Abstract

A method applied in previous work [E. K. Ifantis, P. D. Siafarikas and C. B. Kouris, Conditions for solution of a Linear First-order Differential Equation in the Hardy-Lebesgue space and applications, *J. Math. Anal. Appl.* Vol. **104** (1984), 454-466, E. K. Ifantis and P. D. Siafarikas, An inequality Related the zeros of two Ordinary Bessel functions, *Applicable Analysis* Vol. **19** (1985), 251-263, E. K. Ifantis and P. D. Siafarikas, A Differential equation for the zeros of Bessel functions, *Applicable Analysis* Vol. **20** (1985), 269-281] to the study of the zeros of the ordinary Bessel function $J_\nu(z)$ is here extended and also applied to the zeros of the function $F_\nu(z) = aJ_\nu(z)+(\beta+\gamma z)J'_\nu(z)$, where $J'_\nu(z)$ is the derivative of $J_\nu(z)$. It is proved that in the case where ν is real and $\nu > -1$, the zeros of $F_\nu(z)$ are the same with the zeros of the function $G(x) = -2\nu(1+\nu) - \dfrac{2a(1+\nu)}{\beta+\gamma x} \cdot x + T(x)$, where $T(x)$, in the case of real positive zeros, is meromorphic with poles the positive zeros $j_{\nu,k}$, $k = 1, 2, ...$, of $J_\nu(z)$. Moreover the function $T(x)$ is real for x real and increases as x increases in each of the intervals $(0, j_{\nu,1})$ and $(j_{\nu,k}, j_{\nu,k+1})$, $k = 1, 2, ...$ This result unifies, generalizes and improves, many known results for the zeros of the interesting function $aJ_\nu(z) + \gamma z J'_\nu(z)$.