Statistical Time Series Methods for Structural Health Monitoring

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SHM–In–Action Course
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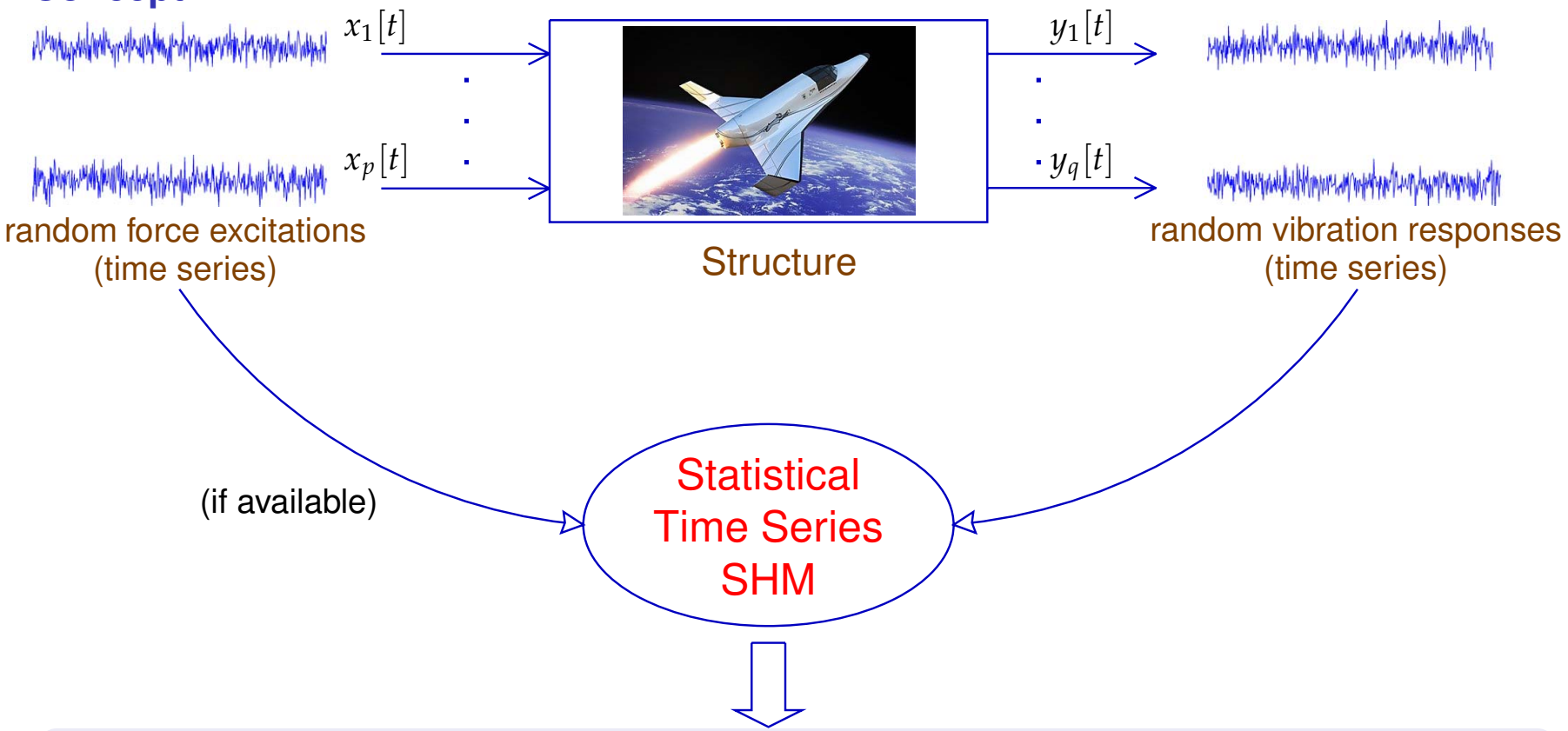
Talk Outline

1. Introduction
2. Statistical Time Series Models: Overview of Concepts
3. Non–Parametric Statistical Time Series SHM Methods
4. Parametric Statistical Time Series SHM Methods
5. Damage Detection and Identification in an Aluminum Truss Structure
6. Recent Developments
7. Conclusions

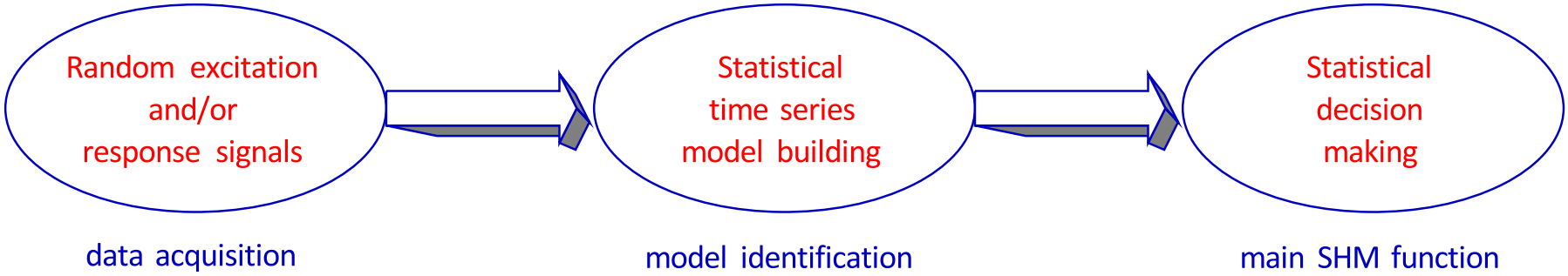
1. Introduction

1.1 Concept



- Level 1 subproblem: Fault detection *(fault presence)*
- Level 2 subproblem: Faut identification *(characterization – localization)*
- Level 3 subproblem: Fault estimation *(fault size)*
- Level 4 subproblem: Remaining life estimation

1.2 What are statistical time series SHM methods?



Why use random signals?

- Actual excitation is often random (buildings under earthquake excitation, aircraft, surface vehicles, etc)
- Natural operating environment (no need to interrupt normal operation)
- They are generally rich in frequency content

What is a statistical time series model?

- A model identified from the available random signal(s) that describes their time evolution and part of the structural dynamics (data based model).

Examples:

- Power Spectral Density (PSD)
- ARMA model
- statistical Neural Network type model

Observation: No physics based model (like finite element model) or complete structural dynamics model needed.

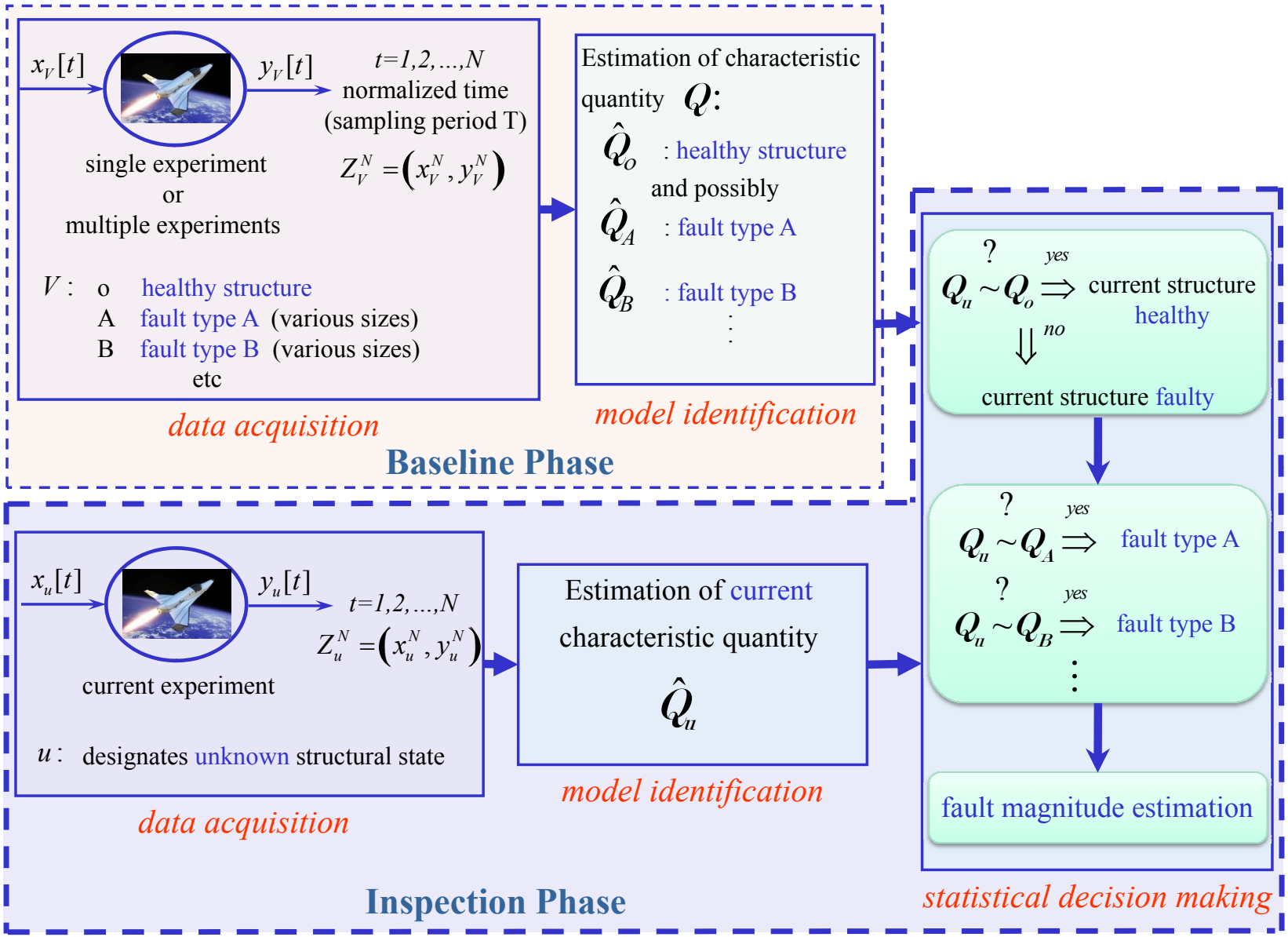
What is statistical decision making?

- Optimal decision making under uncertainty

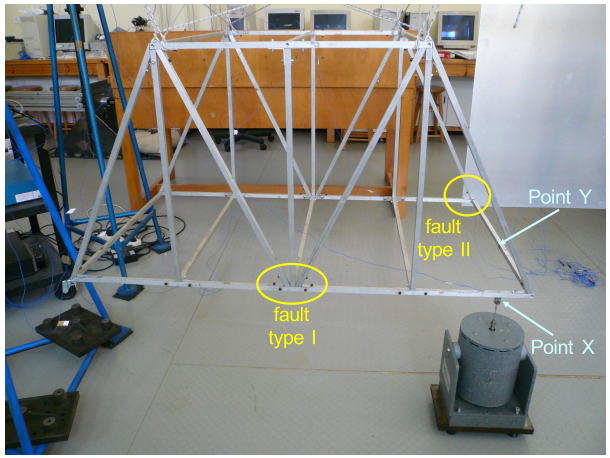
Uncertainty: measurement, environmental, operational, excitation, structural, etc

1.3 How does a typical statistical time series SHM method operate?

The workframe



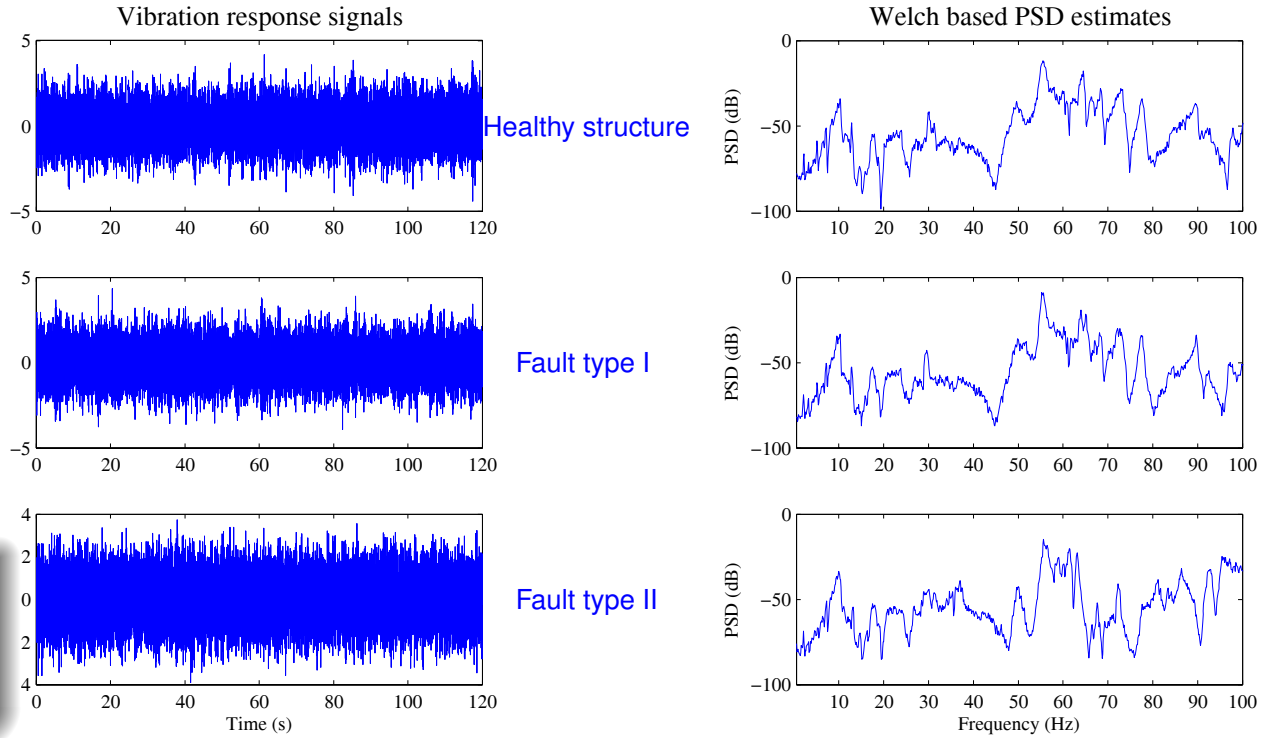
1.4 An example: Fault detection and identification in a laboratory truss structure – Baseline Phase



The structure, fault types and the experimental set-up.

