TECHNOLOGY MEETS MATH EDUCATION: ENVISIONING A PRACTICAL FUTURE FORUM ON THE FUTURE OF TECHNOLOGY IN EDUCATION

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It is the presupposition of this workshop that computers are here to stay—both inside schools and out—and that significant changes in the way we teach and learn will result from their presence. Far be it for me to question that assumption. But in the midst of the exhilaration of being visionary, I want to temper our expectations with a practical view of where schools and classrooms are now and how much they might change in the next decade. I'm certainly not a Luddite and my intent is not to throw cold water on our flights of fancy, but I firmly believe that simultaneously considering both the vast potential of technology and the current realities of schools can lead us to creative solutions to problems we might not otherwise have considered. In this situation, practicality and therefore necessity, may indeed be the mother of invention.

The pairing of mathematics and technology has a long history; for many years a knowledge of mathematics was considered a prerequisite for becoming a programmer and the use of computers was thus available primarily to a small group of mathematically-inclined enthusiasts. Even now, when many people use computers for writing and communication, math-related programs occupy one third of a recent catalogue of educational software (Sunburst, 1999)—much more than any other topic. But this association may have led us down the wrong path, where we've seen computers primarily as machines for calculating and, educationally, for presenting students with exercises in calculation.

This paper insists that, rather than looking at math education from the perspective of the computer, we must look at computers from the perspective of mathematics education. The primary tenet of this paper is that the role of technology in math education must be in service of goals we hold for student's mathematical knowledge and expertise. Of course, technology may dramatically change those goals as well (which could get us in a serious infinite loop), but it is still the aims of mathematics education to which we must return. Broadly speaking, I take these goals to be (based on the NCTM Standards):

- eveloping students' "mathematical literacy" that goes far beyond arithmetic computation—e.g. a thorough knowledge of our number system that underlies computation and estimation, a facility with data that supports the critical analysis of statistical information with which we are bombarded, a comfort with geometric analyses of space, both two and three dimensional, an understanding of what different representations of mathematical quantities—such as graphs—mean and how they relate;

- Supporting students' mathematical "habits of mind," e.g. ability to engage in mathematical proof and argument as a basis for logical thought and discussion;
- Preparing students for the judicious and effective use of computational tools and technologies;
- Nurturing a positive attitude toward and curiosity about mathematics and mathematical thinking that can serve as the basis of lifelong learning;
- Empowering students with the realization that mathematical knowledge (not to mention much else that we learn in school) does not come predigested from teachers and books, but is a product of their own thought and exploration.

These goals take us far beyond the arithmetic that often occupies the majority of elementary school mathematics education and beyond, as well, the formulaic approaches to algebra ("solve this polynomial") and geometry ("prove this theorem") with which many of us are familiar. These goals are best served by *the creation of communities of learners in which students are actively engaged in the process of mathematical sense-making.* In this paper, the promise of technologies will be measured against this vision.

In this context, we can see that the present and future roles of technology in math education are both powerful and problematic; we need to paint a picture that takes advantage of the potential of technology without falling into the technology = computation trap. There are indeed many significant opportunities that go far beyond this impoverished image and I will describe several below. But it is important to note before jumping into descriptions of several compelling uses of technology in math education (says the realist), that the existence of these opportunities does not guarantee that they will be used effectively—or at all. The effects of technology on education and on society in general are emphatically sociotechnical (Bruce, 1999), that is, the technology has an effect only through people's uses and attitudes, in this case, in particular, through pedagogical philosophy. Technology in a vacuum is just that—technology in a vacuum. We will need to figure out how to create the context that will allow this potential to be realized.

The seeds of most of the potential future uses of technology in math education are present in today's possibilities, although we are just beginning to learn how to take advantage of them. In the following sections, I will discuss several categories of technology use, noting the present situation and future possibilities. The structure of the rest of this paper will be:

1. Descriptions of five powerful uses of technology in math education, present and future;
2. A consideration of the factors that are necessary to fulfill this potential;
3. Some concerns about the integration of technology into math education;
4. A brief closing restatement of the dilemma

POWERFUL USES OF TECHNOLOGY IN MATH EDUCATION, PRESENT AND FUTURE

As a way of organizing the ways in which technology may have substantial and significant effects on mathematics education, I have chosen five types of opportunities afforded by computers, calculators, the Internet/Web, and associated input and output devices. In each case, I will give examples of present uses, note how they support the goals identified and project how uses of this technology might grow in the near future.

DYNAMIC CONNECTIONS

Mathematics is most often thought of as an “abstract” topic, populated by symbols and invisible concepts. For many students, this lack of a visual representation makes it difficult to make connections between a mathematical expression and the situation to which it refers. Technology can help here; computers, in spite of their early image as calculating machines, are decidedly visual and provide a medium in which visual representations can be made dynamic. Students do not have to be stuck with a description in words and symbols OR with a diagram in a book that that can’t be examined or explored. Here are two examples of the difference technology can make.

