Abstract

In the present work we study the existence and monotonicity properties of the imaginary zeros of the mixed Bessel function $M_\nu(z) = (\beta z^2 + a)J_\nu(z) + zJ'_\nu(z)$. Such a function includes as particular cases the functions $J'_\nu(z)$ $(a = \beta = 0)$, $J''_\nu(z)$ $(a = -\nu^2), \beta = 1$ and $H_\nu(z) = aJ_\nu(z) + zJ'_\nu(z)$, where $J_\nu(z)$ is the Bessel function of the first kind and of order $\nu > -1$ and $J'_\nu(z)$, $J''_\nu(z)$ are the first two derivatives of $J_\nu(z)$. Upper and lower bounds found for the imaginary zeros of the functions $J'_\nu(z)$, $J''_\nu(z)$ and $H_\nu(z)$ improve previously known bounds.