

*Konstantinos Athanasopoulos*

**Stable and unstable attractors**

The study of compact invariant sets plays a central role in the geometric theory of differential equations and dynamical systems. There are basic difficulties in this study. 1. The compact invariant sets are global objects and so one needs to develop global methods and tools for their study. 2. Their structure may be extremely complicated. 3. Even in the case of a simple compact invariant set, its structure may change dramatically under small perturbations of the system. 4. The structurally stable dynamical systems are not dense. In practice, when studying a parametrized family of differential equations one has to handle all these four problems simultaneously. We will be concerned with the study of the topology and dynamics in compact invariant sets of a continuous flow, and in particular compact minimal sets, in connection with the description of the dynamics around such a set. In particular, we want to examine how the complexity of a compact minimal set affects the behaviour of the flow around it.

*Iakovos Androulidakis*

**The analytic index for elliptic pseudodifferential operators along a singular foliation**

In earlier work (joint with G. Skandalis) we attached to every singular foliation a  $C^*$ -algebra and defined a longitudinal pseudodifferential calculus which realizes it as the algebra of negative-order operators as such. In this talk I will report on the latest development, namely the definition of the analytic index for elliptic pseudodifferential operators as above, and how this index can be obtained in a geometric way from a certain deformation of the foliation. Note that the analytic index is the crucial ingredient in the construction of the Baum-Connes assembly map, which is the next step of this project.

*Mohamed Belkelfa*

**The study of pseudo symmetry in R. Deszcz sense of S Sasakian space forms**

In Soochow journal of Mathematics. 4 (2005), 611-616. For  $S=1$ , M. Belkelfa, R. Deszcz and L. Verstraelen proved that saskian space forms is pseudo symmetric.

The aim of this talk is to study the pseudo symmetry in R. Deszcz sense of S Sasakian space forms.

*Lubjana Beshaj*

### **Computational aspects of genus 3 algebraic curves**

In this talk we will describe some methods of computing with genus 3 curves defined over the complex field. The case of genus 3 hyperelliptic curves is better known. We use previous results of Shioda and other authors to combine such tools in a Maple package. This package computes the automorphism group of such curves, the field of moduli, the minimal field of definition, and an equation of the curve over its field of definition. Similar results we are trying to extend in the case of genus 3 non-hyperelliptic curves, which is obviously harder. This is joint work with T. Shaska and V. Hoxha.

*Giovanni Calvaruso*

### **Geometry of Kaluza-Klein metrics on the sphere $\mathbb{S}^3$**

(Joint work with D. Perrone [2]). *Berger metrics* are well known in Riemannian geometry and have been studied under several different points of view. They are defined as the canonical variation  $g_\lambda$ ,  $\lambda > 0$ , of the standard metric  $g_0$  of constant sectional curvature on  $\mathbb{S}^3$ , obtained deforming  $g_0$  along the fibres of the Hopf fibration. Let  $\{\xi_1, \xi_2, \xi_3\}$  be the unit vector fields on  $\mathbb{S}^3$  corresponding to the standard complex structures  $I, J, K$ . Denoting by  $\theta^1, \theta^2, \theta^3$  the 1-forms dual to  $\xi_1, \xi_2, \xi_3$  with respect to  $g_0$ , an arbitrary Berger metric  $g_\lambda$  on  $\mathbb{S}^3$  may be written as  $g_\lambda = \lambda\theta^1 \otimes \theta^1 + \theta^2 \otimes \theta^2 + \theta^3 \otimes \theta^3$ . It is then natural to generalize such a construction, allowing deformations of the standard metric  $g_0$  not only in the direction of  $\xi_1$ , but also of  $\xi_2$  and  $\xi_3$ . Thus, we consider on  $\mathbb{S}^3$  the three-parameter family of Riemannian metrics of the form

$$\tilde{g}_{\lambda\mu\nu} = \lambda\theta^1 \otimes \theta^1 + \mu\theta^2 \otimes \theta^2 + \nu\theta^3 \otimes \theta^3, \quad \lambda, \mu, \nu > 0.$$