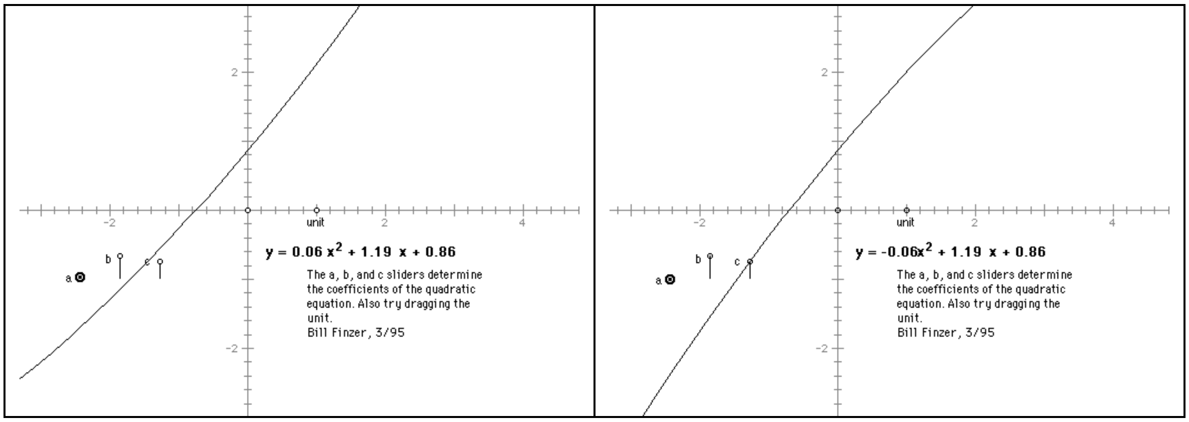
Most students’ notions of geometry are, at worst, of two-column proofs that follow a series of arcane rules, illustrated by one or two static line drawings. Many students who enjoy and succeed in geometry are able to supplement these pictures with some sense of motion, e.g.: if this corner of the square moves here, that angle will grow twice as big. The computer allows everyone to visualize these changes. Several pieces of “dynamic geometry” software have dramatically changed possibilities for geometric exploration. These tools go a long way toward turning mathematics into an experimental science—much closer to the way mathematicians experience mathematics than students usually do.

Figures 1a and b show two views of a dynamic diagram from the Geometer’s Sketchpad that allows a student to explore quadratic equations (containing the square of one variable) and the curves that they define. The power of this diagram lies in the fact that the student can change the curve by moving any of the three sliders on the lower left to change the value of a, b, and c. As the sliders are moved, both the equation and the graph change at the same time, so the relationship between them is visually apparent. It is precisely this dynamic linking that makes this software powerful. There are many explorations possible using this particular diagram; here is one simple one. Figure 1a shows a parabola whose first coefficient is .06. Note that the curve is almost straight. Figure 1b is a similar parabola whose first coefficient is -.06. This curve is also almost straight, but curves in the opposite direction. The student can move the slider back and forth between these two (and beyond), watching how the curvature changes—and what happens when the coefficient becomes 0—the curve becomes a straight line! To most students, parabolas and straight lines aren’t related; after all, one curves and the other doesn’t—but this dynamic diagram shows that a straight line is just a parabola with a 0 coefficient.

Not only can computers draw graphs and other mathematical objects and allow students to “play” with them, they can relate them to images in the “real” world. One way these connections can be made is with digital cameras and videocameras; no longer are the pictures we take static objects, but as digital objects they take on a new life that enables them to be closely linked with mathematical representations.

FIGURE 1a and b

Dynamic Geometry Diagrams To Investigate The Relationship Between Parabolas and Straight Lines (Among Other Things)

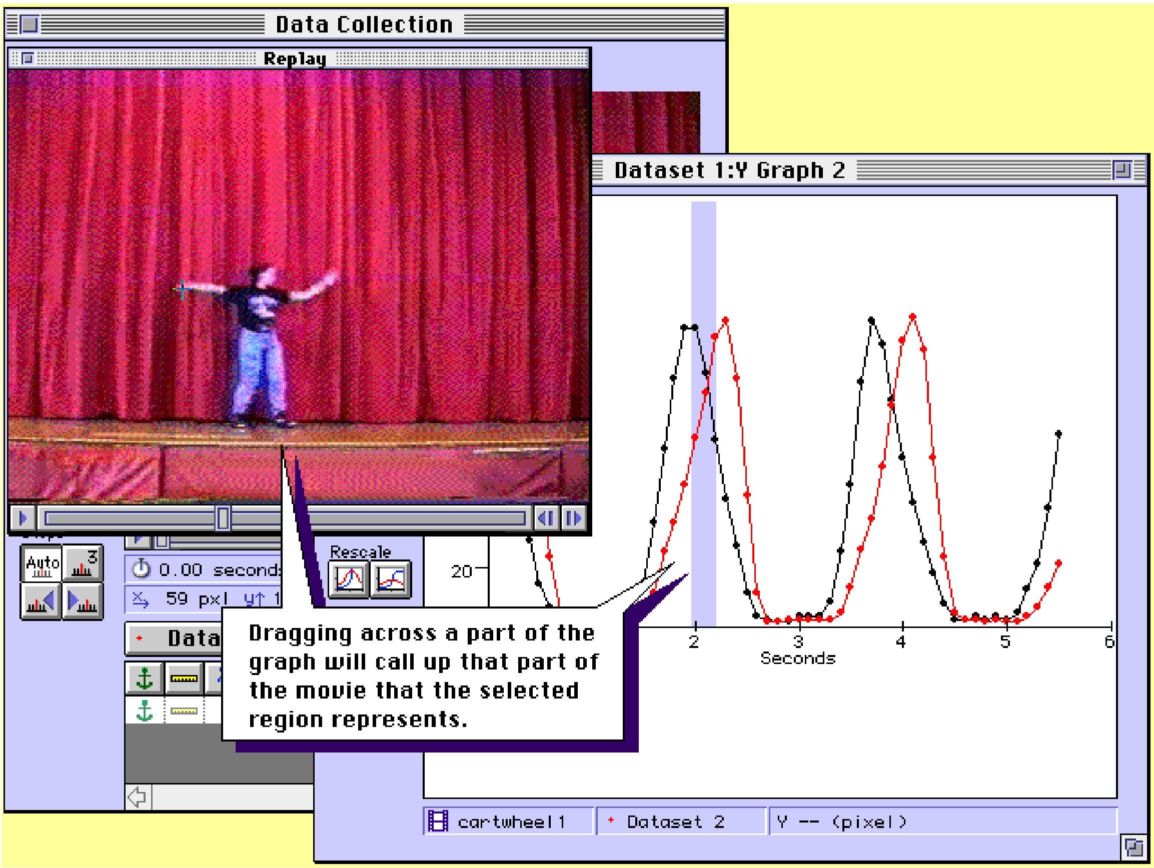


One such piece of software is CamMotion (Rubin and Boyd, 1997). CamMotion (a play on Camera and Motion) provides tools to analyze motion as it is captured on a video camera and to create the corresponding graphs of changes in position or speed over time. First, the video is digitized, so that it can be displayed on and controlled by the computer. Then, the student chooses an object to track (e.g. someone's hand) and clicks on it in each frame as the computer plays the video one frame at a time. The data thus gathered can be seen as a graph; speed can be calculated from the distance between adjacent data points and concepts such as acceleration and deceleration can be explored. But perhaps the most important aspect of this software is that the video and the graph are linked, that is, when the student points with the mouse to a point on the graph, the corresponding frame of the video is displayed. Similarly, when the student plays the video, the corresponding points on the graph are highlighted. So the link between the "real world" and its mathematical representation is made visible in a way that is quite striking.