$f_S = 256$ Hz BWD: $[0.5 - 100]$ Hz
signal length: $N = 30720$ samples (120 s)

segment length $L = 2048$ samples
non-overlapping segments $K = 15$
window: Hamming

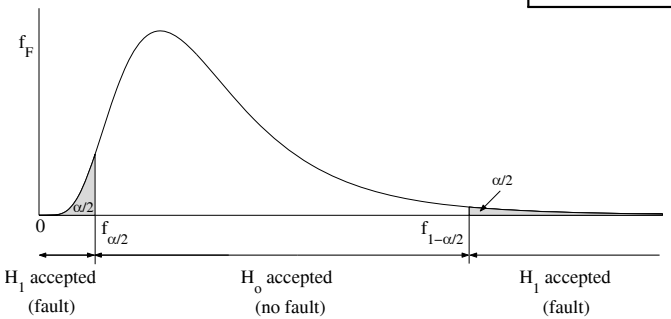


The Structure: Aluminum truss structure $(140 \times 70 \times 80\ mm)$
Faults: Loosening of bolts (types I and II)
Measurements: force (point X) – strain (point Y)

$$F = \frac{\widehat{S}_o(\omega)}{\widehat{S}_u(\omega)} \sim F \text{ distribution under } H_o$$

$$H_o : S_u(\omega) = S_o(\omega) \text{ (healthy structure)}$$
$$H_1 : S_u(\omega) \neq S_o(\omega) \text{ (faulty structure)}$$

Response-only based SHM
$Q = S(\omega)$ response PSD
(Power Spectral Density)

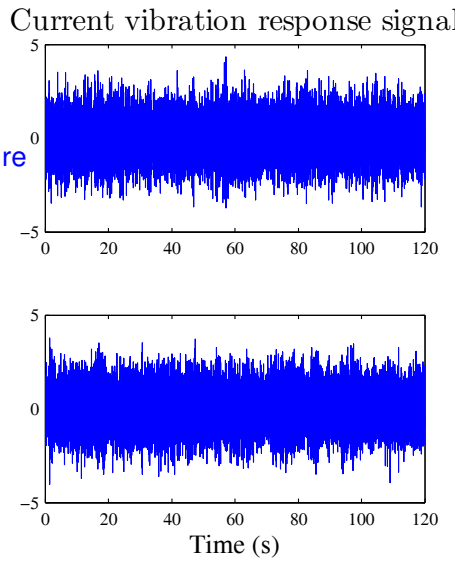


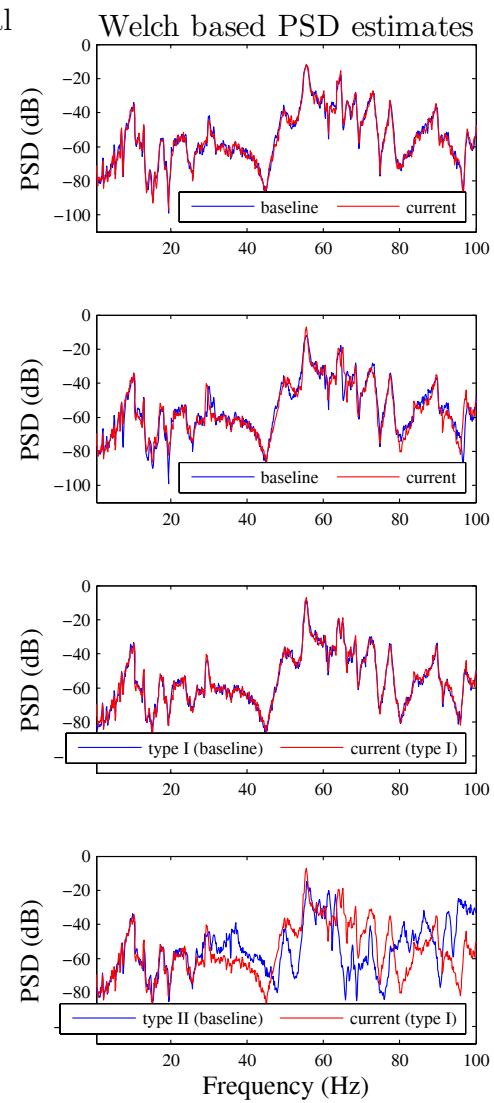
α : type I risk probability
(false alarm probability)

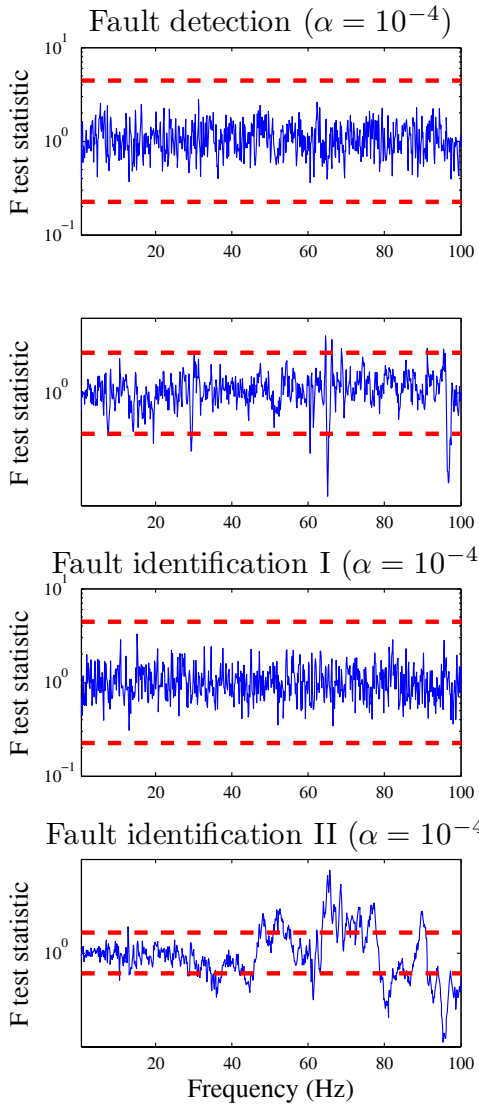
Inspection Phase

Test case I: healthy structure

Test case II: fault type I







$$H_0 : S_u(\omega) = S_0(\omega)$$
$$H_1 : S_u(\omega) \neq S_0(\omega)$$

$$F = \frac{\widehat{S}_0(\omega)}{\widehat{S}_u(\omega)}$$

$$H_0 : S_u(\omega) = S_I(\omega)$$
$$H_1 : S_u(\omega) \neq S_I(\omega)$$

$$F = \frac{\widehat{S}_I(\omega)}{\widehat{S}_u(\omega)}$$

$$H_0 : S_u(\omega) = S_{II}(\omega)$$
$$H_1 : S_u(\omega) \neq S_{II}(\omega)$$

$$F = \frac{\widehat{S}_{II}(\omega)}{\widehat{S}_u(\omega)}$$

1.5 Classes of statistical time series SHM methods

$\underbrace{\text{Excitation–response}}_{Q}$ versus $\underbrace{\text{response–only}}_{Q}$

Q: based on both | Q: based on response only

$\underbrace{\text{Non–parametric}}_{Q}$ versus $\underbrace{\text{parametric}}_{Q}$

Q: based on non–parametric model | Q: based on parametric model

$\underbrace{\text{Linear dynamics}}_{Q}$ versus $\underbrace{\text{non–linear dynamics}}_{Q}$

Q: based on linear model | Q: based on non–linear model

$\underbrace{\text{Stationary (time invariant) dynamics}}_{Q}$ versus $\underbrace{\text{non–stationary (time variant) dynamics}}_{Q}$

Q: based on stationary model | Q: based on non–stationary model

1.6 Why use statistical time series SHM methods?

Advantages

- ✓ No need for **physics-based** or **finite element** (FE) models
- ✓ No need for **complete** models
- ✓ Inherent accounting of uncertainty
- ✓ Statistical decision making with specified performance characteristics
- ✓ Effective use of natural random vibration data records (no need to interrupt normal operation)

Additional benefits (all vibration based methods)

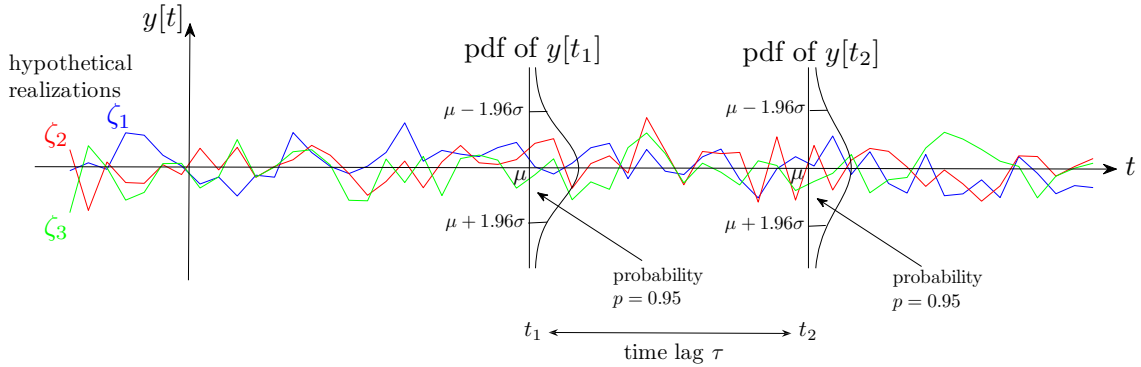
- No need for visual inspection
- "Global" coverage of the structure
- Time and cost effectiveness
- "Automation" capability
- Potential for continuous, real–time, use

Pitfalls

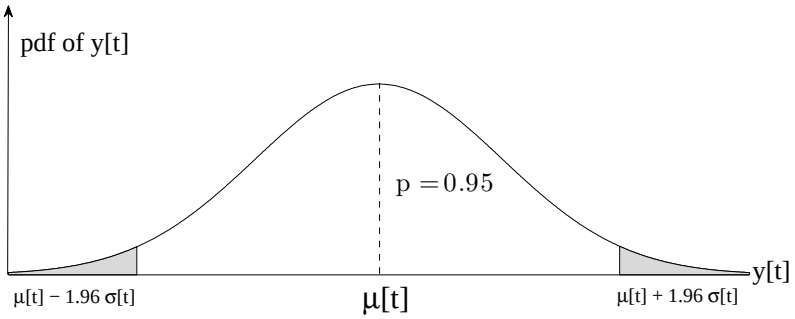
- Faults may be located only to the extent allowed by the type of model used
- Baseline phase requires groundwork under various damage types (fault identification)
- Some user expertise required
- Potential lack of physical insight
- Less sensitive than "local" non–destructive testing type methods

2. Statistical Time Series Models: Overview of Concepts

2.1 Non–parametric models



One–dimensional Gaussian pdf

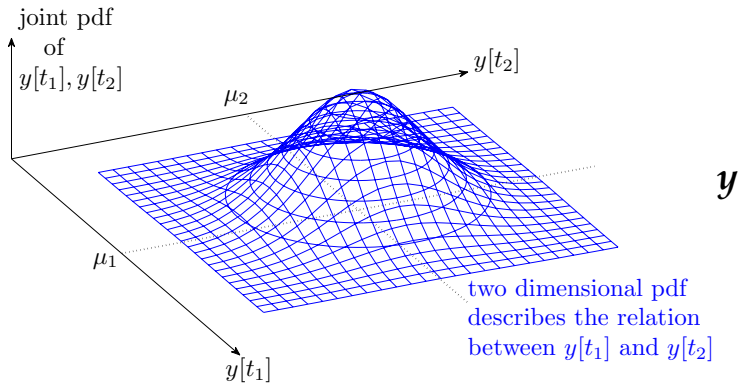


$$f_y(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left\{ -\frac{(y-\mu)^2}{2\sigma^2} \right\}$$

$\mu[t] = \mu$: mean

$\sigma^2[t] = \sigma^2$: variance

Two–dimensional & higher–dim pdfs



$$\boldsymbol{y} = \begin{bmatrix} y[t_1] \\ y[t_2] \end{bmatrix}$$

$$f_y(\boldsymbol{y}) = \frac{1}{\sqrt{(2\pi)^2 \det(\boldsymbol{\Gamma})}} \cdot \exp\left\{ -\frac{1}{2}(\boldsymbol{y}-\boldsymbol{\mu})^T \boldsymbol{\Gamma}^{-1} (\boldsymbol{y}-\boldsymbol{\mu}) \right\}$$

$$\boldsymbol{\mu}[t] = \begin{bmatrix} \mu[t_1] \\ \mu[t_2] \end{bmatrix} \quad \boldsymbol{\Gamma} = Cov[y] = \begin{bmatrix} \gamma[t_1,t_1] & \gamma[t_1,t_2] \\ \gamma[t_2,t_1] & \gamma[t_2,t_2] \end{bmatrix}$$

mean — covariance matrix

Normalized ACF

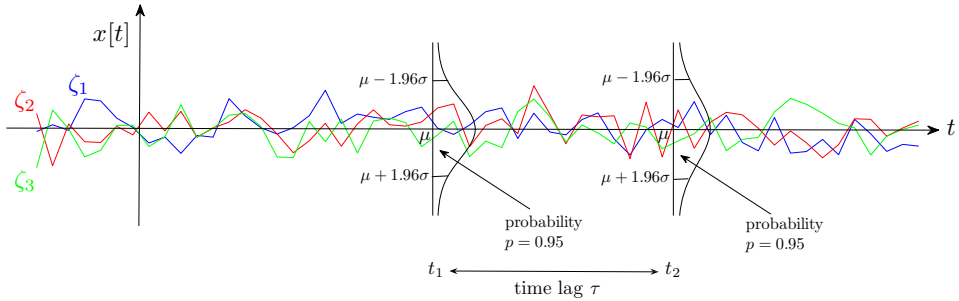
$$\rho[\tau] = \gamma[\tau]/\sigma^2 \qquad |\rho[\tau]| \leq 1$$

