider the network (graph) topology of an one-dimensional lattice of 100 nodes (actors) with degree 4 (which is topologically equivalent to the rectangular grid of 10 x 10 actors with periodic boundaries) and let us take $m = 5$ and $q = 9$ (i.e., 5 features and 10 traits). In these settings and, of course, assuming that there exist no heterophilic actors, after running his simulation 10 times, Axelrod has found an average $D = 3.2$ (Table 7-2, p. 160, in Axelrod [1997]). However, if we suppose that 2 out of the 100 actors are heterophilic and run our simulation (only Step 1) again for 10 times, we find an average $D = 18.6$, which is significantly higher than Axelrod's number of stable regions.

Thus, we observe that in our simulation (even when network elasticity is included too, as we are going to see in next section) just adding a small proportion of heterophilic actors is dramatically increasing the diversity of the field of equilibrium attributes. This means (recalling our previous interpretation of heterophilia) that dropping just a tiny touch of divergent interactions in a population of actors who are predisposed to convergent interactions stabilizes the final level of diversity at a considerably higher value than the one which would be produced by the absence of any 'spirit of disagreement' inside the population of interacting actors.

Let us give a second experiment again without any network elasticity effects (they will be considered in next section). As we want to visualize the initial (random) and the emergent equilibrium patterns, in order to avoid the complexities of large networks, we are going to consider a regular lattice of rather small number of nodes.

Experiment 2 (Non-elastic regular graph, $N = 25$, $n_- = 0.04$): Let us consider an one-dimensional lattice of 25 nodes (actors) with degree 4 (which is topologically equivalent to a rectangular grid of 5 x 5 actors with periodic boundaries) and take $m = 5$ and $q = 14$ (i.e., 5 features and 15 traits). If we assume that 1 actor is heterophilic and 24 are homophilic, after 10 runs we find an average $D = 11.7$, which is again larger than Axelrod's corresponding average number of stable regions $D =$ (approximately) 7 (see Figure 7-2, p. 162, in Axelrod [1997]). Figures 1 and 2 show the initial and the equilibrium patterns of one of these experiments. Notice that in this experiment the equilibrium pattern is composed of ten clusters ($D = 10$).

Figure 1 about here
Figure 2 about here

Next, let us consider the case that the initial social network is a more 'realistic' network than the regular lattices we have been considering in the above experiments. In fact, from modern theories of social networks and complexity studies, we know that social aggregates – at least above a certain size which makes them comparable to large social groups, communities or even society itself – are composing networks of the so-called 'small-worlds' (Milgram, 1967; Watts, 1999). These networks are rather highly clustered but also they possess many shortcuts among their nodes, which place them somehow in the middle of the hierarchy between regular lattices and random graphs (Watts & Strogatz, 1998; Watts, 1999). In our experiments, the small-world networks were produced by the Strogatz-Watts algorithm of links rewiring (Watts, 1999, p. 67), which depends on certain randomness probability.

Experiment 3 (Non-elastic small-world graph, N = 25, n_- = 0.04): In order to compare with Experiment 2, we consider again 25 actors, 1 heterophilic and 24 homophilic (m = 5 and q = 14), linked in a small-world network produced by a randomness probability equal to 0.8. After 10 runs, with no network elasticity effects, we have found at equilibrium an average D = 10.1, which at the level of the corresponding diversity for a regular graph (D = 11.7) but still higher than Axelrod's (about 7). Figures 3 and 4 show the initial and the equilibrium patterns of one of these experiments, in which ten clusters (D = 10) are emerging.

Figure 3 about here
Figure 4 about here

Experiment 4 (Non-elastic random graph, N = 25, n_- = 0.04): Now the 25 actors (among which 1 is heterophilic) are linked in a random network produced by a randomness probability equal to 1 (m = 5 and q = 14). After 10 runs, with no network elasticity effects, we have found at equilibrium an average D = 8.6. The reason that diversity drops is that in a random graph the complete randomness of the patterns of links is pushing towards more homogeneity than in the case of the well-ordered regular graph or the semi-structured small-world graph.

