Abstract

Let $\{P_n(x)\}_{n=0}^{\infty}$ be a system of polynomials satisfying the recurrence relation

$$P_{-1}(x) = 0, \quad P_0(x) = 1, \quad P_{n+1}(x) + h_n P_{n-1}(x) + c_n P_n(x) = x P_n(x),$$

where h_n, c_n are real sequences and $h_n > 0$, $n = 0, 1, 2, ...$ The co-recursive polynomials $\{P_n^*(x)\}_{n=0}^{\infty}$ satisfy the same recurrence relation except for $n = 1$, where $P_1^*(x) = \gamma x - c_0 - \beta$, $\gamma \neq 0$. It is well known that the problem of determining the zeros of $P_n(x)$ is equivalent to the problem of determining the eqigenvalues of a generalized eigenvalue problem $Tf = \lambda A f$, where T and A are symmetric matrices. In this paper the problem of determining the zeros of the co-recursive polynomials is reduced to a perturbation problem of the operators T and A perturbed by perturbations of rank one. A function $\phi(\lambda) = \phi(\lambda, \lambda_1, \lambda_2, ..., \lambda_k)$ is found, $k = 1, 2, ..., n$, whose zeros are the zeros of $P_n^*(x)$, and λ_k are the zeros of the polynomial $P_n(x)$ of degree n, for $\gamma \neq 0$. This function unifies many results concerning interlacing between the zeros of $P_n(x)$ and $P_n^*(x)$ for $\gamma \neq 0$. Moreover we obtain from this function similar results in the unstudied case $\gamma = 0$.