Clearly, all Berger metrics are of the above form, as  $g_\lambda = \tilde{g}_{\lambda 11}$ . Riemannian metrics  $\tilde{g}_{\lambda\mu\nu}$  lie at the crossroad of several interesting topics: Hopf vector fields, the Hopf map, invariant metrics of  $SU(2)$ , Berger metrics, homogeneous spaces and homogeneous structures, natural metrics on the unit tangent sphere bundle, harmonic morphisms, almost contact and contact structures. First of all, they turn out to be related to a class of well known Riemannian  $g$ -natural metrics defined on the unit tangent sphere bundle  $T_1\mathbb{S}^2(\kappa)$ . For this reason, metrics  $\tilde{g}_{\lambda\mu\nu}$  will be called “of Kaluza-Klein type”. Their relationship with Riemannian  $g$ -natural metrics on  $T_1\mathbb{S}^2(\kappa)$  permits to show that for a two-parameter subclass of these metrics, one

can define corresponding harmonic morphisms from  $\mathbb{S}^3$  to  $\mathbb{S}^2$  [1,2], which include the Hopf map between the standard spheres as a special case. On the other hand, these metrics can also be seen as left-invariant Riemannian metrics on the three-sphere group  $SU(2)$ . Hence,  $(\mathbb{S}^3, \tilde{g}_{\lambda\mu\nu})$  is a homogeneous space. The classification of homogeneous structures on  $(\mathbb{S}^3, \tilde{g}_{\lambda\mu\nu})$  (see also [3]) shows how these structures depend on the number of distinct parameters between  $\lambda, \mu, \nu$ . We also introduced a natural almost contact structure on an arbitrary sphere  $(\mathbb{S}^3, \tilde{g}_{\lambda\mu\nu})$  of Kaluza-Klein type, showing that it provides examples of homogeneous almost contact metric manifolds. Several almost contact and contact metric properties of such spheres are investigated.

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*Francesco Saverio de Blasi*

### Some exotic but typical properties in convex and nonconvex geometry

In recent years several unexpected geometric properties of compact sets, compact starshaped sets, compact convex sets,... have been discovered by several authors, including, Gruber, Klee, Zamfirescu, Zhivkov. A common feature of these properties is the fact that, though they are possessed by most sets (in the sense of the Baire's categories) in the corresponding space of sets, yet they are quite difficult to be directly detected and in some cases no concrete examples are known. Some new results in this research area will be discussed.

*Ioannis Delivos*

### Αφινική εικόνα ευθαιογενούς επιφάνειας

Στον Ευκλείδειο χώρο  $E^3$  μελετούμε την αφινική εικόνα  $\Phi^*$  μιας μη κωνοειδούς ευθαιογενούς επιφάνειας  $\Phi$ . Η  $\Phi^*$  είναι πάλι μια ευθαιογενής επιφάνεια με γενέτειρες παράλληλες στις αντίστοιχες της  $\Phi$ . Δίνονται διάφοροι χαρακτηρισμοί ειδικών ευθαιογενών επιφανειών. Ειδικότερα δίνονται ικανές και αναγκαίες συνθήκες, ώστε μια από τις  $\Phi, \Phi^*$  να είναι επιφάνεια Edlinger ή και οι δύο να είναι επιφάνειες Edlinger. Τέλος, εξετάζεται πότε οι επιφάνειες  $\Phi$  και  $\Phi^*$  έχουν παράλληλες αφινικές καθέτους στα αντίστοιχα σημεία.

*Anila Duka*

## **Modular polynomials for genus 2**

In this paper we study modular polynomial of higher genus. They are given as minimal polynomials of the fixed field of the  $\Gamma_0^g(N) \leq Sp(2g, Z)$ , for every level  $N$ . The quotient space of this action is denoted by  $Y_0^g(N)$  and we prove that this is a quasi projective variety of dimension 2. A degree  $N$  isogeny  $\alpha : C \rightarrow C^1$  gives rise to a map  $\beta : \mathcal{P}^1 \rightarrow \mathcal{P}^1$  of ramification  $\sigma$ . The moduli space  $\mathcal{M}(\beta, s)$  of such maps  $\beta$  is a quasi projective variety and it is actually in a one to one correspondence to  $Y_0^g(N)$ . We introduce an algorithm how to compute such spaces. Furthermore, we discuss relations of such spaces with the twisted modular curves.

*Zdeněk Dušek*

## **The existence of homogeneous geodesics**

It is well known that, in any homogeneous Riemannian manifold, there is at least one homogeneous geodesic through each point. For the pseudo-Riemannian case, even if we assume reductivity, this existence problem was open. The standard way to deal with homogeneous geodesics in the pseudo-Riemannian case is to use the so-called “geodesic lemma”, which is a formula involving the inner product. We shall use a different approach, namely, we imbed the class of all homogeneous pseudo-Riemannian manifolds into the broader class of all homogeneous *affine* manifolds (possibly with torsion) and we apply a new, purely affine method to the existence problem. As the main result we prove that any homogeneous affine manifold admits at least one homogeneous geodesic through each point. As an immediate corollary, we have the same result for the subclass of all homogeneous pseudo-Riemannian manifolds.