Structuration of Equilibrium Elastic Networks

Now, we are going to examine our full simulation of the self-organization of plastic actors on an elastic network. We are interested in examining questions like the following:

- How is the diversity index related with the corresponding values it takes in the case of no network elasticity effects (for the same number of heterophilic actors)? Moreover, how is it related with Axelrod's case (no heterophilia, no network elasticity)?
- As actors rewire their links, is the network coherence preserved or does the network fragment in components (clusters), which are not connected with each other?
- In what patterns do heterophilic actors cluster in equilibrium? Of course, by the rules of this simulation, the only way that two heterophilic agents might be linked is when they possess completely dissimilar attributes. But, in the same way, heterophilic actors might link with homophilic ones.

- Do isolated clusters or single actors emerge? Are they heterophilic, homophilic or both? Here, by 'isolated actor' we mean one without links (degree 0) and by 'single actor' we mean one with one link (degree 1).

Experiment 1 (Elastic regular graph, N = 100, n_- = 0.02): This is the network of the above Experiment 1 but under consideration of network elasticity effects (Step 1 and Step 2 in the algorithm of the simulation). After running the simulation 10 times, we have obtained average D = 25.3, which is higher than the diversity index without network elasticity (D = 18.6) and much higher than Axelrod's number of stable regions without any heterophilia (D = 3.2). In our 10 runs, we have observed the following clustering patterns: (i) all clusters are always isolated to (not linked with) each other; (ii) the two heterophilic actors are either both isolated (observed 7 times) or (iii) they are linked to each other and constitute a cluster (observed 3 times).

Experiment 2 (Elastic regular graph, N = 25, n_- = 0.04): Let us consider again the network of the above Experiment 2 but incorporating network elasticity effects this time. In 10 runs, we have found an average D = 6.3, which is lower than the diversity index without network elasticity (D = 11.7) but almost the same with Axelrod's number of stable regions without any heterophilia (about 7). In these 10 runs, we have observed the following clustering patterns: (i) all homophilic clusters are almost always isolated to (not linked with) each other (except in (iii) where a homophilic cluster was connected to the heterophilic actor); (ii) the heterophilic actor is usually isolated (observed 9 times) or (iii) it is linked with a homophilic actor from another cluster (observed 1 time). The latter exceptional case is visualized in Figures 5 and 6. Notice that in this experiments the equilibrium pattern is composed of six clusters (D = 6).

Figure 5 about here
Figure 6 about here

Experiment 3 (Elastic small-world graph, N = 25, n_- = 0.04): This is the small-world network of Experiment 3 together with network elasticity effects. After 10 runs, we have found at equilibrium an average D = 7.3, which is lower than the corresponding diversity without network elasticity (D = 10.1) but slightly higher than the corresponding elastic regular graph (D = 6.3).

Experiment 4 (Elastic random graph, N = 25, n_- = 0.04): If we add network elasticity effects in the random graph of Experiment 4, then, after 10 runs, we find at equilibrium an average D = 6.6, which is still lower than the corresponding diversity without network elasticity (D = 8.6) but again slightly higher than the corresponding elastic regular graph (D = 6.3).

Thus, comparing the small-world or a random graph diversity with the corresponding diversity of a regular graph (with the same number of actors), we observe that the former is (case a) slightly decreasing when no network elasticity effects are considered and is (case b) slightly increasing when network elasticity effects are taken into consideration (in both cases in relation with the latter). To interpret these changes, we need to estimate the counter-balance between network small-worldness or randomness and network elasticity. On the one side, when the underlying network is either a small-world or a random graph, the existing short-cuts of links between actors, which would possess otherwise high geodesic distances between them, are pushing towards more homogeneity than in the case of a regular lattice, for which the average distances between its nodes would have been considerably lower. On the other side, network elasticity appears to be a competing mechanism for the enhancement of diversity because by rewiring links it enables actors to cluster more easily with other similar or dissimilar actors than what the existing network niches might allow them to do. If both network small-worldness or randomness and network elasticity are in effect, then it appears that network elasticity prevails and, thus, diversity tends to increase (case a). If network elasticity is absent, then apparently the homogenization effect of small-worlds or random networks remains without any counteraction and, so, it makes diversity decrease (case b).