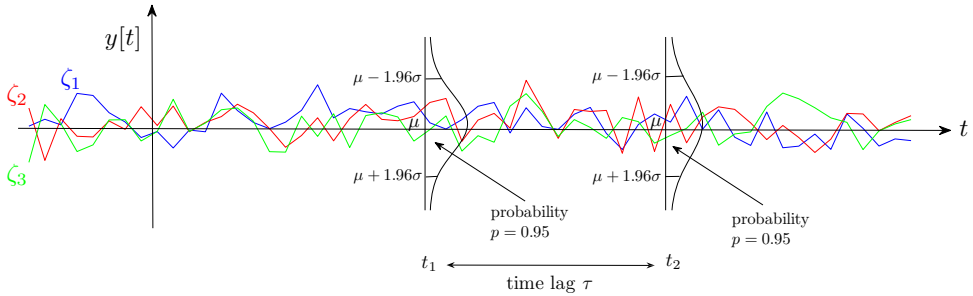
AutoCovariance Function (ACF)

$$\gamma[t_1,t_2] = \gamma[t_1 - t_2] = \gamma[\tau] \qquad \tau : \text{lag}$$

Power Spectral Density (PSD): $S(\omega) = \mathbb{F}\{\gamma[\tau]\} = \sum_{\tau=-\infty}^{+\infty} \gamma[\tau] \cdot e^{-j\omega T\tau}$

T : sampling period





One–dimensional Gaussian pdfs

$$f_x(x), \quad f_y(y)$$

$\mu_x[t] = \mu_x \quad \mu_y[t] = \mu_y$: means

$\sigma_x^2[t] = \sigma_x^2 \quad \sigma_y^2[t] = \sigma_y^2$: variances

Normalized ACF

$$\rho_{xx}[\tau] = \gamma_{xx}[\tau]/\sigma_x^2 \qquad |\rho_{xx}| \leq 1$$

$$\rho_{yy}[\tau] = \gamma_{yy}[\tau]/\sigma_y^2 \qquad |\rho_{yy}| \leq 1$$

$$\rho_{yx}[\tau] = \gamma_{yx}[\tau]/\sigma_y\sigma_x \qquad |\rho_{yx}| \leq 1$$

Two–dimensional & higher–dim pdfs

$$f_x(\boldsymbol{x}), \quad f_y(\boldsymbol{y}), \quad f_{xy}(\boldsymbol{x},\boldsymbol{y})$$

$\gamma_{xx}[t_1,t_2] = \gamma_{xx}[t_1 - t_2] = \gamma_{xx}[\tau]$ ACF

$\gamma_{yy}[t_1,t_2] = \gamma_{yy}[t_1 - t_2] = \gamma_{yy}[\tau]$ ACF

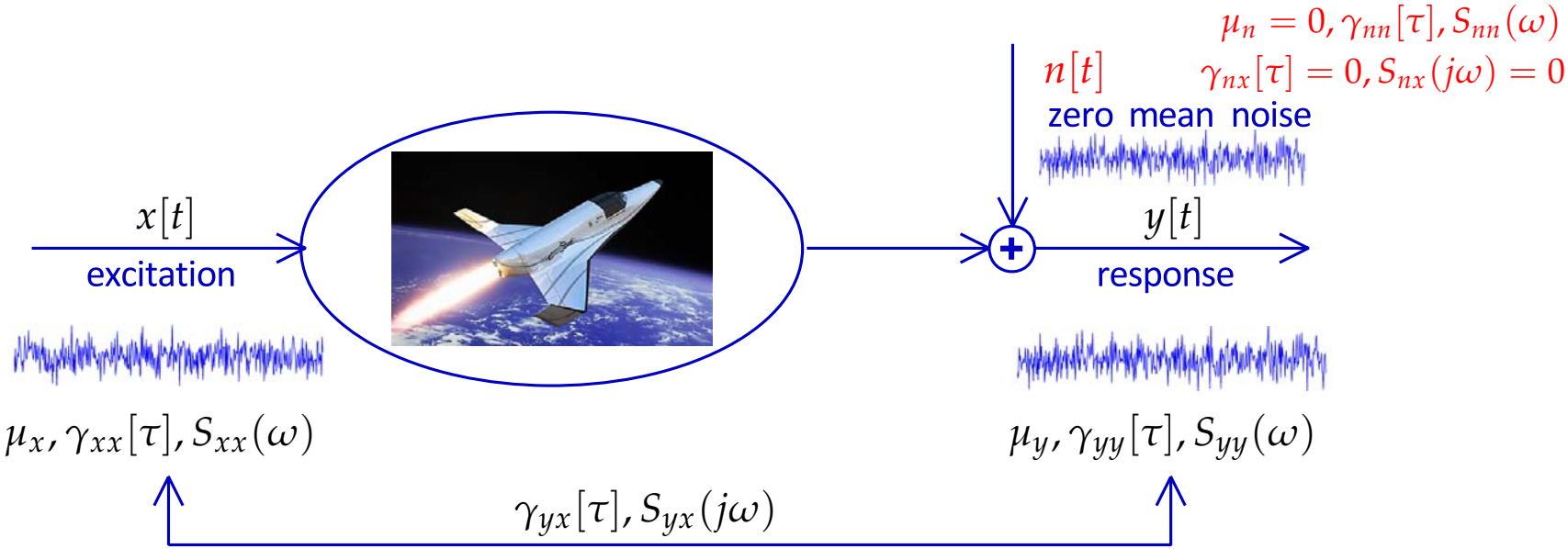
$\gamma_{yx}[t_1,t_2] = \gamma_{yx}[t_1 - t_2] = \gamma_{yx}[\tau]$ CCF

$S_{xx}(\omega) = \mathbb{F}\{\gamma_{xx}[\tau]\}$ PSD

$S_{yy}(\omega) = \mathbb{F}\{\gamma_{yy}[\tau]\}$ PSD

$S_{yx}(j\omega) = \mathbb{F}\{\gamma_{yx}[\tau]\}$ CSD

Relating the ACFs/CCF, PSDs/CSD to the structural dynamics



$$\mu_y = H(j\omega)|_{\omega=0} \cdot \mu_x$$

$$\gamma_{yx}[\tau] = h[\tau] * \gamma_{xx}[\tau] \quad \gamma_{yy}[\tau] = \gamma_{yx}[\tau] * h[-\tau] + \gamma_{nn}[\tau] \Rightarrow \gamma_{yy}[\tau] = h[\tau] * h[-\tau] * \gamma_{xx} + \gamma_{nn}[\tau]$$

$$S_{yx}(j\omega) = H(j\omega)S_{xx}(\omega) \quad S_{yy}(\omega) = H^*(j\omega)S_{yx}(j\omega) + S_{nn}(\omega) \Rightarrow S_{yy}(\omega) = |H(j\omega)|^2 S_{xx}(\omega) + S_{nn}(\omega)$$

Coherence:

$$\gamma^2(\omega) = \frac{|S_{yx}(j\omega)|^2}{S_{xx}(\omega) \cdot S_{yy}(\omega)} = \frac{1}{1 + \frac{S_{nn}(\omega)}{|H(j\omega)|^2 \cdot S_{xx}(\omega)}} \quad \in [0,1]$$

2.2 Linear parametric models

AutoRegressive Moving Average (ARMA) models

$$y[t] + \sum_{i=1}^{na} a_i \cdot y[t-i] = w[t] + \sum_{i=1}^{nc} c_i \cdot w[t-i]$$

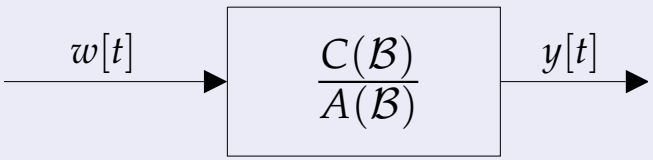
$$w[t] \sim \text{iid } \mathcal{N}(0, \sigma_w^2)$$

ARMA(na, nc) model

or using the backshift operator $\mathcal{B}$ ($\mathcal{B} \cdot y[t] = y[t-1]$):

$$\underbrace{A(\mathcal{B}) \cdot y[t]}_{\text{AR part}} = \underbrace{C(\mathcal{B}) \cdot w[t]}_{\text{MA part}}$$

$$w[t] \sim \text{iid } \mathcal{N}(0, \sigma_w^2)$$



ARMA model

$$A(\mathcal{B}) = 1 + a_1\mathcal{B} + \ldots + a_{na}\mathcal{B}^{na}$$

$$C(\mathcal{B}) = 1 + c_1\mathcal{B} + \ldots + c_{nc}\mathcal{B}^{nc}$$

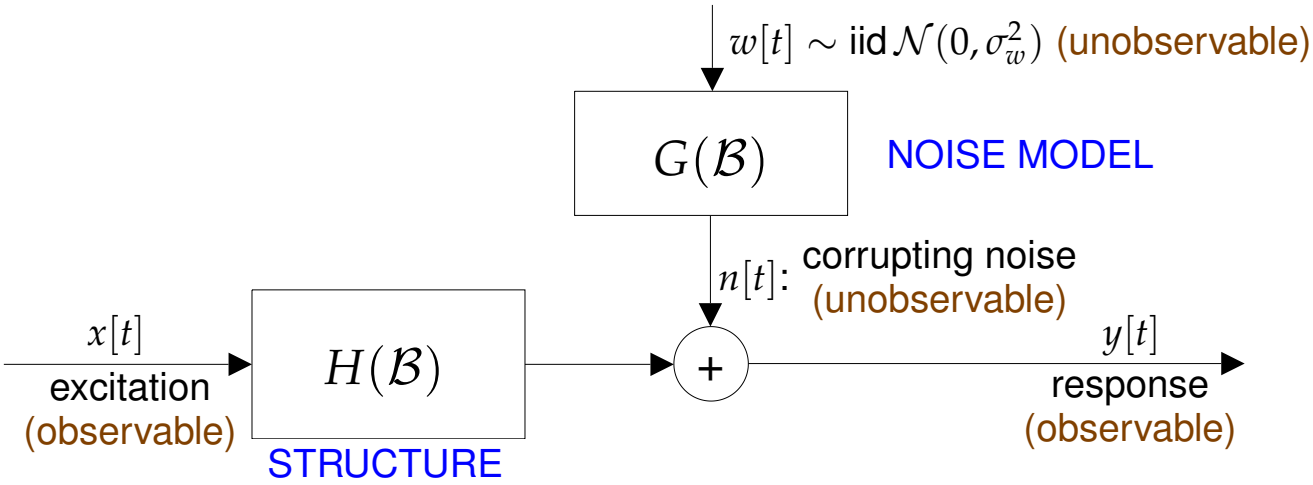
State Space (SS) models

$$\boldsymbol{z}[t+1] = \boldsymbol{A} \cdot \boldsymbol{z}[t] + \boldsymbol{u}[t]$$

$$\boldsymbol{y}[t] = \boldsymbol{C} \cdot \boldsymbol{z}[t] \qquad \boldsymbol{u}[t] \sim \text{iid } \mathcal{N}(0, \boldsymbol{\Gamma}_u)$$

$\boldsymbol{z}[t]$: state vector

Extensions that include exogenous (measurable) excitation



AutoRegressive Moving Average with eXogenous excitation (ARMAX) models

$$A(\mathcal{B}) \cdot y[t] = B(\mathcal{B}) \cdot x[t] + C(\mathcal{B}) \cdot w[t]$$
$$w[t] \sim \text{iid } \mathcal{N}(0, \sigma_w^2)$$

$$B(\mathcal{B}) = b_o + b_1 \mathcal{B} + \ldots + b_{nb} \mathcal{B}^{nb}$$

Box–Jenkins (BJ) models

$$y[t] = \frac{B(\mathcal{B})}{A(\mathcal{B})} \cdot x[t] + \frac{C(\mathcal{B})}{D(\mathcal{B})} \cdot w[t]$$
$$w[t] \sim \text{iid } \mathcal{N}(0, \sigma_w^2)$$

$$D(\mathcal{B}) = 1 + d_1 \mathcal{B} + \ldots + d_{nd} \mathcal{B}^{nd}$$

State Space (SS) models

$$\boldsymbol{z}[t+1] = \boldsymbol{A} \cdot \boldsymbol{z}[t] + \boldsymbol{B} \cdot \boldsymbol{x}[t] + \boldsymbol{u}[t]$$
$$\boldsymbol{y}[t] = \boldsymbol{C} \cdot \boldsymbol{z}[t] + \boldsymbol{D} \cdot \boldsymbol{x}[t] \qquad \boldsymbol{u}[t] \sim \text{iid } \mathcal{N}(0, \boldsymbol{\Gamma}_u)$$

$\boldsymbol{z}[t]$: state vector

2.3 Remarks on model identification

Model structure estimation

Based on goodness–of–fit criteria and optimal (genetic algorithm based) or suboptimal (backward regression) search algorithms.

$$\text{BIC} = -\frac{1}{N}\ln\mathcal{L}(\widehat{\boldsymbol{\theta}}) + \frac{\ln N}{N} \times (\text{model dim})$$

Canonical **C**orrelation **A**nalysis (**CCA**) may be alternatively used.