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*Rida Farouki and Takis Sakkalis*

## **Rational rotation-minimizing frames on space curves**

An adapted frame along a space curve is an orthonormal vector basis that incorporates the unit curve tangent at each point. Such a frame is said to be rotation-minimizing if its angular velocity maintains a zero component in the direction of the curve tangent, i.e., the normal-plane vectors exhibit no instantaneous rotation about the tangent. Rotation-minimizing frames have useful applications in computer animation, swept surface constructions, path planning for robotics, and re-

lated fields. Recently, the possibility of constructing space curves with exact rational rotation-minimizing frames, as a subset of the spatial Pythagorean-hodograph (PH) curves, has been recognized. The underlying theory and construction of such RRMF curves is presented, and alternative characterizations for them (in terms of the quaternion and Hopf map representations of spatial PH curves) are derived and compared.

*D.N. Georgiou and A.C. Megaritis*

### **Covering dimension and finite spaces**

The class of finite topological spaces was first studied by P.A. Alexandroff in 1937 in [1]. There is a strong relationship between finite spaces and finite simplicial complexes (see [6]). On the other hand, all compact Hausdorff spaces arise by approximation using finite  $T_0$ -spaces (see [5]). Finally, finite spaces play an important role in dimension theory (see [8]). Together with the theory of continua, dimension theory is the oldest branch of topology. It is possible to define the dimension of a topological space  $X$  in three different ways, the small inductive dimension  $\text{ind}(X)$ , the large inductive dimension  $\text{Ind}(X)$ , and the covering dimension  $\text{dim}(X)$ . The three dimension functions coincide in the class of separable metric spaces. In larger classes of spaces the dimensions  $\text{ind}(X)$ ,  $\text{Ind}(X)$ , and  $\text{dim}(X)$  diverge. The small and large inductive dimensions for finite topological spaces are studied by some other authors (see, for example, [2], [3], [7]). A topological space  $X$  is *finite* if the set  $X$  is finite. In what follows we denote by  $X = \{x_1, \dots, x_n\}$  a finite space of  $n$  elements and by  $\mathbf{U}_i$  the smallest open set of  $X$  containing the point  $x_i$ ,  $i = 1, \dots, n$ . Also, we denote by  $\omega$  the first infinite cardinal. The space  $X$  is  $T_0$  if and only if  $\mathbf{U}_i = \mathbf{U}_j$  implies  $x_i = x_j$  for every  $i, j$  (see [1]). Let  $X = \{x_1, \dots, x_n\}$  be a finite space of  $n$  elements. The  $n \times n$  matrix  $T = (t_{ij})$ , where  $t_{ij} = 1$  if  $x_i \in \mathbf{U}_j$  and  $t_{ij} = 0$  otherwise, is called the *incidence matrix* of  $X$  (see [4]). We observe that  $\mathbf{U}_j = \{x_i : t_{ij} = 1\}$ ,  $j = 1, \dots, n$ . For the following notions see for example [4]. Let  $X$  be a space. A *cover* of  $X$  is a non-empty set of subsets of  $X$ , whose union is  $X$ . A family  $r$  of subsets of  $X$  is said to be a *refinement* of a family  $c$  of subsets of  $X$  if each element of  $r$  is contained in an element of  $c$ . Define the *order* of a family  $r$  of subsets of a space  $X$  as follows:

- (a)  $\text{ord}(r) = -1$  if and only if  $r$  consists the empty set only.
- (b)  $\text{ord}(r) = k$ , where  $k \in \omega$ , if and only if the intersection of any  $k + 2$  distinct elements of  $r$  is empty and there exist  $k + 1$  distinct elements of  $r$ , whose intersection is not empty.
- (c)  $\text{ord}(r) = \infty$ , if and only if for every  $k \in \omega$  there exist  $k$  distinct elements of  $r$ , whose intersection is not empty. We denote by  $\text{dim}$  the function, calling *covering dimension*, with as domain the class of all spaces and as range the set  $\omega \cup \{-1, \infty\}$ ,

satisfying the following condition:  $\dim(X) \leq k$ , where  $k \in \{-1\} \cup \omega$  if and only if for every finite open cover  $c$  of the space  $X$  there exists a finite open cover  $r$  of  $X$ , refinement of  $c$ , such that  $\text{ord}(r) \leq k$ .

In this paper we study the covering dimension for finite topological spaces. In particular, we give an algorithm for computing the covering dimension of a finite space  $X$  using the notion of the incidence matrix of  $X$ .