There is another very important finding from all the above experiments of network elasticity that we need to reflect upon. This is the fact that in all such experiments the equilibrium networks were found to be 'cluster-wise' disconnected, in the sense that they were all composed of fragmented clusters (which were not linked with each other). Of course, this was due to that at every iteration of the simulation the Step 2 of links rewiring was applied. However, if this step of links rewiring is applied 'cumulatively,' not at every iteration but after a (constant) number of iterations that is regularly repeated until the simulation terminates (reaches equilibrium), then such a fragmentation might be reduced at a considerable degree or even disappear. This is what we have seen in all the above experiments but for the sake of brevity we are going to describe it only for the first experiment:

Experiment 1 (Cumulatively elastic regular graph, $N = 100$, $n_- = 0.02$): This is the network of Experiment 1 with network elasticity effects computed every 500 iterations. After running the simulation 10 times, we have obtained an average D = 7.3, which is lower than the diversity index without network elasticity (D = 18.6) and much lower than the corresponding index with network elasticity computed at each iteration (D = 25.3) but still higher than Axelrod's number of stable regions without any heterophilia (D = 3.2). In our 10 runs, we have observed the following clustering patterns: (i) either all clusters were linked to each other (observed 7 times) or an isolated homophilic actor was emerging (observed once) or an isolated heterophilic actor was emerging (observed twice); (ii) the two heterophilic actors were either one isolated and the other single (observed twice) or both single (observed 4 times) or one single and the other linked more than once with homophilic actors (observed 3 times) or both were linked more than once with homophilic actors (observed once). However, in this experiment, we have never observed the two heterophilic actors to be linked to each other forming a cluster of their own.

Diffusion of Rigid Actors' Attributes

Let us now assume that certain actors are 'rigid' or permanently 'persistent' in their attributes: they do not change their attributes whenever they are picked by the simulation to do so. Of course, another actor, which might interact, with a rigid actor could possibly adapt its attributes in relation with the rigid actor's attributes. Therefore, we might expect rigid actors' attributes to diffuse over the social network. Undoubtedly, such diffusion would depend on the structural opportunities for interactions with rigid actors (i.e., the network positions of rigid actors) and on the field of initial attributes (i.e., the distances of non-rigid actors' attributes from the rigid actors' attributes). To simplify our investigation of such a possible diffusion process, let us assume here that the network is non-elastic, i.e., we are not considering any network elasticity effects and we are just restricted by actors' plasticity (only Step 1 of the simulation).

Furthermore, without any loss of generality, let us assume that rigid actors are always homophilic. Then, as expected, any heterophilic actor linked with a rigid actor will be constantly resisting to the adoption of the attributes of the rigid actor and only if an heterophilic actor is not directly linked with a rigid one might possibly give in to its attributes. But all these would depend on the network positions of rigid (homophilic) actors and the heterophilic ones as well as on the distribution of attributes over the actors of the network.

Thus, what we have found in our experiments under these settings are (details will appear in another paper):

- The more central (either in degree or betweenness) a rigid actor is and the more distant it is from heterophilic actors, then the more easily its attributes diffuse over the network.
- Resistant heterophilic actors play the role of 'gate-keepers' in trying to impede the adoption of the attributes of rigid actors into certain parts of the network.

Conclusions and Further Directions

In Axelrod's adaptive culture model, it is assumed that culture disseminates through convergent local interactions, which eventually produce a global cultural landscape. However, when convergent interactions are formalized as mimetic modifications of actors' attributes, it is not surprising that the diversity levels in such an emerging global landscape tend to be rather low. In fact, the theory of dynamical systems in mathematics suggests that contractive modes of dynamical transformations are always attracted by simple equilibrium states of low complexity. But, in this formal (mathematical) sense, a route to produce chaos and complexity would be through a 'hyperbolic' mixing of both contractive and expansive modes of interaction. This suggestion seems to work out in our simulation too: By increasing the range of possible plastic deformations through the mixing of consensual with oppositional actors, even if the latter are very few, the emergent equilibrium patterns are much more complex than without the presence of the latter. Of course, to explain or interpret what role the effects of contentious behavior play in cultural dynamics, political processes, social communication or collective action is beyond the scope of this paper. The claim we make here that an infinitesimal disturbance to local norms suffices to produce higher global diversity of the equilibrium outcomes of an agent-based simulation (of networked actors' attributes) cannot be considered as bearing any normative implications. It is simply a theoretical observation of the immanent structural instability of such a stochastic and nonergodic simulation, which was manifested in the computer experiments we have performed.

