Abstract

Let $H_1(\Delta)$ be the Banach space of all functions $f(z)=\sum_{n=1}^{\infty} f_n z^{n-1}$ which are analytic in the unit disc $\Delta=\{z\in \mathbb{C}:|z|<1\}$ and satisfy the condition $\sum_{n=1}^{\infty}|f_n|<+\infty$. In this paper we prove that the cokernel of the operator corresponding to the singular differential equation $z^2y''(z)+a(z)y(z)=b(z)$ in the complex plane, consists of functions which belong to $H_1(\Delta)$. Also we prove that the solutions of the differential equation $w''(z)+q(z)w(z)=0$ are elements of $H_1(\Delta)$ under the assumption that $q(z)$ belongs to $H_1(\Delta)$, i.e. the Hilbert space of functions $h(z)=\sum_{n=1}^{\infty} h_n z^{n-1}$ which are analytic in Δ and satisfy the condition $\sum_{n=1}^{\infty}|h_n|^2<\infty$. These results improve previously known results.