Model parameter estimation – main estimator families

- **P**rediction **E**rror (**PE**) / **L**east **S**quares (**LS**) methods

$$\widehat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} f(w[t,\boldsymbol{\theta}]) \overset{e.g.}{=} \arg\min_{\boldsymbol{\theta}} \frac{1}{2}\sum_{t=1}^{N} w^2[t,\boldsymbol{\theta}]$$

- **M**aximum **L**ikelihood (**ML**) method

$$\widehat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} \mathcal{L}(\boldsymbol{\theta}) \qquad \mathcal{L}(\boldsymbol{\theta}) \text{ : likelihood function of the data}$$

- **M**ulti–**S**tage (**MS**) methods
- Subspace methods

Estimator property

Under proper conditions: $\sqrt{N}(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}^*) \overset{N\to\infty}{\longrightarrow} \mathcal{N}(0,\boldsymbol{\Gamma})$ $\qquad Cov[\widehat{\boldsymbol{\theta}}] = \frac{1}{N}\boldsymbol{\Gamma}$

$\boldsymbol{\Gamma}$ may be also minimal for certain estimators (Cramer–Rao bound – estimator efficiency).

3. Non–Parametric Statistical Time Series SHM Methods

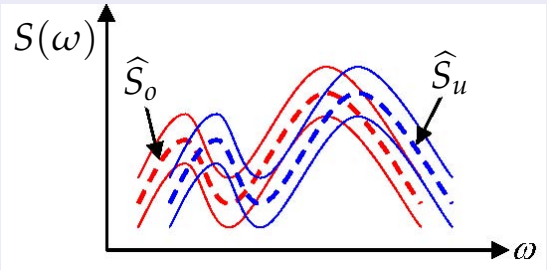
3.1 Power Spectral Density (PSD) based method (response–only case)

(Sakellariou et al. 2001, Liberatore & Carman 2004*, Kopsaftopoulos & Fassois 2008)

Main idea

Fault detection, identification and estimation are associated with changes in the response PSD between the baseline and inspection phases.

characteristic quantity: $Q = S_{yy}(\omega) = S(\omega)$



Baseline Phase

$y_o^N \rightsquigarrow$ signal pre-processing

Welch PSD estimate for healthy structure: $\widehat{S}_o(\omega)$

$$\frac{2K \cdot \widehat{S}_o(\omega)}{S_o(\omega)} \sim \chi^2(2K)$$ *(Kay 1988, p.76)*

Inspection Phase

$y_u^N \rightsquigarrow$ signal pre-processing

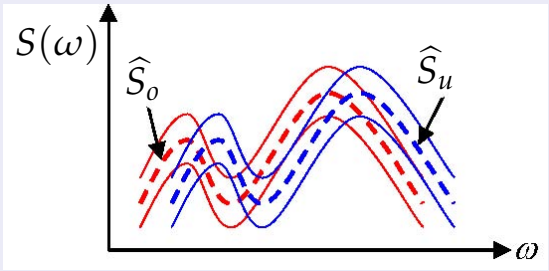
Welch PSD estimate for current structure: $\widehat{S}_u(\omega)$

$$\frac{2K \cdot \widehat{S}_u(\omega)}{S_u(\omega)} \sim \chi^2(2K)$$

Inspection Phase (cont'd)

Is S_u different from S_o?

- no → structure is healthy
- yes → structure is faulty



To answer set up the statistical hypothesis testing problem:

$$H_o : S_u(\omega) = S_o(\omega) \quad \text{null hypothesis – healthy structure}$$
$$H_1 : S_u(\omega) \neq S_o(\omega) \quad \text{alternative hypothesis – faulty structure}$$

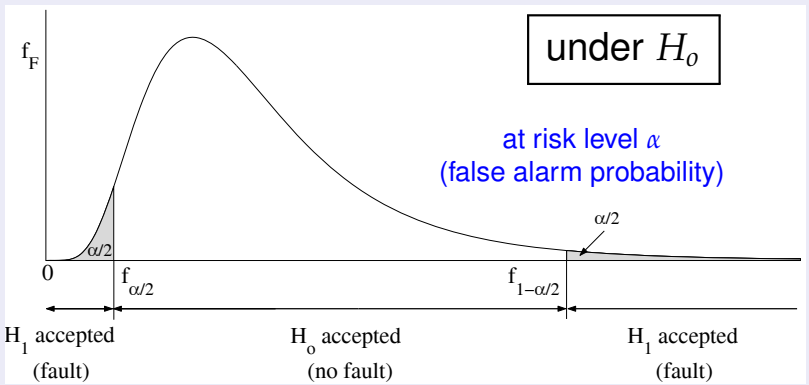
Form the statistic:

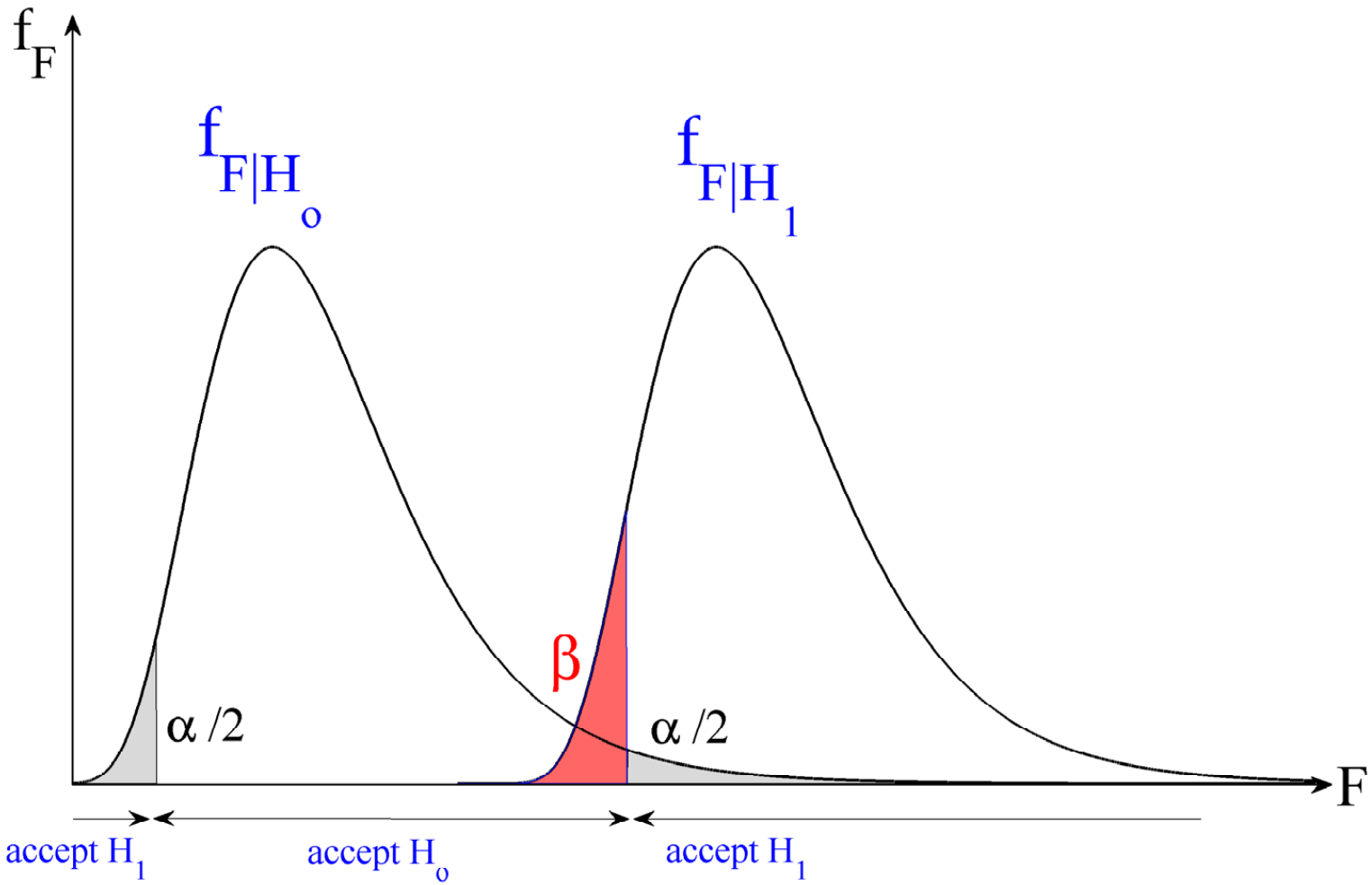
$$F = \frac{\widehat{S}_o(\omega)/S_o(\omega)}{\widehat{S}_u(\omega)/S_u(\omega)} \sim F(2K, 2K)$$

F distribution with $(2K, 2K)$ degrees of freedom
(ratio of normalized χ^2 distributions)

Under H_o $(S_u(\omega) = S_o(\omega))$:

$$F = \frac{\widehat{S}_o(\omega)}{\widehat{S}_u(\omega)} \sim F(2K, 2K)$$





Type I error (false alarm) probability: α

Type II error (missed fault) probability: β

3.2 Frequency Response Function (FRF) based method (excitation-response case)

(Rizos et al. 2001, Hwang & Kim 2004*, Fong et al. 2008, Kopsaftopoulos & Fassois 2008)

Main idea

Fault detection and identification are associated with changes in the FRF.

Characteristic quantity:

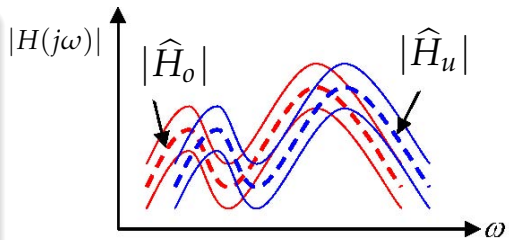
$$Q = |H(j\omega)|$$

Baseline Phase

$$y_o^N \rightsquigarrow |\widehat{H}_o(j\omega)|$$

$$|\widehat{H}_o(j\omega)| \sim \mathcal{N}(|H_o(j\omega)|, \sigma_o^2(\omega))$$

Welch estimator
(K segments)



Inspection Phase

$$y_u^N \rightsquigarrow |\widehat{H}_u(j\omega)|$$

$$|\widehat{H}_u(j\omega)| \sim \mathcal{N}(|H_u(j\omega)|, \sigma_u^2(\omega))$$

Welch estimator
(K segments)

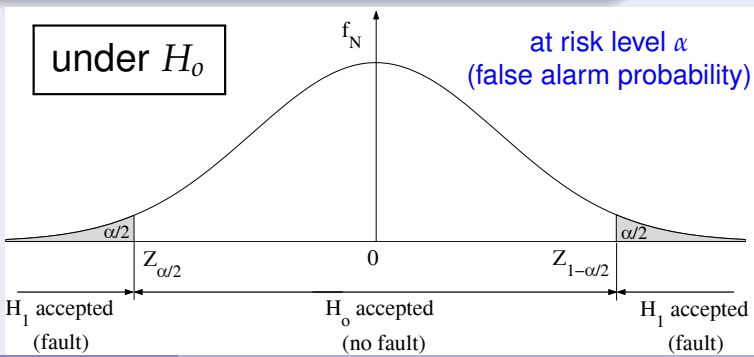
Inspection Phase (cont'd)

Statistical hypothesis testing problem

$$\begin{array}{lcll} H_o & : & |H_u(j\omega)| - |H_o(j\omega)| = 0 & \text{null hypothesis – healthy structure} \\ H_1 & : & |H_u(j\omega)| - |H_o(j\omega)| \neq 0 & \text{alternative hypothesis – faulty structure} \end{array}$$

Under H_o the statistic:

$$Z = \frac{\delta|\widehat{H}(j\omega)|}{\sqrt{2\sigma_o^2(\omega)}} \sim N(0,1)$$



3.3 A coherence measure based method (excitation–response case)

(Rizos et al. 2001, 2008)

Main idea

Fault detection, identification and estimation are associated with decreases in the "overall" coherence due to the introduction of nonlinear effects.