However, in our model, broadening the range of actors' plastic deformations in order to encompass divergent interactions too was not the only mechanism enhancing the equilibrium global diversity patterns. We have been also considering the effects of network elasticity resulting from a possible rewiring of their links that actors might make in order to cluster together with other actors, either more similar or more dissimilar with them, depending on whether actors are what we have called ‘homophilic’ or ‘heterophilic.’ The dynamics of such network elasticity could be nicely explained on the basis of Harrison White’s (n.d.) concepts of ‘catness’ and ‘netness,’ the way they have been used in Charles Tilly’s (1978) ‘mobilization model’ of collective action: On the one side, people are sharing some common characteristics and belong to certain categories of actors. On the other side, people are linked by interpersonal relations in their own networks. According to Tilly, the probability that people will be mobilized to act collectively depends on whether they form a ‘catnet,’ i.e., “a set of individuals comprising both a category and a network” (1978, p. 63). In this view, both homophilic and heterophilic actors in our model of network elasticity are trying to break their ‘netness’ links and to create new links based on their ‘catness.’ Thus, the equilibrium network structuration that we have observed in our simulation is a manifestation of this process of emergence of ‘catnets,’ through which actors are reshaping their interpersonal bonds in such a way that will facilitate them to mobilize better their attributes over their interactions.

The final remarks that we would like to make concern some possible extensions of the present simulation. In our model, both actors’ plasticity and network elasticity have been conceptualized as mechanisms of ‘networked influences.’ What we mean by this is that, in this simulation, an actor might influence and be influenced by its structural environment (the network). However, actors might have their own preferences, which might play a role in the way they interact in the network and possibly reconfigure the network. In other words, a second source of actors’ plasticity might be stemming from their own individual attitudes that actors might possess. These individual attitudes and preferences might mold actors’ attributes in a different and competing way than the networked influences. Thus, it would be interesting to simulate the rules of actors’ interactions based both on their preferences and their networked influences. In a different setting but with a similar aim to balance individual choices with network effects, Brian Arthur (1989) had studied a selection problem between two competing technologies under increasing returns from the network. In particular, Arthur was interested in explaining how ‘historical small events’ (or path dependence) might create lock-ins (inflexibility) of the one of the two competing technologies. Similarly, by introducing rules of interaction taking into account actors’ preferences together with networked influences, it would be very interesting to investigate the emergent patterns from such an extension of the present simulation. Would inflexibility (lock-ins) and nonergodicity (path dependence) still be possible in an elastic network of plastic actors?

