Abstract

Let the orthogonal polynomials $P_n(x)$ be defined by

$$P_n(x) = (x - c_n)P_{n-1}(x) - \lambda_n P_{n-2}(x),$$

$$P_{-1}(x) = 0, \quad P_0(x) = 1,$$

where $\lambda_{n+1} > 0$, c_n real, $\lim_{n\to\infty} c_n = c$ and $\lim_{n\to\infty} \lambda_n = \lambda$, $(n \geq 1)$.

Blumenthal has proved that the true interval of orthogonality $[\sigma, \tau]$ of the above polynomials is given by $\sigma = c - 2\sqrt{\lambda}$, $\tau = c + 2\sqrt{\lambda}$ and the zeros of $P_n(x)$ are dense in $[\sigma, \tau]$. Blumenthal also asserted that the spectrum of the distribution function ψ corresponding to the polynomials P_n has at most finite points in the complement of $[\sigma, \tau]$. In other words the limit points of the zeros of the polynomials P_n outside the intrval $[\sigma, \tau]$ are finite. The falseness of this assertion has been proved first in 1968 with the use of a series of results concerning chain sequences and a theorem due to Szegö. Now although one can find many other ways of proving this in the literature, a concrete counter example fails to exist. Here a counterexample is given which proves the invalidity of Blumenthal's assertion. This example is also of some interest because it exhibits a particular class of the Pollaczek polynomials where the support of the measure of orthogonality is extented beyond the interval $[-1, 1]$.