Characteristic quantity:

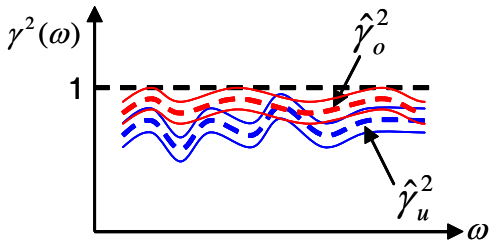
$$Q = \Gamma = \delta\omega \cdot \sum_{i=1}^{n} \gamma^2(\omega_i)$$

Baseline Phase

$$y_o^N \rightsquigarrow \widehat{\Gamma}_o$$

$$\widehat{\Gamma}_o \sim \mathcal{N}(\Gamma_o, \sigma^2_{\Gamma_o})$$

$$\sigma^2_{\Gamma_o} = (\delta\omega)^2 \cdot \sum_{i=1}^{n} \text{var}\{\hat{\gamma}^2_o(\omega_i)\}$$



Inspection Phase

$$y_u^N \rightsquigarrow \widehat{\Gamma}_u$$

$$\widehat{\Gamma}_u \sim \mathcal{N}(\Gamma_u, \sigma^2_{\Gamma_u})$$

$$\sigma^2_{\Gamma_u} = (\delta\omega)^2 \cdot \sum_{i=1}^{n} \text{var}\{\hat{\gamma}^2_u(\omega_i)\}$$

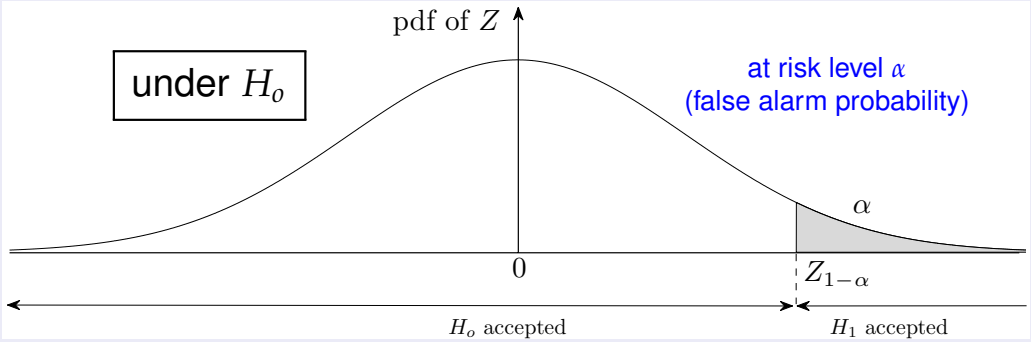
Inspection Phase (cont'd)

Statistical hypothesis testing problem

$$H_o \quad : \quad \delta\Gamma = \Gamma_o - \Gamma_u = 0 \qquad \text{null hypothesis – healthy structure}$$
$$H_1 \quad : \quad \delta\Gamma = \Gamma_o - \Gamma_u > 0 \qquad \text{alternative hypothesis – faulty structure}$$

Under H_o the statistic:

$$Z = \frac{\delta\widehat{\Gamma}}{\sqrt{2(\sigma_\Gamma)^2_o}} \sim \mathcal{N}(0,1)$$



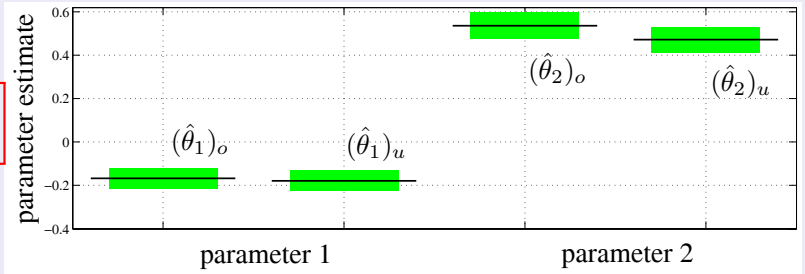
4. Parametric Statistical Time Series SHM Methods

4.1 Model parameter based methods *(Lifshitz & Rotem 1969, Stubbs & Osegueda 1990, Sadeghi & Fassois 1997, Fritzen et al. 1998, Sohn & Farrar 2000, Sakellariou & Fassois 2006, Hios & Fassois 2009)*

Main idea

Fault detection, identification and estimation are associated with changes in the parameter vector $\boldsymbol{\theta}$ of a suitable parametric model.

characteristic quantity: $Q = \boldsymbol{\theta}$ (or a function of it)



Baseline Phase

(x_o^N, y_o^N) $\rightsquigarrow$ parametric model for healthy structure: $\widehat{\boldsymbol{\theta}}_o$

signal pre-processing

$$\widehat{\boldsymbol{\theta}}_o \sim \mathcal{N}(\boldsymbol{\theta}_o, \boldsymbol{\Gamma}_o) \quad \text{as} \quad N \longrightarrow \infty$$

(Söderström & Stoica 1989, p. 198)

Inspection Phase

(x_u^N, y_u^N) $\rightsquigarrow$ parametric model for current structure: $\widehat{\boldsymbol{\theta}}_u$

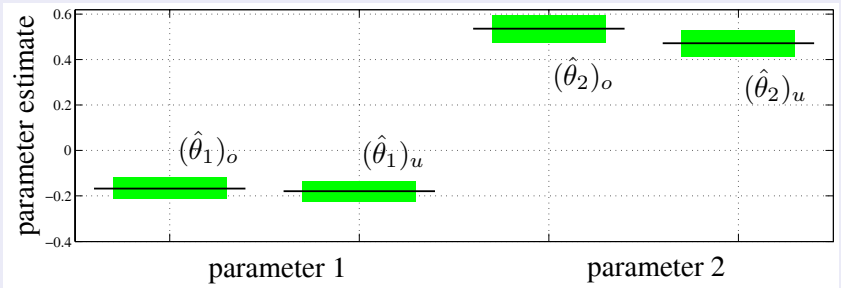
signal pre-processing

$$\widehat{\boldsymbol{\theta}}_u \sim \mathcal{N}(\boldsymbol{\theta}_u, \boldsymbol{\Gamma}_u) \quad \text{as} \quad N \longrightarrow \infty$$

Inspection Phase (cont'd)

Is $\boldsymbol{\theta}_u$ different from $\boldsymbol{\theta}_o$?

- no → structure is healthy
- yes → structure is faulty



To answer set up the statistical hypothesis testing problem:

$$H_o \quad : \quad \delta\boldsymbol{\theta} = \boldsymbol{\theta}_o - \boldsymbol{\theta}_u = \mathbf{0} \qquad \text{null hypothesis – healthy structure}$$
$$H_1 \quad : \quad \delta\boldsymbol{\theta} = \boldsymbol{\theta}_o - \boldsymbol{\theta}_u \neq \mathbf{0} \qquad \text{alternative hypothesis – faulty structure}$$

Form the quantity:

$$\delta\widehat{\boldsymbol{\theta}} = \widehat{\boldsymbol{\theta}}_o - \widehat{\boldsymbol{\theta}}_u \sim \mathcal{N}(\delta\boldsymbol{\theta}, \delta\boldsymbol{\Gamma})$$

$$\delta\boldsymbol{\theta} = \boldsymbol{\theta}_o - \boldsymbol{\theta}_u, \quad \delta\boldsymbol{\Gamma} = \boldsymbol{\Gamma}_o + \boldsymbol{\Gamma}_u$$

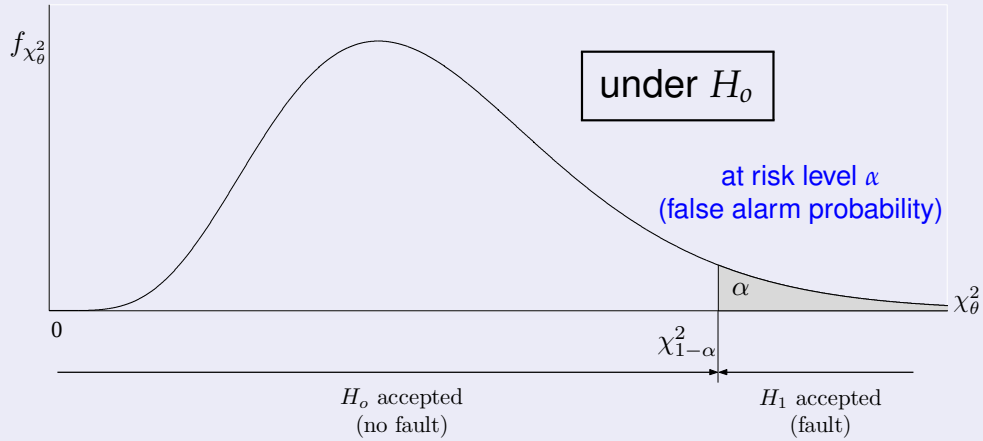
(difference of independent normal variables)

Under H_o $(\boldsymbol{\theta}_u = \boldsymbol{\theta}_o)$:

$\delta\widehat{\boldsymbol{\theta}} \sim N(0, 2\boldsymbol{\Gamma}_o)$ and the statistic:

$$\chi^2_{\boldsymbol{\theta}} = \delta\widehat{\boldsymbol{\theta}}^T \cdot (2\boldsymbol{\Gamma}_o)^{-1} \cdot \delta\widehat{\boldsymbol{\theta}} \sim \chi^2(d)$$

χ^2 distribution with $d = dim(\boldsymbol{\theta})$ degrees of freedom
(sum of squares of independent Gaussian variables)



4.2 Model residual based methods

Method A: residual variance based method *(Sohn & Farrar 2001, Fugate et al. 2001, Sohn et al. 2003, Poulimenos & Fassois 2004, Lu & Gao 2005, Zhang 2007)*

Main idea

(structural state u)

$x_u[t]$ current excitation

$y_u[t]$ current response

model $\mathcal{M}_V$

(structural state V)

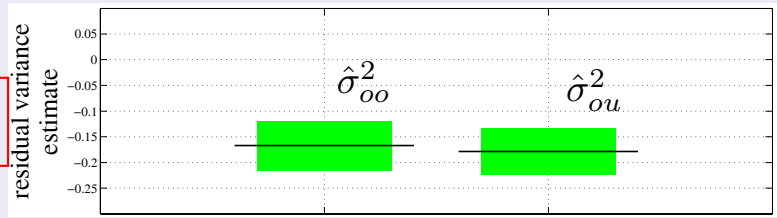
$e_{Vu}[t]$ residual

$$e_{Vu}[t] \sim \mathcal{N}(0, \sigma^2_{Vu})$$

for fault detection the model corresponds to the healthy structural state o

$e_{Vu}[t]$: white with minimal variance if the model accurately describes the structure $\Leftrightarrow$ $u \equiv V$

characteristic quantity: $Q = \sigma^2_{Vu}$ (σ^2_{ou} for fault detection)



Baseline phase

(x_o^N, y_o^N) $\rightsquigarrow$ parametric model identification for healthy structure: $\widehat{\mathcal{M}}_o$ (incl. $\hat{\sigma}^2_{oo}$)

signal pre-processing

$$\frac{(N-d)\hat{\sigma}^2_{oo}}{\sigma^2_{oo}} \sim \chi^2(N-d)$$

χ^2 distribution with $N-d$ degrees of freedom ($d = d(\theta)$)

Inspection phase

$(x_u^N, y_u^N) \rightsquigarrow$ computation of $e_{ou}[t]$ using $\widehat{\mathcal{M}}_o \rightsquigarrow$ estimation of σ_{ou}^2 : $\widehat{\sigma}_{ou}^2$

signal pre-processing

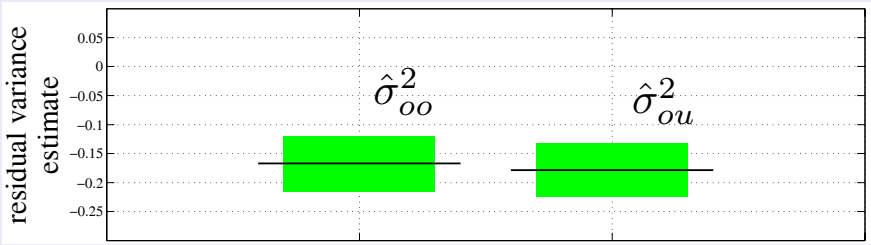
$$\frac{N\widehat{\sigma}_{ou}^2}{\sigma_{ou}^2} \sim \chi^2(N)$$

χ^2 distribution with N degrees of freedom

Is σ_{ou}^2 minimal $(= \sigma_{oo}^2)$?

yes → structure is healthy

no → structure is faulty



To answer set up the statistical hypothesis hypothesis testing problem:

H_o : $\sigma_{oo}^2 = \sigma_{ou}^2$ null hypothesis – healthy structure

H_1 : $\sigma_{oo}^2 < \sigma_{ou}^2$ alternative hypothesis – faulty structure

Inspection phase (cont'd)

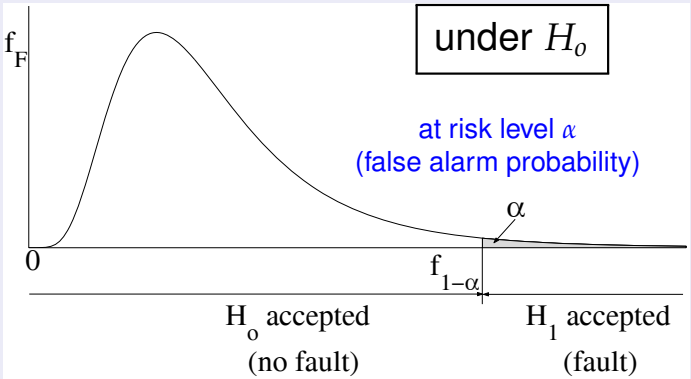
Form the statistic:

$$F = \frac{\frac{N\widehat{\sigma}^2_{ou}}{N\sigma^2_{ou}}}{\frac{(N-d)\widehat{\sigma}^2_{oo}}{(N-d)\sigma^2_{oo}}} \sim F(N, N-d)$$

F–distribution with $(N, N-d)$ degrees of freedom
(ratio of two independent and normalized χ^2 random variables)

Under H_o $(\sigma^2_{ou} = \sigma^2_{oo})$:

$$F = \frac{\widehat{\sigma}^2_{ou}}{\widehat{\sigma}^2_{oo}} \sim F(N, N-d)$$



Method B: likelihood based method

(Kopsaftopoulos & Fassois 2008, Spiridonakos & Fassois 2009)

Main idea

(structural state u)

$x_u[t]$ current excitation

$y_u[t]$ current response

model $\mathcal{M}_V$

(structural state V)

for fault detection the model corresponds to the healthy structural state o

$$L(y_u^N, \boldsymbol{\theta}_V / x_u^N) = \text{prob}\,(y_u^N, \boldsymbol{\theta}_V / x_u^N)$$

The structural state $u \equiv V$ iff the likelihood L above is greater or equal to a selected threshold:

$$L \geq l$$

characteristic quantity: $Q = L(y_u^N, \boldsymbol{\theta}_V / x_u^N)$

Likelihood of y_u^N given x_u^N for a given model with parameter vector $\boldsymbol{\theta}_V$ *(Box et al. 1994, p. 226)*:

$$L(y_u^N, \boldsymbol{\theta}_V / x_u^N) = \prod_{t=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} exp\left[-\frac{e^2[t, \boldsymbol{\theta}_V]}{2\sigma^2}\right] \qquad \left\{\begin{matrix} e[t, \boldsymbol{\theta}] \equiv e_{Vu}[t] \\ \sigma^2 = \sigma^2_{Vu} \end{matrix}\right\}$$

Method C: residual uncorrelatedness based method *(Sakellariou & Fassois 2008, Kopsaftopoulos & Fassois 2008)*