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*Zohreh Jafari*

## **Bi-invariant metrics and Ricci flow**

Each compact Lie group has a bi-invariant metric and there is 1-1 corresponding between all the bi-invariant metrics and *ad*-invariant scalar products on Lie algebra. On the other hand, Killing form is *ad*-invariant and therefore every compact semi-simple Lie group has a bi-invariant metric which is the same minus Killing form. This metric is obviously Einstein and according to Wolf theorem all the bi-invariant metrics on compact Lie groups are unique up to scaling and as consequence fixed points of Ricci Flow. In this paper, we focus on uncompact Lie groups. For example,  $SL(n, R)$  doesn't have any bi-invariant metric. To be precise, its bi-invariant metrics are not positive definite. In this case, we calculate components of Ricci Curvature of semi-metrics and obtain the Ricci flow equation which is a PDE and solvable in some special cases.

*Nikolaos Kadianakis and Fotis Travlopanos*

## **On the kinematics of hypersurfaces**

We study the kinematics of a hypersurface in a Riemannian manifold, producing evolution equations of geometric objects such as the metric, the unit normal, the shape operator etc. For this, we use the polar decomposition theorem which has been traditionally used in Continuum Mechanics where it is providing a clear geometrical description of a deforming body, by introducing suitable kinematical quantities. We present variation formulas concerning the most general type of motion and containing either geometrical or kinematical quantities.

*Ioannis Kaffas*

### **Σχετική εικόνα ευθαιογενούς με ειδική καθετοποίηση**

Στον Ευκλείδειο χώρο  $E^3$  μελετούμε τη σχετική εικόνα  $\Phi^*$  μιας ευθαιογενούς επιφάνειας  $\Phi$ , που αντιστοιχεί σε μια συνάρτηση στήριξης της μορφής  $q = \frac{f(u)}{w}$ , όπου  $u$  είναι φυσική παράμετρος της σφαιρικής εικόνας των γενετειρών,  $f(u)$  συνάρτηση της κλάσεως διαφορισιμότητας  $C^2$  και  $w^2$  η ορίζουσα του μετρικού τανυστή της  $\Phi$ . Βρίσκουμε αναλλοιώτους της  $\Phi^*$  και το διανυσματικό πεδίο του Tchebychev, με τη βοήθεια του οποίου μελετούμε ιδιότητες της  $\Phi$  σε σχέση με διακεκριμένες οικογένειες καμπυλών, όπως οι ασυμπτωτικές γραμμές, οι γραμμές καμπυλότητας, κ.ά. Η σχετική εικόνα  $\Phi^*$  είναι επίσης μια ευθαιογενής επιφάνεια, και μάλιστα οι γενέτειρές της είναι παράλληλες σε εκείνες της  $\Phi$ . Θεωρούμε σχετική καθετοποίηση της  $\Phi^*$  της ίδιας μορφής. Συνεχίζοντας έτσι, δημιουργείται μία ακολουθία επιφανειών, τις οποίες συσχετίζουμε με τη βοήθεια των θεμελιωδών αναλλοιώτων τους.

*Anastasios Kartsaklis*

### **Γεωμετρία και φυσική**

Κλασική Μηχανική (Newton, Lagrange, Hamilton). Ειδική Θεωρία της Σχετικότητας (Voigt, Lorentz, Poincare, Einstein). Γενική Θεωρία της Σχετικότητας. Κβαντομηχανική. Χωροχρόνοι. Εξισώσεις του πεδίου βαρύτητας των Hilbert-Einstein. Γνωστές λύσεις αυτών. Το καθιερωμένο πρότυπο. Θεωρία υπερχορδών. M-Theory. Έννοια των συμβάντων κατά τον Αριστοτέλη. Κβαντική υπολογιστική επεξεργασία (Quantum computing).

*Nikos Lampropoulos*

### **The influence of the geometry on Sobolev type inequalities on compact Riemannian manifolds**

In this article we study the most interesting aspects of the Sobolev inequalities from the Geometrical point of view. By developing particular geometrical proper-

ties of the manifold, we can calculate the precise values of the best constants in the presented Sobolev inequalities. This result of this analysis represents an improvement over the classic analysis and allows us to prove the existence of solutions for elliptic differential equations of scalar curvature of the generalized type with supercritical exponents. In the first part of the lecture, I will provide a review of the history and the development of Sobolev type inequalities, presenting the most interesting examples and discuss the critical role of geometry in their solution as well as their applications. In the second part, I will present new results concerning Sobolev inequalities, Nash inequalities and Logarithm Sobolev inequalities where the exponents are in the critical of the supercritical case. The lecture is completed with the resolution of the famous problem  $\Delta u + |u|^{4/(n-2)}u = 0, u \in C^2(\mathbb{R}^n), n \geq 3$ , that is founded in Mathematical Physics.