Figure 2 shows an example of CamMotion being used to analyze the motion of a girl doing a cartwheel. The student has clicked on the position of the girl's left and right hands for each frame of the video, than made this graph by displaying the height of her left hand and the height of her right hand on the same graph. You can see that each hand follows a similar path going up and down, with one hand ahead of the other. The displayed video frame corresponds to the highlighted part of the graph; both of her hands are at about the same height, midway between the lowest (floor) and highest points they reach. It is also possible to see how quickly her hands are moving from this graph. When is her hand moving most quickly? Most slowly? How could you tell from this graph?

FIGURE 2

A Cammotion Graph of The Motion of a Girl's Hands Doing a Cartwheel



Embracing a student's real world like this is especially important when we consider technology's place in math education, because the general effect of computers is to separate students' concrete experiences from their digital representations. Being able to capture students' physical experience is more than just a new "input device;" it allows us to turn some of our mathematics pedagogy on its head. As just one example, imagine if the student's task is to move in a way that matches a particular graph, which, in turn, may have come from another video—we might call this life imitating math imitating life.

As digital devices make it increasingly simple to capture representations of the analog world on the computer, there will be more opportunities to treat the world as a grand data base, whose secrets and rules are waiting to be discovered. These systems provide a more "intimate" connection to the mathematics that can counteract the general effect of computers to separate students' concrete experiences from their digital representation, In addition, they create an environment in which mathematics is an experimental science, in which trying things out and noting what happens is an acceptable—and even preferable—approach. Having shared mathematical representations—displayed on a screen that is visible by several students at a time—also supports collaboration, since it provides objects to refer to, talk about, and investigate and goes a long time toward creating a mathematical community.

SOPHISTICATED TOOLS

Many authors have noted the growing importance of numeracy in our lives; few jobs are immune from a need for mathematical sense-making. As a result, many workplaces now provide workers with an integrated tool set (spreadsheet, calculator, graphing calculator, graphing/data analysis tools) and expect that they will have the expertise to use them effectively. Part of the responsibility of math education is to "keep up" with these developments in order to prepare students for the work world. Having such a set of tools widely available to students also has the potential to significantly change the curriculum—to give students access to mathematical topics and insights by removing computational barriers to inquiry.

This is an area of some controversy; many people who grew up mastering pencil-and-paper algorithms fear that if students use calculators they will never learn the basics of computation and will be lost without this tool. In fact, in many ways, the opposite is true: knowing how to use a calculator appropriately requires the student to know which numbers to enter, what operations to carry out and how to interpret the answer—all more important and often more demanding than doing the calculation itself. There is plenty of evidence that the <u>appropriate</u> use of calculators can improve students' mathematical achievement, as well as lead to more positive attitudes toward mathematics. In addition, calculators can add significant richness to students' mathematical experiences. Here are some examples:

The Range Game, developed by Grayson Wheatley, asks students to start with a number (say 37) and, using a calculator, find numbers that when multiplied by it give a number in the range from 500 to 600. In the conversation reported by him, students talk about their estimates, their results, whether they "have them all," and the largest and smallest numbers that would work (which leads to a discussion of decimals and limits.) This kind of conversation, which exercises students' number sense and even leads them into unfamiliar mathematical territory (e.g. limits), would be impossible if calculators were not available.

Graphing calculators have been more consistently praised as enhancing mathematics education. The ease with which they can produce complete pictures for a variety of functions means that students can graph functions, zoom in for greater detail, zoom out to see the function as X increases or decreases and compare graphs of one function with those of others. Simple models—e.g. of population growth—can be built and run on these calculators. In essence, much of the power of programs that a few years ago ran on microcomputers has been captured on personal, portable, affordable technology. Palm Pilots are the latest example of these personal aids; one of the most exciting uses of these hand-held computers is as a data collection device that can go where the student goes, rather than being stuck in a classroom.

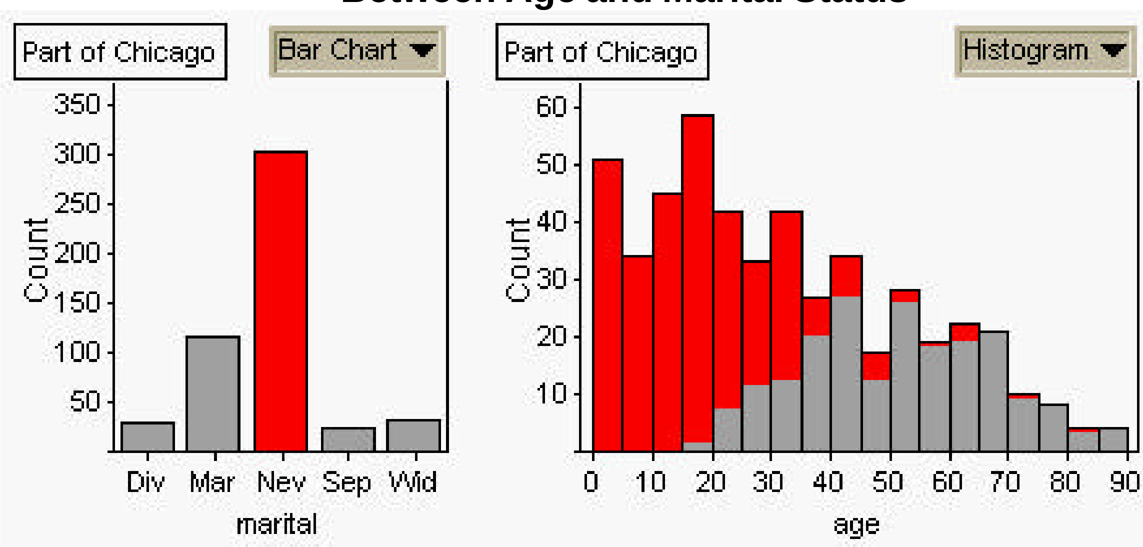
The use of calculators makes possible significant changes in the mathematics curriculum. While it is still important for students to understand computation and be fluent in carrying out problems of reasonable size, there is no reason for students to spend time dividing 5-digit numbers, adding long columns of numbers or finding square roots. The emphasis can instead be on problem-solving and developing number sense. Graphing calculators call into question the emphasis on algebraic symbol manipulation; many of the skills students learn symbolically (e.g. factoring complex polynomials to figure out the roots) are more effectively taught by working with the graphs of the functions.