Main idea

(structural state u)

$x_u[t]$ current excitation

$y_u[t]$ current response

model $\mathcal{M}_V$

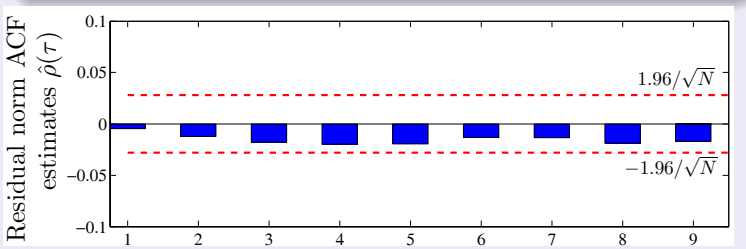
(structural state V)

$e_{Vu}[t]$ residual

$$e_{Vu}[t] \sim \mathcal{N}(0, \sigma^2_{Vu})$$

for fault detection the model corresponds to the healthy structural state o

$e_{Vu}[t]$: white with minimal variance if the model accurately describes the structure $\Leftrightarrow$ $u \equiv V$



characteristic quantity: $Q = \rho[\tau] \quad (\tau = 1, 2, \ldots, r)$

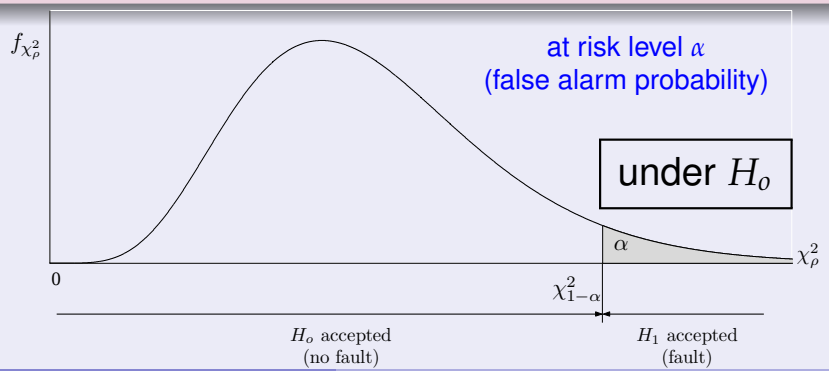
H_o : $\rho[\tau] = 0$ $\tau = 1, 2, \ldots, r$ null hypothesis – healthy structure

H_1 : $\rho[\tau] \neq 0$ for some τ alternative hypothesis – faulty structure

Form the statistic:

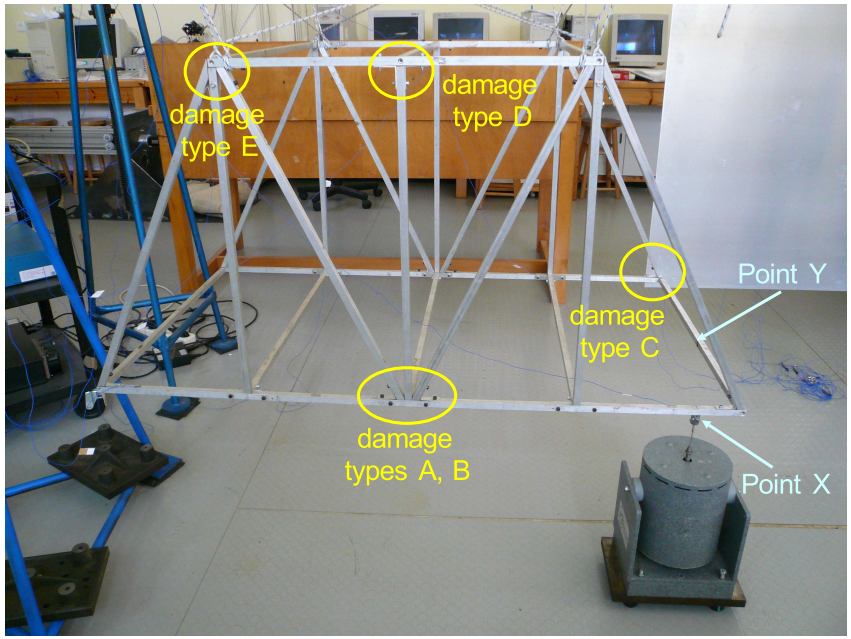
$$\chi^2_\rho = N \cdot (N+2) \sum_{\tau=1}^{r} (N-\tau)^{-1} \cdot \hat{\rho}^2[\tau]$$

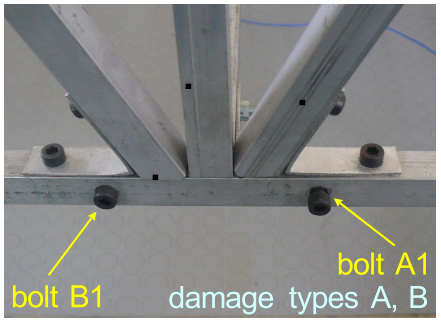
Under H_o: $\chi^2_\rho \sim \chi^2(r)$

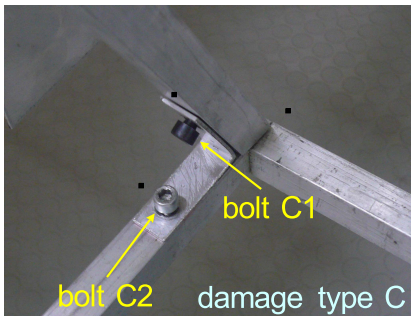


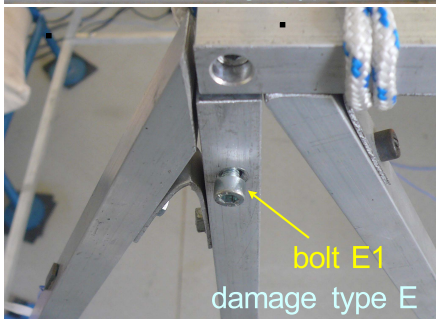
5. Damage detection and identification in an aluminum truss structure

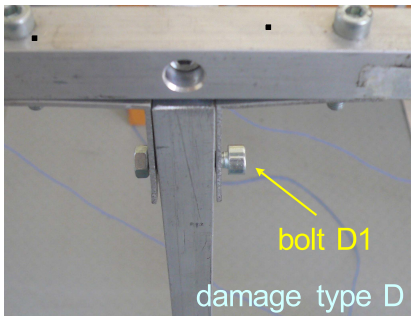
(Kopsaftopoulos & Fassois 2008)











The Structure:	Aluminum truss structure $(140 \times 70 \times 80\ mm)$
Damages:	Loosening of bolts A, B, C, D, E (five cases)
Measurements:	force (point X) – strain (point Y)
Methods:	➢ Power Spectral Density (PSD) based method
	➢ FRF based method
	➢ model parameter based method
	➢ model residual based methods

Equipment

Exciter: Modal 50A, MB Dynamics Inc.

Amplifier: Labworks PA-138-1

Signal conditioner: PCB 481A02 16 ch.

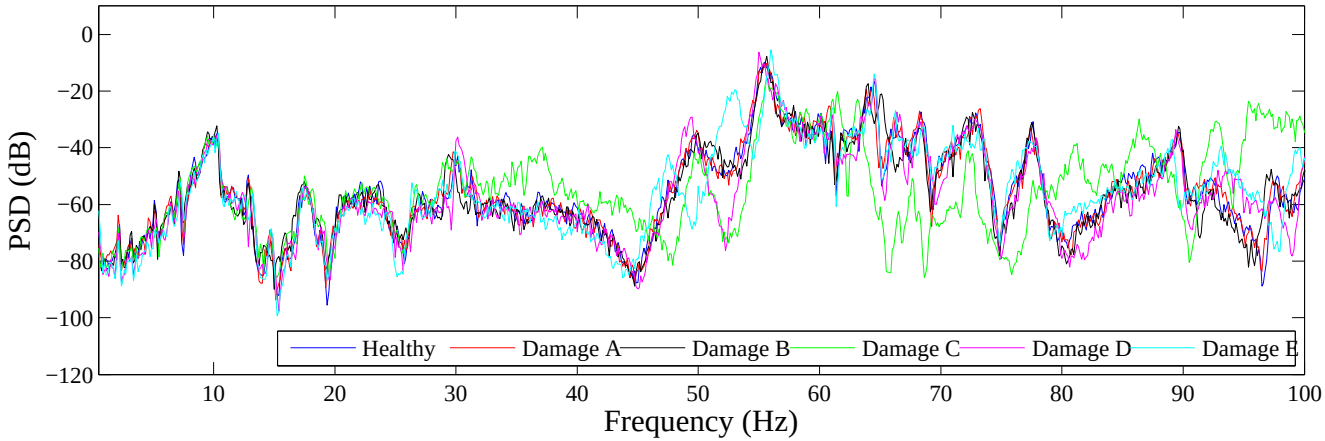
Strain sensors:
PCB ICP 740B02, Piezotronics Inc.

Force sensor:
PCB 288D01 impedance head

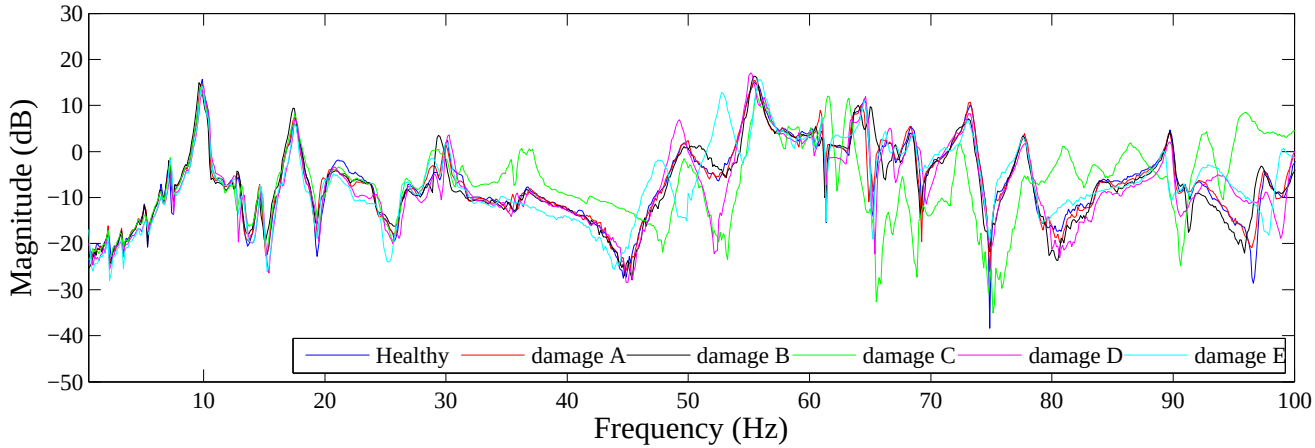
DAQ: Siglab model 20-40

Non–parametric methods

Welch PSD estimates



Welch FRF estimates



Signals

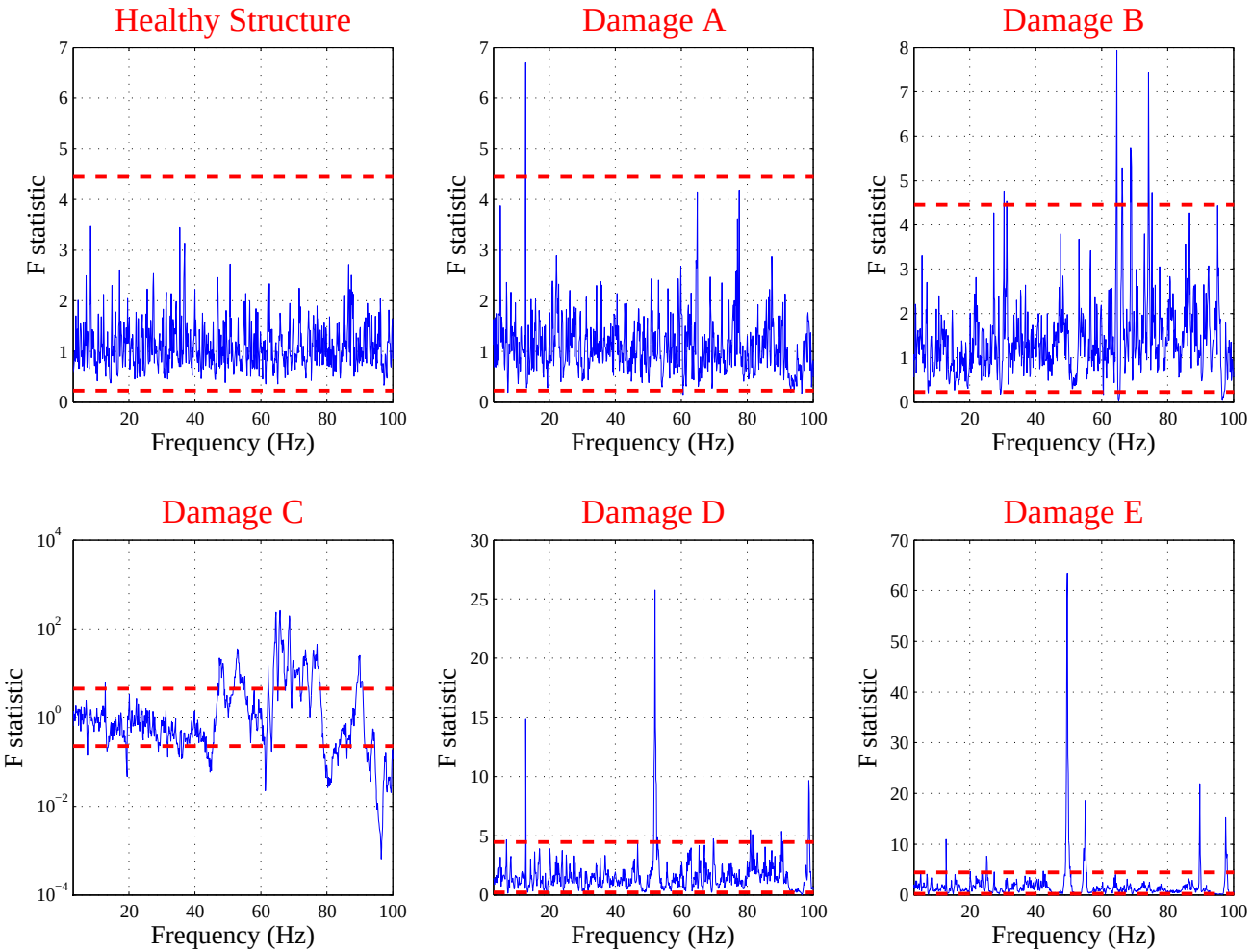
Sampling frequency	:	$f_S = 256$ Hz
Sampling bandwidth	:	$[0.5 - 100]$ Hz
Signal length	:	$N = 30720$ (120 s)

