Abstract

Let $j_{\nu,k}$ be the kth positive zero of the Bessel function $J_\nu(z)$ of the first kind and order $\nu > -1$. We prove the differential inequalities

$$\frac{dj_{\nu,k}}{d\nu} < \frac{j_{\nu,k}}{\nu+l+1} + \frac{2}{j_{\nu,k}} \sum_{n=0}^{l-1} \left\{ 1 - \frac{\nu+n+1}{\nu+l+1} \right\} h_{n,\nu+1}^2 \left(\frac{1}{j_{nu,k}} \right), \nu > -1, l = 0, 1, 2, ...,$$

where $h_{n,\nu}(x)$, $n \geq 0$, are the Lommel polynomials (Watson, 1966). As a consequence we obtain some new lower and upper bounds for $j_{\nu,k}$ improving previously known results.