In addition to calculators, other tools have the potential to change the math curriculum. Sophisticated data analysis tools, for example, expand what students can learn about statistical reasoning. The newest tools do more than produce fancy graphs; they facilitate the discovery of patterns through exploratory data analysis. These tools are well-suited for complex data bases with many variables and employ new kinds of graphs, many of which are interactive in ways similar to the dynamic geometry software described above. Many of these tools are actually used by statisticians, and several are designed with educational purposes as well. One such tool is Fathom, a sophisticated tool that provides students with many ways to look at—and therefore understand—complex data bases. Here is a relatively simple example, illustrated in Figure 3:

The data set is of a large number of people in Chicago—two of the variables are age and marital status. To see how old people are in each marital category (divorced, married, never married, separated, widowed), we can select that bar in the left hand bar graph and all the matching people are highlighted in the histogram graph on the right; this graph shows that most (but not all) of the people who have never been married are young. We can investigate related questions by choosing other bars in the bar graph (e.g. how old are the people who are divorced?) or by choosing one or more of the bars in the histogram and seeing how people in that age group are distributed among the marital categories. (e.g. What portion of people in the 50–60 age category are divorced?) In general, Fathom makes it easy to do intelligent data analysis: the student can research a question with a few simple commands and the resulting graphs will provide at least a partial answer to the question—and inevitably pose additional ones.

FIGURE 3

Fathom Graphs Exploring the Relationship Between Age and Marital Status



The value of such tools is that they create a boardwalk over the computational swamp, allowing students to see patterns they would never glimpse if they had to do the calculations or even draw the graphs themselves. In this way, exploratory data analysis software (and other visualization techniques) play a similar role to calculators and graphing calculators in emphasizing the meaning in mathematical objects and the beauty of the patterns they exhibit.

RESOURCE-RICH MATHEMATICAL COMMUNITIES

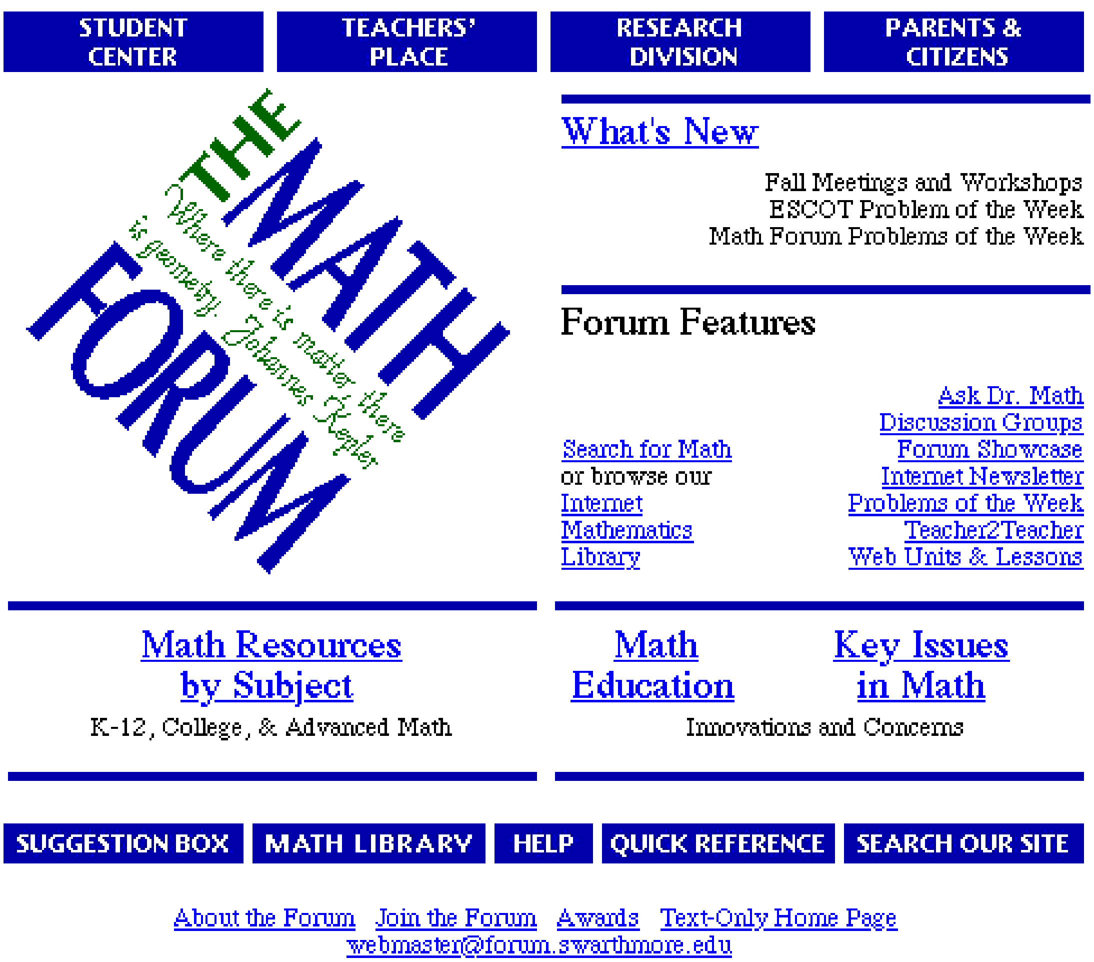
More than any other recent development, the Web has changed the public's view of technology; while few people had even heard of the Web five years ago, it is now almost impossible to watch a television ad that does not mention a Web site. The amount of information available on the Web continues to expand exponentially as more and more diverse organizations—profit-making, non-profit, large, composed of one person—are getting into the act.