Estimation

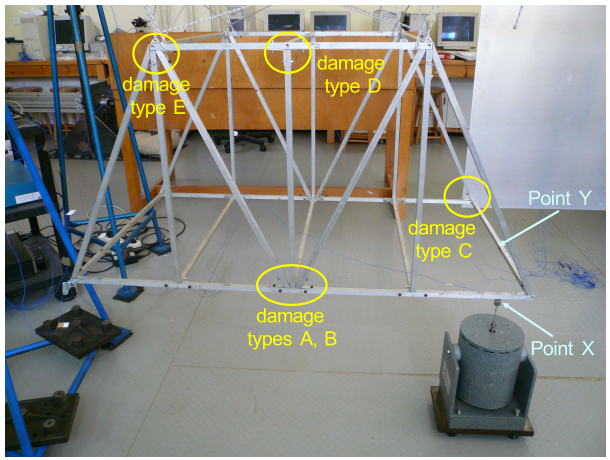
Method	:	Welch
Segment length	:	2048 samples
Window type	:	Hamming
Overlap	:	0 %
Frequency resolution	:	0.125 Hz
Number of segments	:	15

The Power Spectral Density (PSD) based method – damage detection ($\alpha = 10^{-4}$)

$$H_o \quad : \quad S_u(\omega) = S_o(\omega) \quad \text{(healthy structure)} \quad \Longleftrightarrow \quad f_{\frac{\alpha}{2}} \leq F \leq f_{1-\frac{\alpha}{2}}$$
$$H_1 \quad : \quad S_u(\omega) \neq S_o(\omega) \quad \text{(damaged structure)} \quad \Longleftrightarrow \quad \text{otherwise}$$



Damage detection results for the six test cases (one healthy and five damaged). The critical points at the $\alpha = 10^{-4}$ risk level are shown as dashed lines - - - -).

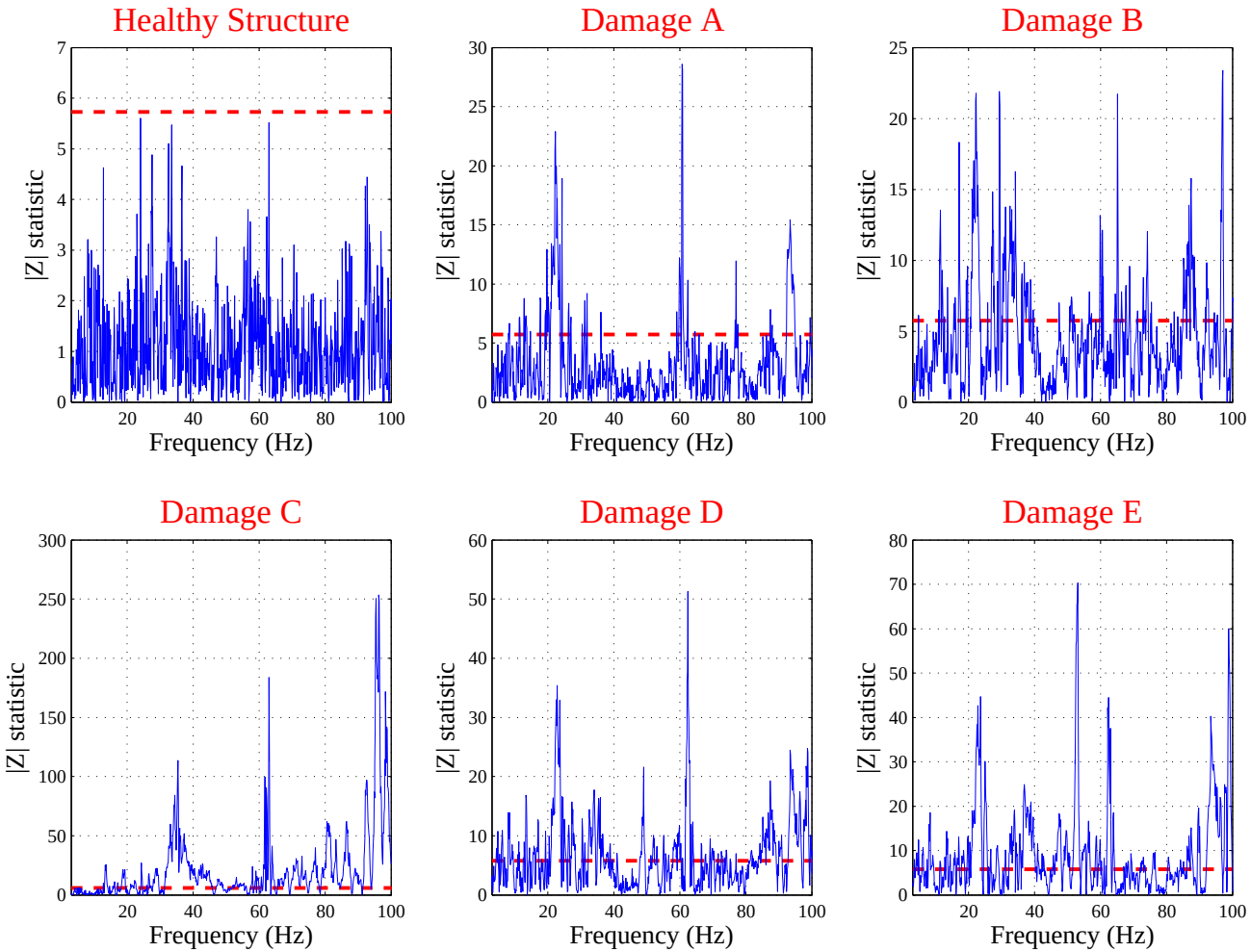


Method	: Welch
Segment length	: 2048 samples
Non-overlapping segments	: $K = 15$
Window type	: Hamming

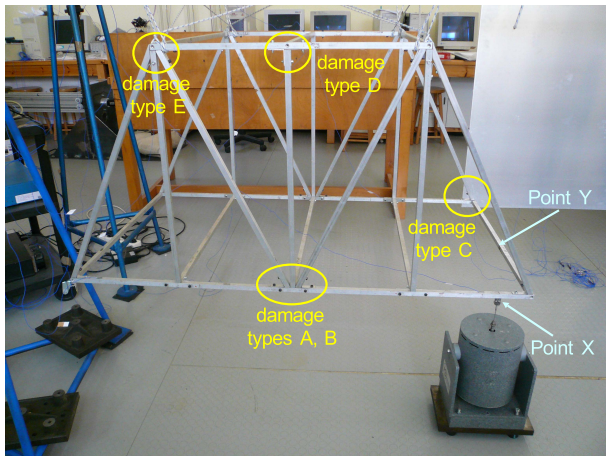
A damage is detected if F exceeds the critical points (dashed lines - - - -).

The Frequency Response Function based method – damage detection ($\alpha = 10^{-8}$)

$$H_o : |H_o(j\omega)| - |H_u(j\omega)| = 0 \quad \text{(healthy structure)} \iff |Z| \leq Z_{1-\frac{\alpha}{2}}$$
$$H_1 : |H_o(j\omega)| - |H_u(j\omega)| \neq 0 \quad \text{(damaged structure)} \iff \text{otherwise}$$



Damage detection results for the six test cases (one healthy and five damaged). The critical points ($\alpha = 10^{-8}$) are depicted in dashed lines (- - - -).

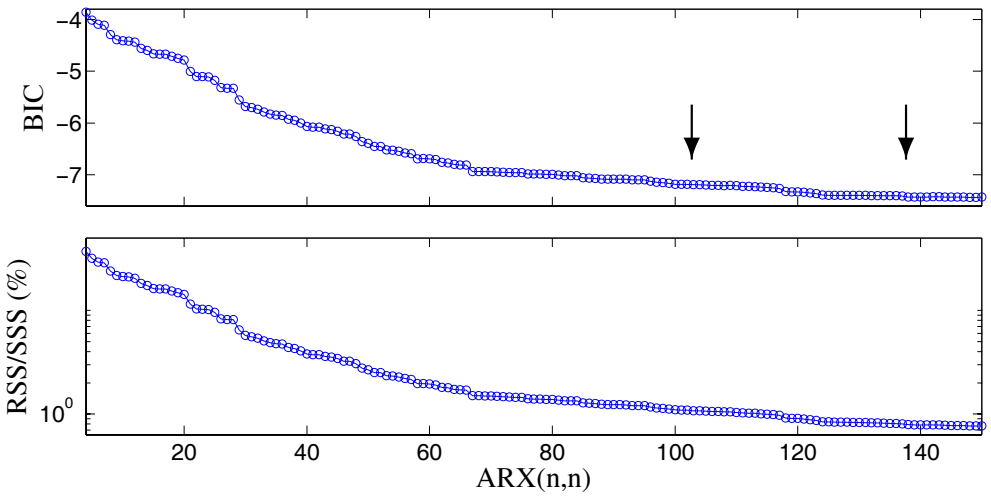


Method	: Welch
Segment length	: 2048 samples
Non-overlapping segments	: $K = 15$
Window type	: Hamming

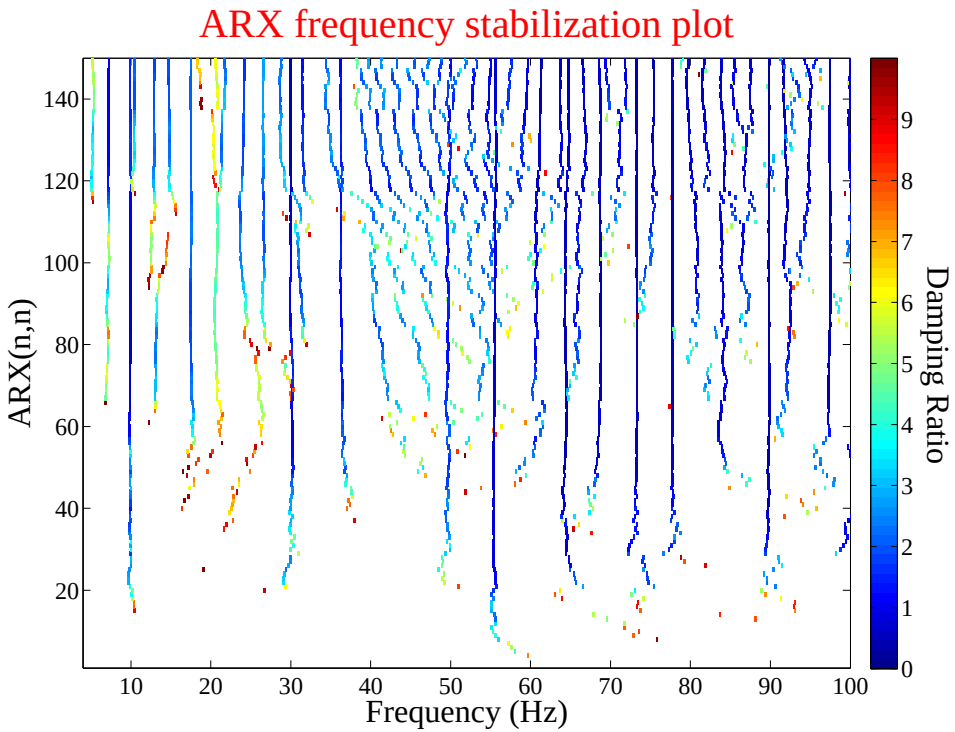
A damage is detected if $|Z|$ exceeds the critical points (dashed lines - - - -).