*Dionysios Lappas*

### **Δίσκοι, συμπλεκτικοί δίσκοι και αποτελέσματα ανυψώσεων**

Η κύρια λειτουργία του δίσκου (slice) έγκειται στην τοπική αναγωγή της δράσης, σε δράση μιας συμπαγούς ομάδας στο δίσκο. Η ύπαρξη δίσκων αποδείχθηκε πολύ νωρίς για γνήσιες δράσεις συμπαγών ομάδων Lie και στη συνέχεια ο Palais (1961) το απέδειξε γενικά για γνήσιες δράσεις ομάδων Lie. Έκτοτε, δίσκοι αναζητήθηκαν και επετεύχθησαν σε μια ποικιλία περιπτώσεων, όπως π.χ. σε συμπλεκτικές δράσεις. Η τοπική ύπαρξη δίσκων και η εξάρτηση από τις ομάδες ισοτροπίας κάνει το χειρισμό των δίσκων δύσκολο όταν μελετώνται ολικά προβλήματα. Σε ειδικές περιπτώσεις, όπως ανυψώσεις δράσεων σε χώρους επικάλυψης ή στην εφαπτομένη (συνεφαπτομένη) δέσμη, υπάρχουν ικανοποιητικά αποτελέσματα, με μερικά από τα οποία και τις εφαρμογές τους θα ασχοληθούμε στην παρούσα επισκόπηση.

*Mixalis Marias*

### **Analysis on locally symmetric spaces**

*Romanos Malikiosis*

### **Επεκτάσεις των θεωρημάτων του Minkowski στα διαδοχικά ελάχιστα**

Στην ομιλία αυτή θα παρουσιαστούν μερικές επεκτάσεις των θεωρημάτων του Minkowski. Θα αναλυθούν τα αποτελέσματα διακριτών αναλόγων αυτών των θεωρημάτων, που πρώτα διατυπώθηκαν από τους Betke, Henk, και Wills το 1993, όπου ο όγκος αντικαθίσταται από το πλήθος των σημείων με ακέραιες συντεταγμένες. Θα περιγραφεί η απόδειξη στις τρεις διαστάσεις, όπως και η απόδειξη μιας ασθενέστερης ανισότητας στην γενική περίπτωση (αποδεδειγμένα από τον ομιλητή).

*Stelios Markatis*

### **A model for the set of spherical triangles classifies cubic harmonics**

The action of the orthogonal group  $O(3)$  on a configuration of the unit sphere, consisting of three great circles, generates orbits containing all equal configurations. In this work, it is proved that each orbit contains a unique, up to equality, spherical triangle having sum of its two longest sides less than or equal to  $\pi$ . This triangle is chosen as a representative of its orbit and a tetrahedron is constructed as a model for the set containing all these triangles. The vertices, the edges and the faces of the tetrahedron correspond to special spherical triangles, including the degenerate ones, while its interior points correspond to generic triangles. As an application, the cubic harmonic polynomials, defined in  $\mathbb{R}^3$ , are classified by associating the Maxwell's poles of the harmonics to the vertices of a spherical triangle of the model.

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*Michael Markellos*

### **The bienergy of unit vector fields**

The bienergy of a unit vector field  $V$  on a Riemannian manifold  $(M, g)$  is defined to be the bienergy of the mapping  $V : (M, g) \mapsto (T_1M, g_S)$ , where the unit tangent sphere bundle  $T_1M$  is equipped with the Sasaki metric  $g_S$ . Especially, we determine the Euler-Lagrange equation, that is the critical point condition, for the variational problem related to the bienergy functional  $E_2$  restricted to unit vector fields. We say that a unit vector field  $V$  is *biharmonic* if and only if the corresponding map is a critical point for the bienergy functional  $E_2$ , only considering variations among maps defined by unit vector fields. Furthermore, we prove that a unit vector field  $V : (M, g) \mapsto (T_1M, g_S)$  defines a biharmonic map if

and only if it is a biharmonic vector field and it is satisfied an additional condition which involves the Riemann curvature tensor of  $(M, g)$  and its covariant derivative. In the sequel, we completely determine biharmonic invariant unit vector fields in three dimensional unimodular Lie groups and the generalized Heisenberg groups  $H(1, r), r \geq 2$ , equipped with a left-invariant metric. In the case of three dimensional non-unimodular Lie groups, we provide examples of biharmonic invariant unit vector fields. Finally, we construct examples of biharmonic unit vector fields on the  $n$ -dimensional Poincaré half - space  $H^n$  by using the notions of homogeneous structures and infinitesimal models.