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Figures

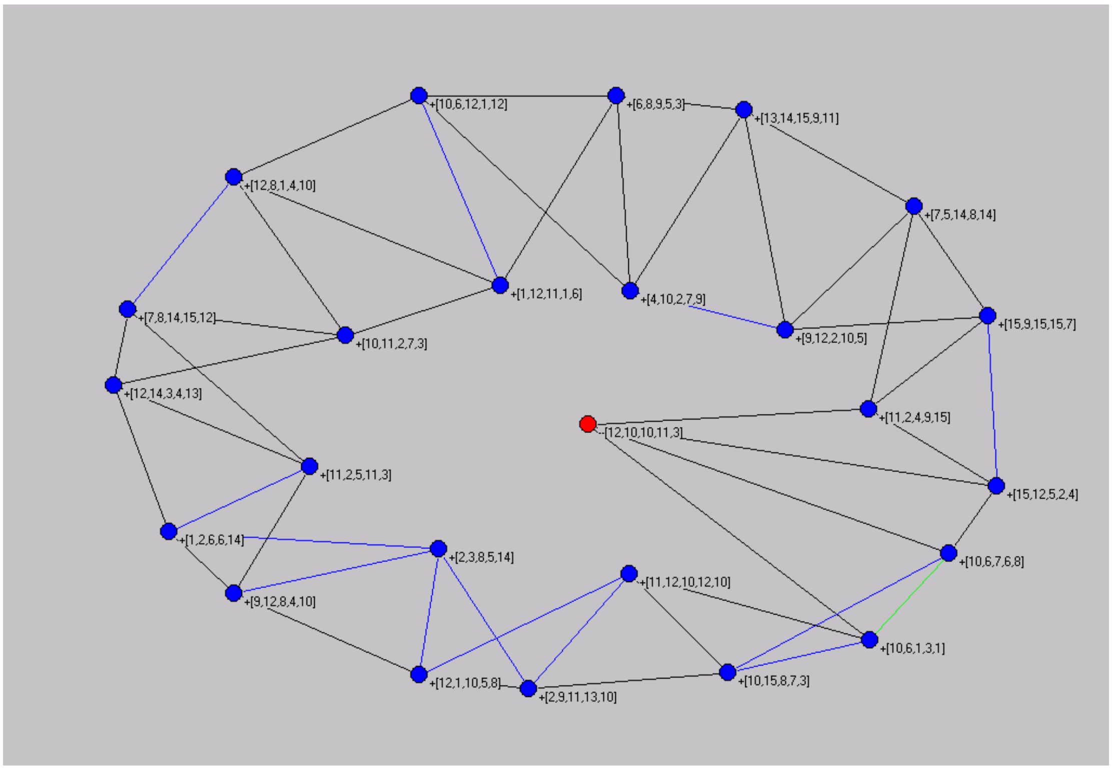

Figure 1: The field of initial (random) attributes (N = 25, one heterophilic actor, no network elasticity).

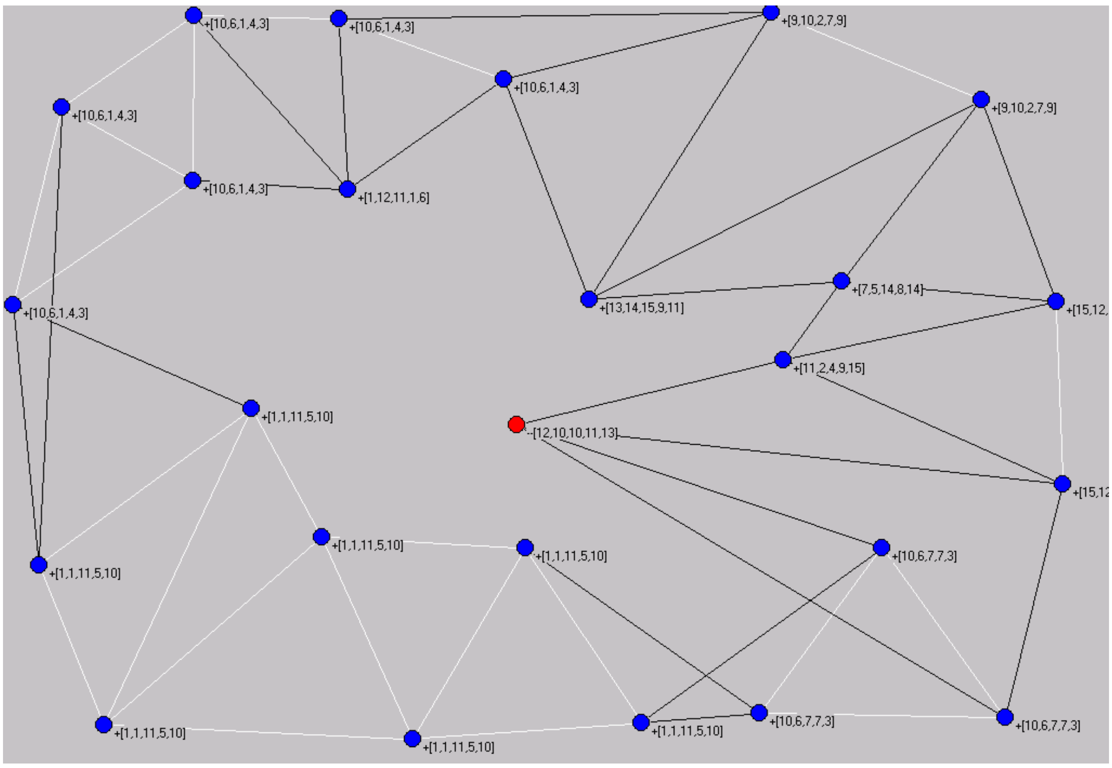

Figure 2: The emerging field of equilibrium attributes with 10 clusters (D = 10).