This extraordinary growth has led to several developments that have important implications for mathematics education:

Resource sites. The best known of these is the Math Forum (www.mathforum.org), whose home page is shown as Figure 4.

FIGURE 4

The Home Page of the Math Forum



The Math Forum site includes a large list of (screened for quality) resources for K through college math teaching, including interactive activities; recommendations of software; examples of classroom activities and links to related discussion groups; a conversation space for teachers (Teacher2Teacher); an extensive Internet Math Library, which contains even more resources than the resource list; an Ask Dr. Math feature, in which an expert answers students' questions (the most recent topic is rounding); Problems of the Week at a variety of levels of difficulty; discussion groups on topics such as discrete math and a multi-lingual discussion on the history of mathematics; A Forum Showcase that highlights recently added sites, e.g. the following:

> Exploring Data—Pages for finding and displaying data sets, designed to support workshops on statistics given by the Math Forum for the Urban Systemic Initiative (Philadelphia and San Diego). Included are: links by level to relevant statistics Standards (NCTM, California, Philadelphia); lesson plans for collecting, analyzing, and/or displaying data; sources of data sets, general information, courses, and statistics software on the Web; and an "Oceans of Data" page with a data set (diving records) to download, instructions for making ClarisWorks graphs, suggested questions for discussion, and related 'ocean links' (NOAA, SeaWifs, tide tables, etc.). and on-line exhibits, e.g. of symmetrical patterns in Oriental carpets.

The site is impressively complete and well-organized and continues to grow as more materials are produced. It has served as an important portal for mathematics educators and as a kind of social center for the mathematics education community.

On-line professional development. In addition to the professional development materials included on sites like the Math Forum, there are entire courses being developed for delivery online. Lesley College in Cambridge, for example, teaches a semester-long online course on technology in education (much of which is focused on mathematics) for pre- and in-service teachers. Other organizations have put together repositories of professional development materials, some of which contain digitized video segments of classrooms and interviews with teachers. This material is of varied quality, of course; being on the Web is no guarantee that something has been well-designed, but it is much easier for teachers to find the resources and judge for themselves than it would be if the materials had to be ordered.

Mathematical communities for students. One of the hopes for the Internet and Web is that they would provide students with a sense of community and audience for the math they are doing—and that these communities might even be international. There have been some successes in this regard (e.g. the Problem of the Week on the Math Forum). While these uses have the potential to engage large numbers of budding mathematicians, they require some human infrastructure to organize students' participation and their impact is still unclear.

Possibility of home-school connections. Another vision put forward for the Web is the possibility that it could enable homes and schools to communicate more effectively. Possible scenarios that have been proposed are: parents and teachers could now have access to the same information about students and might communicate via email; schools might post homework on the Web or send comments to parents about children's work; schools might post materials for parents to use with their children at home to reinforce what they have done in school. There is potential here, but the uses suggested so far don't seem to make a significant leap from the status quo and few of them have been implemented in enough places to assess their effect.

Availability of data. The Web opens up a huge world of data to every student. Data bases that one could never imagine accessing before (seismographic data, weather data, environmental data) are now out there for the taking. Some Web sites have even been created with the express purpose of compiling and supplying databases for statistical analysis (e.g. the CHANCE data base out of Dartmouth). Students can also participate in creating large databases with information culled from a variety of classrooms; if these data are correlated with some geographical variable, students have a particular interest in comparing their data with that from distant schools. The National Geographic Kids Network was one of the first such projects; measurements of the pH of rainfall and local bodies of water were analyzed to track the sources and effects of acidity. Other examples have been: classroom air quality, butterfly migration studies, and measurements of shadows from different places at the same time. These kinds of projects make students members of a larger mathematical community and give them the opportunity to engage in a mathematical activity that reflects what real scientists do.

The Web will only get larger, with faster connections and more information. We can anticipate that as the amount of material on the Web increases, the difficulty of sifting through all the resources will increase as well. In addition, many of the powerful uses of the Web require human infrastructure as a foundation—organizing a coordinated data collection activity or a math competition must begin with personal contact (albeit over email) that can then make the best use of the Web's capabilities. Getting schools connected to the Internet has been a major policy goal for the past several years; now that we've come a long way toward achieving that goal, it's time to look more critically at the possible uses we might make of these electronic connections.

CONSTRUCTION AND DESIGN TOOLS

The increasing power and versatility of computers makes possible uses that are dramatically different from those described above. As computers become potentially smaller and more portable, they can play a central role in the design and construction of artifacts that have personal meaning for students. This approach to technology, which extends beyond mathematics education, has been called "constructionism." Its roots are in the LOGO community; LOGO is a powerful yet accessible programming language in which it is particularly easy to create pictures, animations and simple robot command sequences. Programming in LOGO incorporates math explicitly at times (e.g. figuring out what angles form a regular pentagon), but also introduces students to more general mathematical concepts such as iteration and recursion.

Design activities can engage a wide variety of students, since the actual projects are individually conceived and created. (Witness the unfortunate popularity of Barbie Fashion Designer.) In one recent project that dealt more explicitly with mathematics, students designed and programmed computer games that would teach other students about fractions. Some chose a video game format, others a more narrative approach. (These characteristics were, in general, correlated with gender.) Figure 5 is an example of a screen from one of these games.