*Evangelia Moutafi*

### **Curvature of $(\kappa, \mu, \nu)$ - contact metric 3-manifolds and Ricci solitons**

(Joint work with F. Gouli-Andreou). 1) We study curvature conditions for 3-dimensional  $(\kappa, \mu, \nu)$ -contact manifolds, i.e.  $R(X, Y)\xi = \kappa(\eta(Y)X - \eta(X)Y) + \mu(\eta(Y)hX - \eta(X)hY) + \nu(\eta(Y)\phi hX - \eta(X)\phi hY)$  ( $\kappa, \mu, \nu$  smooth functions), when the Ricci tensor  $S$  of  $M$  is cyclic-parallel, i.e.  $(\nabla_Z S)(X, Y) + (\nabla_X S)(Y, Z) + (\nabla_Y S)(Z, X) = 0$ , or  $\eta$ -parallel, i.e.  $(\nabla_Z S)(\phi X, \phi Y) = 0$  for all vector fields  $X, Y, Z$ .

2) As part of a common research with Prof. J. Inoguchi of Yamagata University of Japan, we study:

a)  $(M^3, \varphi, \xi, \eta, g)$   $(\kappa, \mu, \nu)$ - contact metric Ricci solitons  $\tau + 2Ric + 2mg = 0$ ,  $\tau = L_\xi g$  ( $m$  constant), and

b)  $(M^3, \varphi, \xi, \eta, g)$  contact metric  $\eta$ - Ricci solitons  $\tau + 2Ric + 2\mu g + 2\nu\eta \otimes \eta = 0$ ,  $\tau = L_\xi g$  ( $\mu, \nu$  constants) to be pseudo-symmetric in the sense of R. Deszcz.

*Konstantina Panagiotidou and Philippos Xenos*

### **Real hypersurfaces in $\mathbb{C}P^2$ and $\mathbb{C}H^2$**

A complex  $n$ -dimensional Kaehler manifold of constant holomorphic sectional curvature  $c$  is called a **complex space form**, which is denoted by  $M_n(c)$ . A complete and simply connected complex space form is complex analytically isometric to a complex projective space  $\mathbb{C}P^n$ , a complex Euclidean space  $\mathbb{C}^n$  or a complex hyperbolic space  $\mathbb{C}H^n$  if  $c > 0, c = 0$  or  $c < 0$  respectively. A **real hypersurface**  $M$  is a submanifold of  $\mathbb{C}P^n$  or  $\mathbb{C}H^n$ , which codimension is equal to 1. It is called **Hopf hypersurface**, when its structure vector field  $\xi$  is principal, i.e.  $A\xi = \alpha\xi$ , where  $A$  is the shape operator of  $M$  and  $\alpha$  is a smooth function. The classification problem of real hypersurfaces was initiated by Takagi, who classified homogeneous real hypersurfaces in  $\mathbb{C}P^n$ , ( $n \geq 2$ ). Cecil and Ryan showed that they can be regarded as tubes of constant radius over Kaehlerian submanifolds, when they are Hopf

hypersurfaces. Real hypersurfaces in  $\mathbb{C}H^n, (n \geq 2)$ , were investigated by Berndt, who showed that those with constant principal curvatures are realized as tubes of constant radius over certain submanifolds, when they are Hopf hypersurfaces. Many authors have studied real hypersurfaces whose structure Jacobi operator  $l$ , ( $l = R(\cdot, \xi)\xi$ , where  $R$  is the Riemannian curvature tensor), satisfies certain conditions. They were led to either classification or non-existence of them. We present two results in the classification of real hypersurfaces in  $\mathbb{C}P^2$  and  $\mathbb{C}H^2$ . First, we classify real hypersurfaces whose structure Jacobi operator is  $\xi$ -parallel, ( $(\nabla_\xi l)X = 0$ , for any vector field  $X$ ). Secondly, we give the classification of real hypersurfaces whose structure Jacobi operator is cyclic parallel ( $\mathfrak{S}g((\nabla_X l)Y, Z) = 0$ , for any vector field  $X, Y$  and  $Z$ ). In both cases we get the following types of real hypersurfaces: if  $\alpha \neq 0$

- in  $\mathbb{C}P^2$ , a tube of radius  $r$  over a hyperplane  $\mathbb{C}P^1$ , where  $0 < r < \frac{\pi}{2}, r \neq \frac{\pi}{4}$
- in  $\mathbb{C}H^2$ , ( $A_0$ ) a horosphere in  $\mathbb{C}H^2$ , i.e. a Montiel tube,  
( $A_1$ ) a geodesic sphere and a tube over the hyperplane  $\mathbb{C}H^1$ .

