Abstract

Let $j_{\nu,1}$ be the smallest (first) positive zero of the Bessel function $J_\nu(z)$, $\nu > -1$, which becomes zero when ν approaches -1. Then $j^2_{\nu,1}$ can be continued analytically to $-2 < \nu < -1$, where it takes on negative values. We show that $j^2_{\nu,1}$ is a convex function of ν in the interval $-2 < \nu \leq 0$, as an addition to an old result [A. Elbert and A. Laforgia, SIAM J. Math. Anal. **15** (1984, 206-212)], stating this convexity for $\nu > 0$. Also the monotonicity properties of the functions $\dfrac{j^2_{\nu,1}}{4(\nu+1)}$, $\dfrac{j^2_{\nu,1}}{4(\nu+1)}\sqrt{\nu+2}$ are determined. Our approach is based on the series expansion of Bessel function $J_\nu(z)$ and it turned out to be effective, especially when $-2 < \nu < -1$.