FIGURE 5

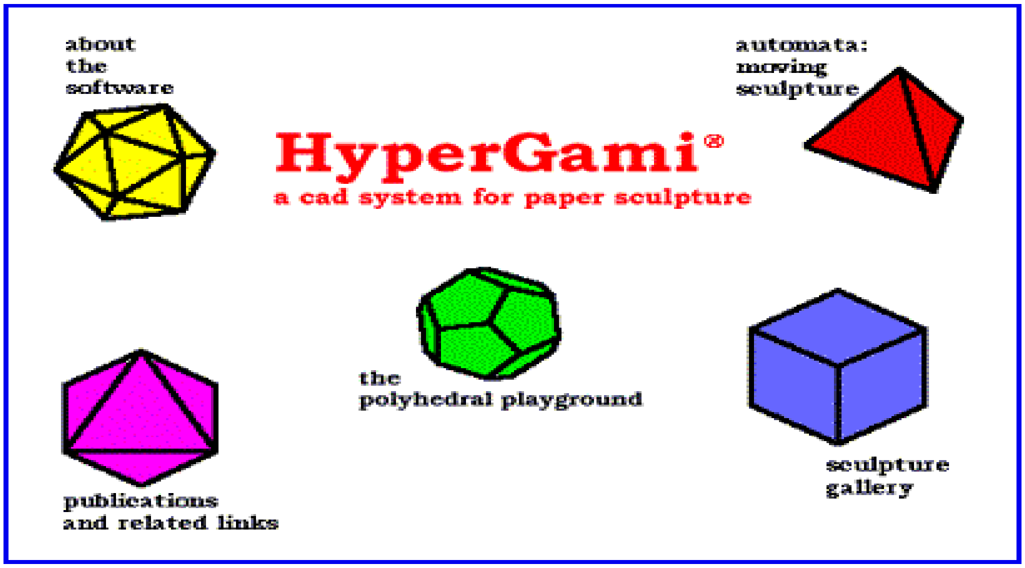
Screen from a Fraction Game Designed for Kids By Kids

Recently, design activities have been extended even further to bridge the gap between software and physical objects. The most prominent of these is the Lego/LOGO connection, in which students can write programs in LOGO that control Lego constructions. Students have created robots that back up when they hit a wall or other obstacle, miniature amusement park rides, dump trucks, and many examples of kinetic art. A more recent development in the same vein is the introduction of crickets, simple computers the size of 9-volt batteries that contain their own programs and can thus be left on their own to perform tasks such as recording the changing temperature in a room or on a walk, counting the number of times a refrigerator door is opened (and, potentially, catching a midnight snacker), or controlling a light that goes on when someone enters a room.

Physical materials that serve as input or output to software further expand the range of mathematically engaging design possibilities. Scanners provide geometric figures to modify; various visualizing techniques (including MRI's) provide 3D models to be manipulated and analyzed. There are also interesting developments in what can be the output of a software design activity. One recent project that moves in this direction is Hypergami (Eisenberg); in this program the student designs a 3-dimensional origami sculpture using a customized geometric tool, which then creates and prints out the patterns that, when folded, would produce that sculpture. Figure 6 is the opening screen of Hypergami.

FIGURE 6

The Opening Screen of Hypergami, a Cad System for Paper Sculpture



In the relatively near future, we may have even more imaginative and sophisticated output devices; there are currently laser cutters that can be computer controlled to create custom wood parts for a sculpture. (In an interesting twist, Barbie Fashion Designer has moved in this direction; the material on which the program prints has a fabric-like texture.)

While these materials are just beginning to be used in schools, they have great potential to engage students at their own level, and to encourage collaboration and mathematical discussion. They expand our concept of mathematical thinking to encompass the subtle understandings students must have to actually design something that works—skills they are likely to need later in life and in work.

TOOLS FOR EXPLORING COMPLEXITY

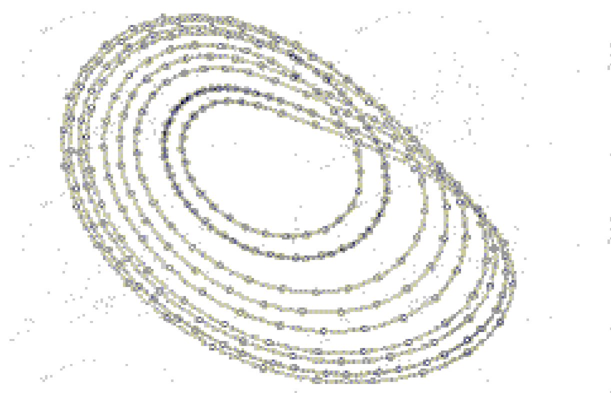
For mathematicians, one of the most important developments in technology has been the increased number of tools for dealing with complexity. From Mathematica, a general algebraic tool, to specific modeling systems (e.g. Agent Sheets) to specially designed languages for exploring large-scale parallel models (e.g. Star LOGO), areas of mathematics that had previously been off-limits for almost everyone are now accessible to students as well as mathematicians.

One type of investigation made possible by such tools is simulation. There are certain natural systems (most popularly, predator/prey systems) for which the basic structure is expressed by a relatively compact set of rules but the behavior can be vastly different depending on the value of a few variables such as the birth and death rates of the predators. Having a tool with which to explore these patterns not only gives students the opportunity to learn about a biological interaction, but teaches them about functions, variables, cyclical functions and sensitivity analysis. Exploring these concepts by hand is practically impossible, since the number of calculations necessary to see any kind of pattern is astronomical. A whole new area of mathematics is suddenly available to middle and high school students.