*Ioannis Papadoperakis*

**Οι ομοιομορφισμοί στο χώρο των γεωδαισιακών laminations και οι ομοιομορφισμοί μιας επιφάνειας**

Έστω  $S$  επιφάνεια με μια υπερβολική μετρική  $d$ . Μια γεωδαισιακή lamination  $\Lambda \subset S$  είναι ένα συμπαγές υποσύνολο του  $S$  που αποτελείται από απλές γεωδαισιακές ξένες ανα δύο. Συμβολίζουμε με  $\mathcal{GL}(S)$  το σύνολο των γεωδαισιακών laminations. Στο  $\mathcal{GL}(S)$  θεωρούμε την μετρική Hausdorff  $d_H$  που επάγεται από την  $d$ . Είναι άμεσο ότι κάθε ομοιομορφισμός  $h : S \rightarrow S$  επάγει κατά φυσικό τρόπο έναν ομοιομορφισμό  $h_* : \mathcal{GL}(S) \rightarrow \mathcal{GL}(S)$ . Εμείς δείχνουμε το ακόλουθο: Για κάθε ομοιομορφισμό  $f : \mathcal{GL}(S) \rightarrow \mathcal{GL}(S)$  υπάρχει ένας ομοιομορφισμός  $h : S \rightarrow S$  έτσι ώστε  $h_* = f$ .

*Saima Parveen*

**Braid groups in complex projective spaces**

We describe the fundamental groups of ordered and unordered  $k$ -point sets in  $CP^n$  generating a projective subspace of dimension  $i$ . We apply these to study connectivity of more complicated configurations of points.

*Ioannis Platis*

**Methods for quasiconformal mappings of the Heisenberg group**

A method based on the moduli of domains in the Heisenberg group is adopted

to solve problems of extremality of the distortion and the mean distortion for families of quasiconformal mappings. This is a joint work with Zoltán M. Balogh and Katrin Fässler, University of Bern.

*Andreas Savas-Halilaj*

### **Rigidity of minimal hypersurfaces**

A classical result of Beez-Killing states that a hypersurface in the Euclidean space is rigid if its type number, i.e., the rank of its Gauss map, is at least 3. Hence, the interesting case is that of hypersurfaces with type number less than three. Inspired by works of V. Sbrana and E. Cartan, M. Dajczer and D. Gromoll proved that, at least locally, every hypersurface in a Euclidean space can be described in terms of its Gauss map. This kind of description is called the “Gauss parametrization” and has interesting applications in the study of rigidity problems for hypersurfaces with type number 2. However, the Gauss parametrization provides a description of the hypersurface at neighborhoods where the rank of the Gauss map is constant and there is no discussion on the manner in which hypersurfaces with different type numbers can be produced or joined together. In this talk, I shall present some new results about the classification of minimal hypersurfaces with type number less than 3 in a 4-dimensional Euclidean space.

*Tanush Shaska*

### **Theta functions of algebraic curves**

Theta functions are a classical area of mathematics which have been studied extensively in the 19-th century. The topic has had renewed interest in the last few decades due to advances of computational algebraic geometry, and the use of theta functions of algebraic curves in cryptography.

In this talk we will discuss some new results in theta nulls of cyclic algebraic curves and discuss theta nulls of non-hyperelliptic genus three curves.

*Stylianos Stamatakis*

### **Περί του διανυσματικού πεδίου του Tchebychev στη σχετική διαφορική γεωμετρία**

Ασχολούμαστε με σχετικά καθετοποιημένες υπερεπιφάνειες στον Ευκλείδειο χώρο  $\mathbb{R}^{n+1}$ . Θεωρούμε μια σχετική καθετοποίηση  $\bar{y}$  μιας υπερεπιφάνειας  $\Phi$  και αναλύουμε το διάνυσμα Tchebychev της  $\Phi$ , που αντιστοιχεί στην  $\bar{y}$ , σε δυο συνιστώσες (όχι απαραίτητα γραμμικά ανεξάρτητες), εκ των οποίων η μια είναι παράλληλη στο διάνυσμα του Tchebychev της Ευκλείδειας καθετοποίησης και το άλλο παράλληλο

στην ορθή προβολή του  $\bar{y}$  πάνω στο εφαπτόμενο επίπεδο της  $\Phi$ . Με τη βοήθεια της ανάλυσης αυτής μελετούμε ιδιότητες της  $\Phi$ , που σχετίζονται με την καμπυλότητα του Gauss, τη σχετική καθετοποίηση, το διανυσματικό πεδίο του Tchebychev, κ.ά.

