

(Exert from the Introduction)

## INTRODUCTION

This book started as Lecture Notes for a first year course in The Department of Mathematics -University of Patras. The title of the course was «**A Mathematical Journey**» which have been incorporated into the new Program of Study of the Mathematics Department in 1983.

The target of this course is to give to the new student of mathematics, a global conception for mathematics and their methods. In addition, on the one hand it tries to prepare the student from a technical, methodological and psychological point of view, in order to face successfully the subsequent studies, and on the other hand to give him/her a minimum of a mathematical culture, indispensable for the creation of an appropriate climate, on which a mathematical thinking can be developed.

The development of the intuition, and in general the non-analytic aspects of mathematics constitute a basic objective of the book. The mathematics as «pemptousia» (see Figure 1.25) does not restrict them only into their analytic aspects.

The expedition inception of the course, certainly is not exhausted with a «sewing together» certain mathematical pieces, but mainly it is materialized by approaching and grasping in perspective, the mathematical substance, exactly through these pieces of mathematics and the related remarks and comments.

In the wilderness of the analytical mathematical knowledge and the in the confusion of the splintering of mathematical substance, we are looking, using synthetic reasoning, for the interrelationship of the mathematical concepts and theories, but also for their relationship with thw real objective reality.

All these lead and transform the «dry» technical mathematical knowledge into a synthetic mathematical wisdom. It is exactly then where mathematics and philosophy are united into a «perfect knowledge».

In the last 15 years this book have been printed into many preliminary editions, mainly for the service of the students of the Department of Mathematics. After all these experiences and after many «publication adventures» the old Notes have been rewritten to form the present book:

### **INTRODUCTION TO MATHEMATICAL THOUGHT**

which finally shall be developed into two volumes:

***Vol.1 Mathematical Journeys.***

***Vol.2 The Content, the Method, and the Philosophy of Mathematics.***

In the present book (vol. 1), Chapters 0,1 and 3, together with a crash introduction to Chapter 2, constitute the material of the lectures. In addition we run a non compulsory, weekly seminar, where we held free discussions concerning mathematics. In the «exercises session» we were focused on the Problem Solving techniques. ...

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