Even more spectacular is the rise of the field of chaos. Formerly the province of a few mathematicians, chaos is now the subject of books for the lay person and many non-mathematicians have now seen examples of the beautiful patterns created by chaotic systems. Figure 7 is a picture of the Roessler attractor, one of the more basic patterns that arises in the exploration of chaos. This is not the only connection between mathematics and art that technology has facilitated; many middle school students create tessellations with Tessellmania or fractal images with simple programming languages. The implications of the rise of chaos in the mathematical community have yet to filter through the educational system, but the addition of such an engrossing and artistic topic may turn out to be an opportunity to engage and challenge more students.

FIGURE 7

A Roessler Attractor



These examples support the practice of mathematics as an experimental science, an exploratory process of sense-making. As with the dynamic geometry software, rather than giving students a result or theorem and asking them to prove it, these systems provide laboratories in which students can investigate patterns on their own; armed with their discoveries, students can enter actively into a more formal consideration of the underlying mathematical structures.

Systems for modeling complex or dynamical systems can also change the curriculum, as they make advanced topics, whose symbolic representations are inaccessible, available to young students. This is similar to—but perhaps even more striking than—the kinds of changes graphing calculators can make to the algebra curriculum. Other topics in the curriculum may be similar susceptible to the effects of technology. For example, a group of tools under development (SimCalc) makes calculus accessible to younger students (and more understandable to older ones), in part by building technologies that allow students to connect simulations of changes in position and velocity on a computer with similar changes in motions of toy cars off the computer. A classroom set up to use this LBM (Line Becomes Motion) technology integrates graphing calculators with a computer simulation with computer-controlled cars. Between a re-visioning of calculus and the creative use of technology, the SimCalc developers aim to create an elementary through undergraduate curriculum that thoroughly explores the mathematics of change and variation, in ways that every student can understand.

The study of complexity, understandably, has great appeal for mathematicians and students alike; we can explore it today like never before, given the large crop of new tools that have appeared. We don't know, yet, how much this will affect K-12 education—how much more sophisticated mathematics can be made accessible to students or how much these new experiences might engage students who otherwise might not "take to" math.

NECESSARY CONDITIONS FOR TECHNOLOGY TO HAVE A SIGNIFICANT EFFECT ON MATH EDUCATION

It's the realist's turn now. In fact, with all the potential described above, effective uses of technology in math education are not always easy to find. The increasing presence of computers in our schools does not automatically translate into educational improvement. Why? I see four major conditions for change that need to be more consistently met to reap the gains of technological development.

More support for development. Who has paid for the development of the software described above? In some cases, the National Science Foundation has provided funds for prototyping, leaving developers to find other support for bringing the software to market (which often involves a great deal of time and money; prototypes are often quite far from marketable products). Geometer's Sketchpad and Fathom required a significant investment from a commercial publisher; SimCalc is not yet commercially available; the developers of CamMotion were never able to get it published; the Math Forum was begun by and continues to be partially supported by the NSF. Where software has not been published, it is often because no commercial publisher is willing to invest funds into what appears to be (and mainly is) a small market. We have a serious Catch-22 situation here: if there is only a small selection of available math education software, schools will not see that as a reason to buy technology; if there is a small base of computers in schools, publishers will continue to be cautious about putting their own money into development. (This is closely related to the issue of curriculum integration, discussed below.)

Another manifestation of the same problem is that of keeping up with the changing state of hardware. Many teachers bemoan the fact that Apple IIe's are no longer in their schools, as many pieces of software that ran on them do not work on more recent machines. As schools upgrade their machines, software that worked just fine stops working when a new operating system in installed, often at most inconvenient times. Keeping software up to date requires considerable resources (Microsoft and Apple spend large parts of their budgets on compatibility) and educational materials publishers, through whom some software is distributed, often do not have the infrastructure for making software work on different platforms and revising it as systems change.

Curriculum integration. Another critical prerequisite for successful use of technology is its integration into the curriculum. While the most motivated and informed teachers will figure out themselves how technology fits into their teaching, most do not have the expertise to do so. In this regard, the situation is almost uniformly poor. Even the new NSF-funded reform curricula make little use of technology beyond calculators. Only one of the three elementary curricula integrates computer technology at all—and this only minimally. Of the five middle school curricula, one is heavily based on technology, while the other four contain little beyond recommendations for related software. All of the secondary curricula integrate graphing calculators and several make suggestions for using other software, but here, again, there is not much support for teachers who have little experience using technology in their mathematics teaching. Some good curriculum units exist that make use of particular pieces of software, such as dynamic geometry software like Geometer's Sketchpad or data analysis software like Fathom, but these are published separately from the standard curricula.

This situation makes the Catch-22 mentioned above even worse. There are even fewer examples of good software-curriculum integration than there are pieces of software, so schools are less likely to see the value of spending money on technology. Curriculum writers are then loathe to link their products too closely to technology, lest schools without computers decide not to use them.

Beyond content integration, most curricula do not provide support for the logistics that successful use of technology requires. Secondary schools often have self-contained computer labs, which makes logistical planning easier, but elementary schools may not; a common arrangement is for many of the computers to be located in classrooms, in groups of three to five. Even if schools have computer labs, scheduling time in them is often difficult. In order for teachers to fit technology into their teaching, they must have the curricular flexibility to accommodate these different arrangements. One of the NSF-funded elementary curricula, *Investigations in Number, Data, and Space*, includes detailed instructions for teachers about using computers in different configurations, but this kind of support is unusual.

Considering curricular requirements could actually influence decisions about hardware purchases and how they are arranged, but this seldom happens. Schools may set as their goal to "have a computer lab" or "have one computer in every classroom" or "be connected to the Internet," but these plans are often not informed by the pedagogical context in which the technology will be used.

Professional development. Following closely on the heels of curriculum integration as a necessary condition for technology implementation is professional development. This is a difficult task, because teachers need to learn about both the computer itself—how to install software, and troubleshoot hardware and software problems—and how to use the software effectively in their teaching. Professional development, where it exists, usually focuses on the former. Learning how to best use software is itself a complex task, because effective integration may require changes in teachers' pedagogical approaches as well as knowledge of content. The NSF-funded reform curricula confront the same dilemma—i.e. the pedagogy assumed is quite different from the way many teachers tend to approach mathematics teaching—and the situation becomes even more complex with the addition of technology. In many school systems, professional development money ranks third in technology-related expenditures after hardware (the major portion of money spent) and software.