*Hiroshi Tamaru*

### **Parabolic subgroups and geometry of solvable Lie groups**

The solvable parts of parabolic subgroups of semisimple Lie groups provide a rich sources of geometry of noncompact homogeneous manifolds. In this talk, I will talk about Einstein solvmanifolds, cohomogeneity one actions on symmetric spaces of noncompact type, and polar actions.

*Theoharis Theofanidis and Philippos Xenos*

### **Real hypersurfaces in complex space forms in terms of the Jacobi structure operator**

The study of real hypersurfaces has been an active field over the last decades. It is impossible to accomplish a classification of real hypersurfaces without another condition because of the complicated differential equations that arise. Our research focuses on the **Jacobi structure operator**:  $lX = R(X, \xi)\xi$  where  $R$  is the curvature Riemann of real hypersurface. This operator has been studied by many researchers and many points of view. In our study we have the following results:

- 1) There exist no real hypersurfaces in non - flat complex space forms  $M_n(c)$  ( $n \geq 2$ ) with **recurrent Jacobi structure operator**:  $(\nabla_X l)Y = \omega(X)lY$ , where  $X, Y$  are vector fields on  $M$  and  $\omega$  is a 1-form.
- 2) There exist no real hypersurfaces in  $M_n(c)$ ,  $n \geq 2$  equipped with **Jacobi structure operator of Codazzi type**:  $(\nabla_X l)Y = (\nabla_Y l)X$ , where  $X, Y$  are vector fields on  $M$ .
- 3) Let  $M$  be a real hypersurface of a complex space form  $M_n(c)$ , ( $n > 2$ ) ( $c \neq 0$ ), satisfying  $\phi l = l\phi$ . If  $\nabla_\xi l = \mu\xi$  on  $\ker(\eta)$  or on  $\text{span}\{\xi\}$ , then  $M$  is a Hopf hypersurface. Furthermore, if  $\eta(A\xi) \neq 0$ , then  $M$  locally congruent to a model space of type A.
- 4) Let  $M$  be a real hypersurface of a complex space form  $M_n(c)$ , ( $n > 2$ ) ( $c \neq 0$ ), satisfying  $\phi l = l\phi$ . If  $lA = Al$  on  $\ker(\eta)$  or on  $\text{span}\{\xi\}$ , then  $M$  is a Hopf hypersurface. Furthermore, if  $\eta(A\xi) \neq 0$ , then  $M$  locally congruent to a model space of type A.

*Georgios Tsapogas*

### **On the mapping class group of a Heegaard splitting**

The Mapping Class group of a Heegaard splitting in a 3–manifold  $M$  consists of the isotopy classes of orientation preserving homeomorphisms of  $M$  that preserve the Heegaard splitting. The mapping class group of a Heegaard splitting is known to be finitely presented ([Akb], [Cho], [Sch]) only for  $M = S^3$  and for a genus 2 Heegaard splitting. In aiming to examine the corresponding open questions for Heegaard splittings of genus  $\geq 3$  in  $S^3$  as well as for certain classes of hyperbolic 3–manifolds  $M$ , we define a complex  $\mathcal{I}(M)$  which, endowed with the combinatorial metric, is shown to be Gromov hyperbolic. This combinatorial complex encodes the complexity of the mapping class group, namely, it is shown that the group of automorphisms of  $\mathcal{I}(M)$  is isomorphic with the mapping class group of the Heegaard splitting of  $M$ .

[Akb] E. Akbas, *A presentation for the automorphisms of the 3-sphere that preserve a genus two Heegaard splitting*, Pacific J. Math. 236 (2008), no. 2, 201–222.

[Cho] S. Cho, *Homeomorphisms of the 3-sphere that preserve a Heegaard splitting of genus two*, Proc. Amer. Math. Soc. 136 (2008), no. 3, 1113–1123.

[Sch] M. Scharlemann, *Automorphisms of the 3-sphere that preserve a genus two Heegaard splitting*, Bol. Soc. Mat. Mexicana (3) 10 (2004), Special Issue, 503–514.

*Charalambos Tsihlias*

### **Locally classification of contact metric manifolds**

In the present paper all contact metric structures are constructed locally. Also as an application of this construction, K-contact manifolds are locally classified.