Abstract

It is proved that the differential equation

$$x^2 f''(x) + (\beta_0 x^2 + \beta_1 x) f'(x) + (\gamma_0 x^2 + \gamma_1 x + \gamma_2) f(x) = 0$$

for arbitrary β_0, γ_0, γ_1 and $\beta_1 \neq 0, \pm 1, \pm 2, -3, -4, ...$ has an entire solution if and only if the condition $\beta_1 - 2 - \gamma_2 = n^2 + (\beta_1 - 3)n$, $n = 1, 2, ...$ is satisfied. The entire solution $f(x)$ for $n = k$, $k \geq 1$ is of the form $f(x) = x^{k-1} \sum_{k=1}^{\infty} \alpha_k x^{k-1}$, $\alpha_1 = 1$.