Many schools have hired technology coordinators to deal with hardware and software issues, but they rarely work in concert with other curriculum specialists, so there is a frequent disconnect between content issues and technology issues. A more common model is that a class goes to the computer lab for a self-contained lesson and at times the teacher doesn't even go along. A more productive collaboration among teachers, curriculum specialists, technology coordinators and curriculum developers is necessary before technology implementation will be truly successful.

Public education. The general public sometimes suffers from one of two inaccurate visions of technology and, thus, their support in terms of money, expertise, and even political goodwill can be unreliable. Some are over-optimistic about the potential effects of technology, see it as a panacea, and imagine it reducing the costs of schooling while at the same time increasing its effectiveness. They imagine computers that are individually attuned to each student's needs with limited effort on the part of teachers—or students immediately and productively connected to the world's knowledge via the Internet. This point of view can lead to unrealistic expectations and, in the end, to disillusionment with the entire enterprise.

On the other hand, there are those who see technology as a threat to quality education. They view calculators as tools that undercut students' mathematical growth or computers as overused crutches. (This perspective is often related to the belief that reform math is a dangerous proposition because it erodes students' computational abilities.) These views are even more damaging to the future of technology in mathematics education than the over-optimistic ones, since they are not even willing to see what happens before condemning the possibilities.

The images of computers in the media are clearly one reason for these attitudes, especially for the over-optimistic view. Some parents are not aware of the opportunities that technology offers beyond drill and practice of mathematical facts and computational skills; others are not aware of the complexity of using the Web productively. Educating the media more carefully so that they can educate the public should create a more supportive atmosphere for the kinds of creative uses of technology we need to be educationally successful.

CONCERNS AND CAVEATS ABOUT TECHNOLOGY AND MATH EDUCATION

Equity concerns. There is a history of inequity across gender and race in mathematics achievement in this country. While this gap is closing, particularly with respect to gender, technology may be taking its place as the great divider. Just a few statistics illustrate the seriousness of the situation:

- Schools with more than 90% minority enrollment have 16% fewer computers per capita than other schools.
- A recent study of elementary math software games found that only 12% of gender-identifiable characters in the games were girls.
- In a 1998 study, girls consistently rated themselves significantly lower than boys on computer ability.
- Sixty-nine percent of computer scientists are men; 31% are women.

We cannot simply ignore these differences; girls and women are already being cast as the users who will see the computer as a word processor, but will not write challenging software; Mattel has even come out with different computers for girls (Barbie Computer) and boys (Hot Wheels Computer). Plans for integrating technology into the public schools need to take into account the needs and experiences of a variety of groups across gender, class and other group characteristics. (AAUW, 1998)

Some school systems have offered girls-only technology classes; some community technology centers have as well—and organizations such as the Girl Scouts are venturing into the world of computers. Community technology centers also offer access to computers for populations who are less likely to have computers at home. But these efforts are just getting started—and they are not widely known or adopted.

Risks of inappropriate use. While there are many valuable ways to use technology in math education, there are also those that are detrimental. While many of the fears about calculators and graphing calculators are unfounded, it is important that they be used with thought. For example, before students become reliant on graphing programs, they should have the opportunity to construct graphs on their own, including making decisions about the appropriate representation to use, where to put each variable, how to scale the graph, etc. If they do not have this experience, using a graphing program may amount to little more than choosing a graph type from a small menu.

Calculators can also be used inappropriately: it is important that students know how to carry out relatively simple calculations without electronic support, and we do not do students a service by providing calculators all the time. On the other hand, the hardest part of using calculators in real world situations is figuring out which numbers to type in, what operations to use and how to interpret the answer. No amount of practice with rote operations will lead to that kind of expertise.

Another risk is that of using computers to automate drill and practice—also known as drill and kill. Besides being a considerable waste of computational power, this kind of software only reinforces the view that learning mathematics is primarily about calculating quickly, an attitude that we know cuts out many students from being mathematically engaged. In fact, there is evidence that students who use such programs do not do any better in math than those who do not (while those who use simulation programs do appear to do better). Drill and practice software also has the effect of isolating students who are working individually and certainly not engaging them in mathematical discourse.

Trying to replace teachers. It is worth being suspicious of any use of software that proposes to replace teachers, for it must be based on an impoverished view of mathematics education. In this technological age, teachers are more necessary than ever; this is why professional development is so important, as teachers' roles are changing from being lecturer to facilitator or, as one common slogan goes, from being "the sage on the stage to the guide on the side." It is easier to imagine a computer giving a lecture with multiple choice questions than it is to imagine a computer engaging students in a productive discussion of the concept of limits or even of the concepts of odd and even. Most of the attempts at intelligent tutoring systems are, in my view, rather lame—and even if they are successful, they deal with quite circumscribed and relatively minor amounts of mathematics. We must guard against the public conception that teachers are expendable and that the way to save money in schools is to trim their ranks.

Succumbing to Web ecstasy. While there are many valuable uses of the Internet and the Web as discussed above, there is also a danger that excitement over the Web will cloud our judgment about particular activities. People in other disciplines (e.g. language arts and social science) have noticed—and complained about—students' reliance on the Web for information, to the exclusion of libraries and physical books. In a similar vein, using the Internet to carry on mathematical discussions does not make sense if they could also be happening in person, within a single classroom or a single school. Analyzing a large data set made up of information gathered from around the country may be exciting, but if there is no additional information uncovered than there would be in a survey of the school, it may be poorly utilized effort. It is not always possible to know the value of such a data collection activity before it is done—but there is good reason to think carefully about it before embarking. Journeys on the Web are quite prone to getting lost.

CONCLUSION (QUITE BRIEF)

So there it is: rich potential, significant obstacles, and important concerns. Even though they still eye one another with suspicion, the visionary and the realist rest their cases, sit back and watch what happens.

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