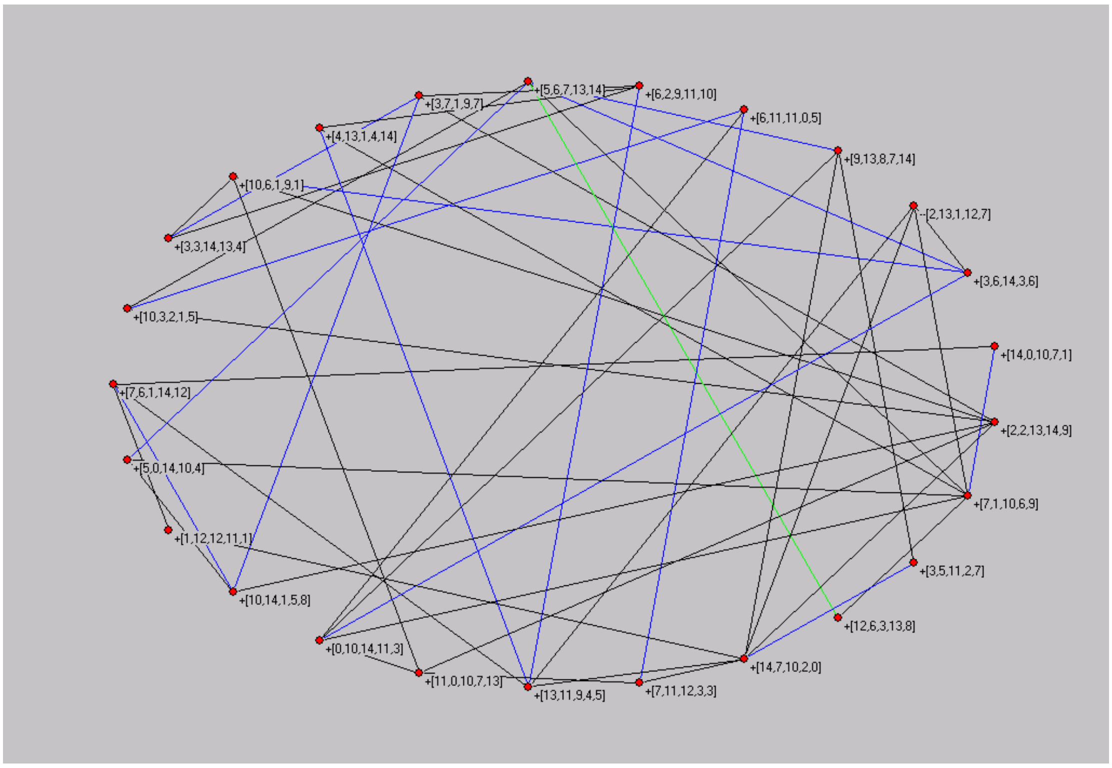

Figure 3: The field of initial (random) attributes (N = 25, one heterophilic actor, no network elasticity) on a small-world network.

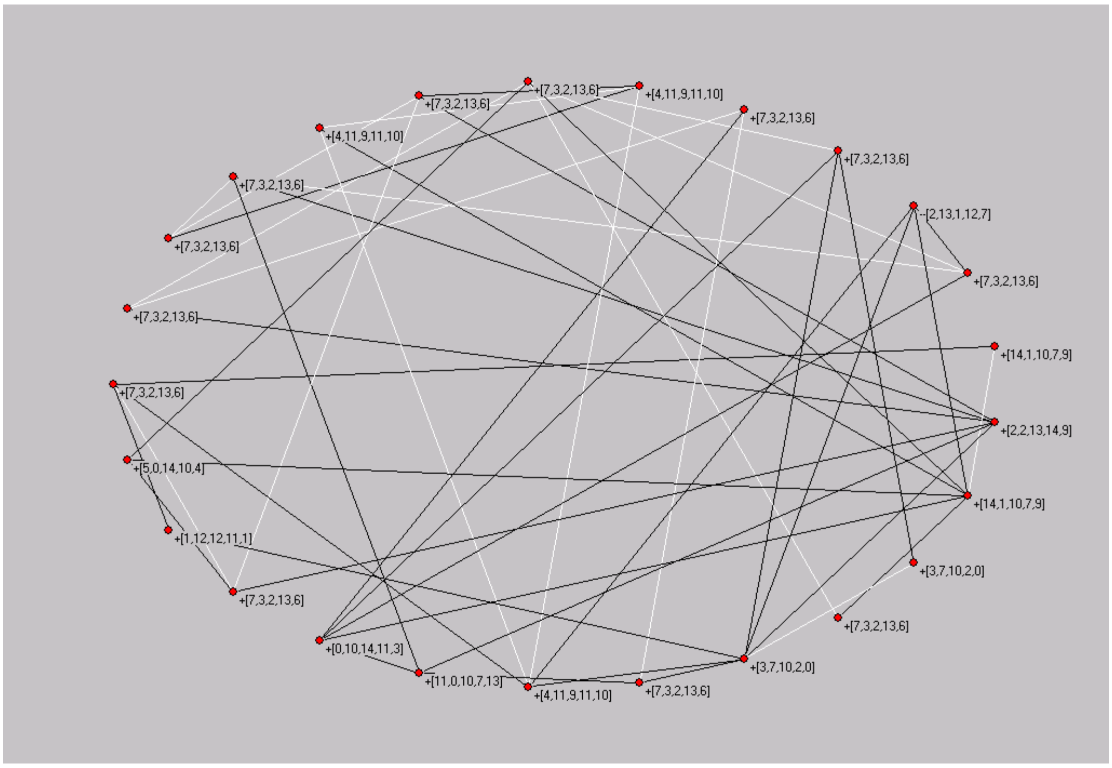

Figure 4: The emerging field of equilibrium attributes with 10 clusters (D = 10) on the previous small-world network.

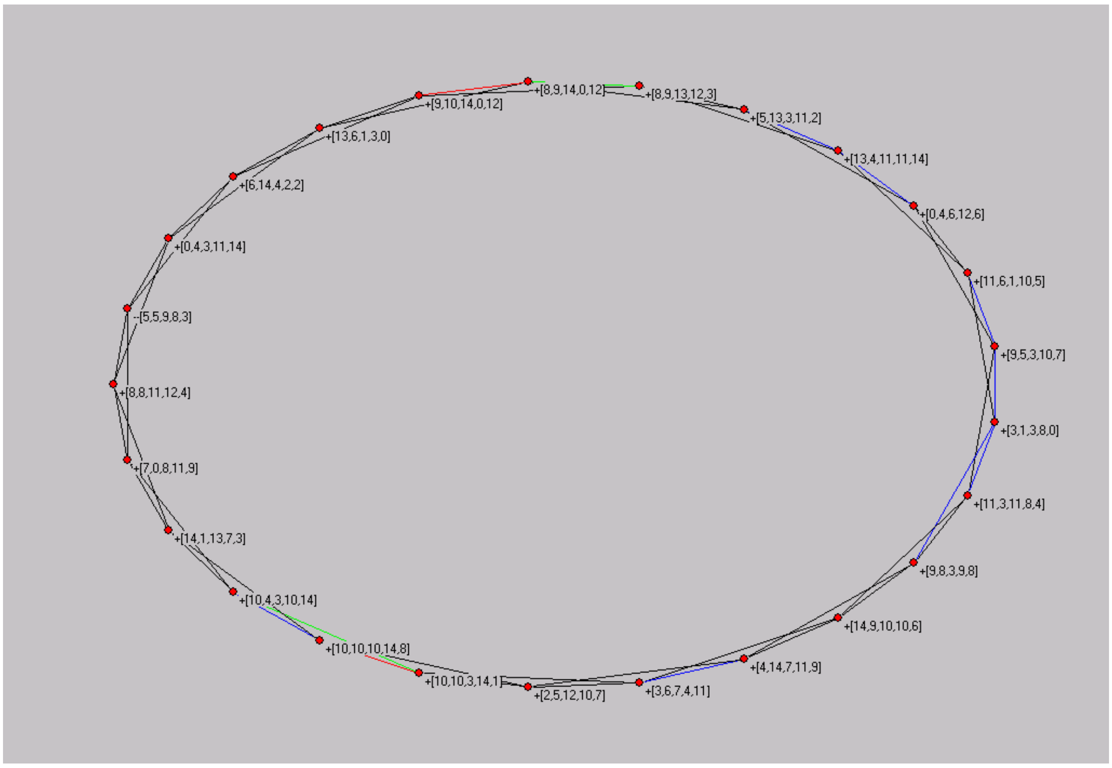

Figure 5: The field of initial (random) attributes (N = 25, one heterophilic actor, with network elasticity).

Figure 6: The emerging field of equilibrium attributes with 6 clusters (D = 6).