Parametric methods

Baseline Phase: ARX(n,n) model identification



BIC and RSS/SSS order selection criteria for ARX(n,n) type models (healthy structure)



Modal frequency stabilization diagram for ARX(n,n) type models (healthy structure)

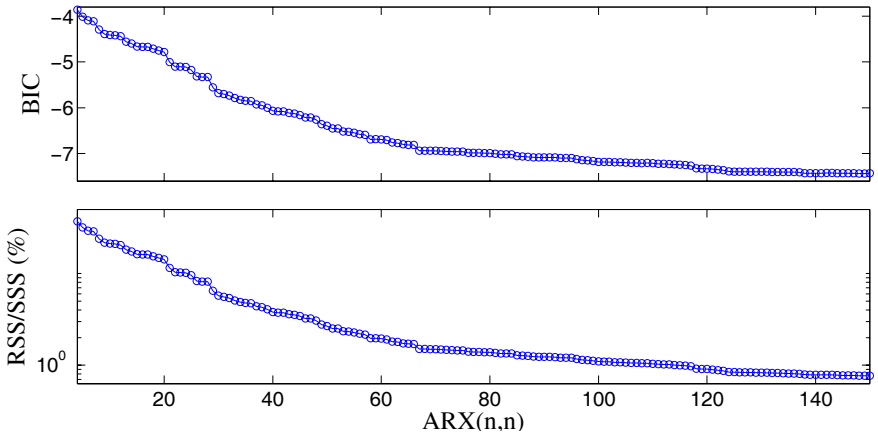
Selected models: ARX$(103,103)$, ARX$(138,138)$
Linear Least Squares estimation
QR implementation

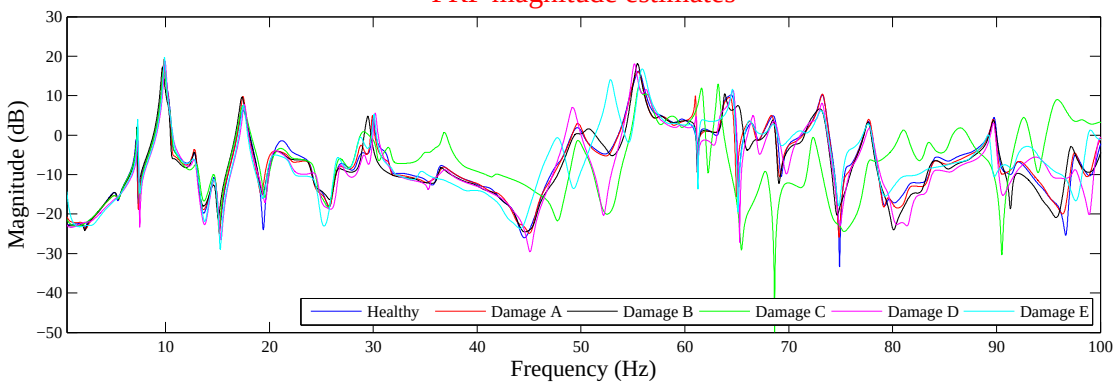
Signals

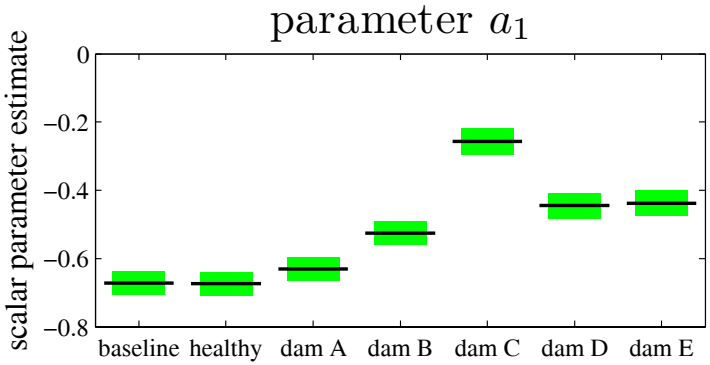
Sampling frequency	: 256 Hz
Signal length N	: 10000 samples (39 s)
Frequency range	: $[0.5-100]$ Hz

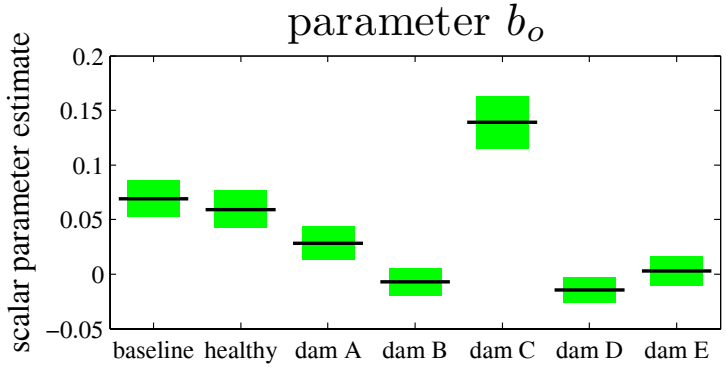
Model parameter based method – damage detection $(\alpha = 10^{-12})$

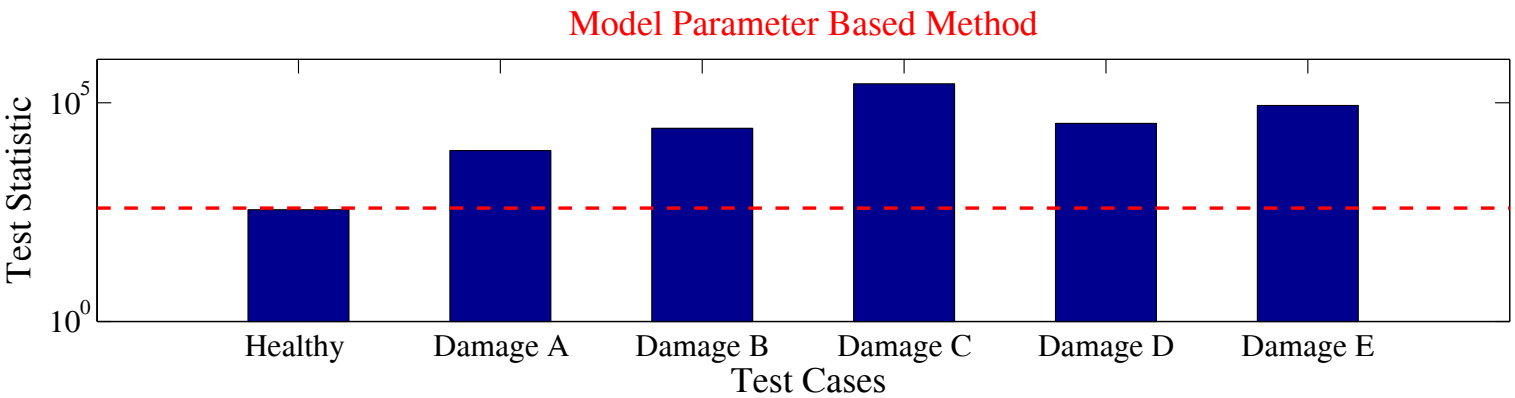
ARX$(103, 103)$ model







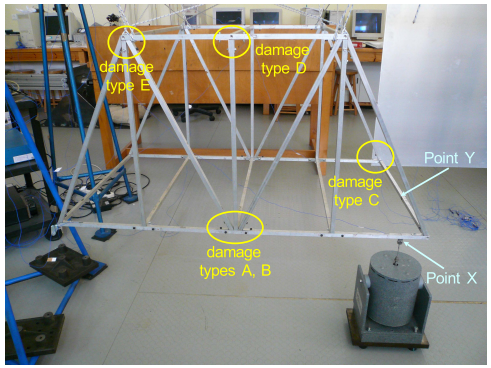




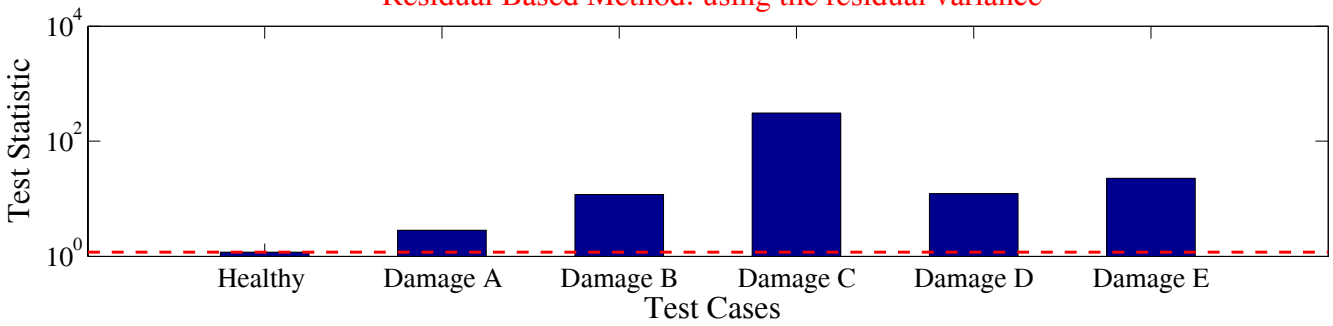
Model residual based methods – damage detection $(\alpha = 10^{-12})$

The test cases

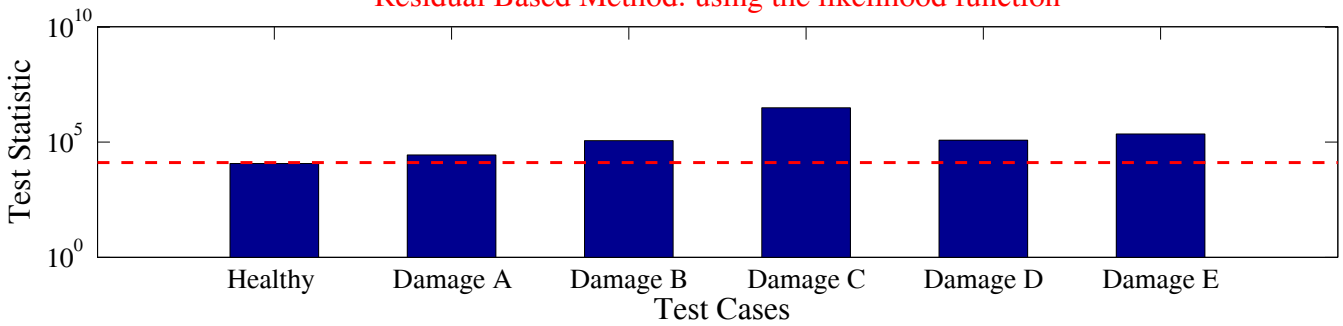
Test case I	:	*Healthy*	Test case II	:	Damage A
Test case III	:	Damage B	Test case IV	:	Damage C
Test case V	:	Damage D	Test case VI	:	Damage E



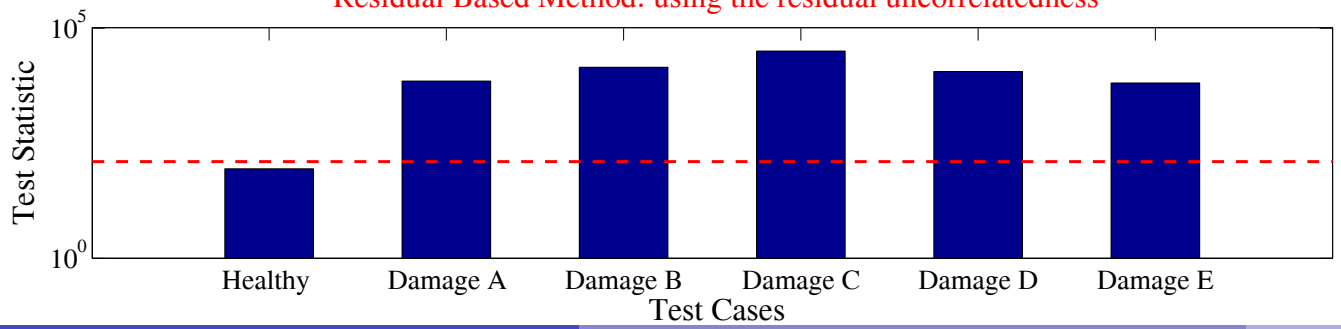
Residual Based Method: using the residual variance



Residual Based Method: using the likelihood function



Residual Based Method: using the residual uncorrelatedness



Summary Results

DAMAGE DETECTION AND IDENTIFICATION SUMMARY RESULTS

39 healthy experiments 31×5 damaged experiments

Method	Damage Detection						Damage Identification				
	False alarms	Missed damage					Damage misclassification				
		dam. A	dam. B	dam. C	dam. D	dam. E	dam. A	dam. B	dam. C	dam. D	dam. E
PSD based	0/39	0/31	0/31	0/31	0/31	0/31	0/31	0/31	0/31	0/31	0/31
FRF based	1/39	0/31	0/31	0/31	0/31	0/31	2/31	0/31	0/31	0/31	0/31
Mod. parameter†	0/39	0/31	0/31	0/31	0/31	0/31	0/31	0/31	0/31	0/31	0/31
Res. variance†	0/39	0/31	0/31	0/31	0/31	0/31	0/31	0/31	0/31	0/31	0/31
Res. likelihood†	0/39	0/31	0/31	0/31	0/31	0/31	0/31	0/31	0/31	0/31	0/31
Res. uncorrelatedness†	0/39	0/31	0/31	0/31	0/31	0/31	0/31	0/31	0/31	0/31	0/31

† adjusted α

6. Recent Developments

6.1 Functional Model (FM) based methods for damage detection, localization & estimation

(Kopsaftopoulos & Fassois 2006, 2007, Sakellariou & Fassois 2008)

Vector–dependent Functionally Pooled ARX (VFP–ARX) model for damage mode A

$$\mathcal{M}^A(\boldsymbol{k}): \quad \underbrace{y_{\boldsymbol{k}}[t] + \sum_{i=1}^{na} a_i(\boldsymbol{k}) \cdot y_{\boldsymbol{k}}[t-i]}_{\text{AR part}} = \underbrace{\sum_{i=0}^{nb} b_i(\boldsymbol{k}) \cdot x_{\boldsymbol{k}}[t-i]}_{\text{X part}} + e_{\boldsymbol{k}}[t], \quad e_{\boldsymbol{k}}[t] \sim \text{N.I.D.}\left(0, \sigma_e^2(\boldsymbol{k})\right)$$

$x_{\boldsymbol{k}}[t], y_{\boldsymbol{k}}[t]$: *excitation/response* vibration signals (under $\boldsymbol{k}$)
$e_{\boldsymbol{k}}[t]$: innovations (residual) sequence (under $\boldsymbol{k}$)

$$\boldsymbol{k} = \begin{bmatrix} k^1 \\ k^2 \end{bmatrix} = \begin{bmatrix} \textit{damage magnitude} \\ \textit{damage location} \end{bmatrix}$$

$$a_i(\boldsymbol{k}) \stackrel{\Delta}{=} \sum_{j=1}^{p} a_{i,j} \cdot G_j(\boldsymbol{k}), \qquad b_i(\boldsymbol{k}) \stackrel{\Delta}{=} \sum_{j=1}^{p} b_{i,j} \cdot G_j(\boldsymbol{k})$$

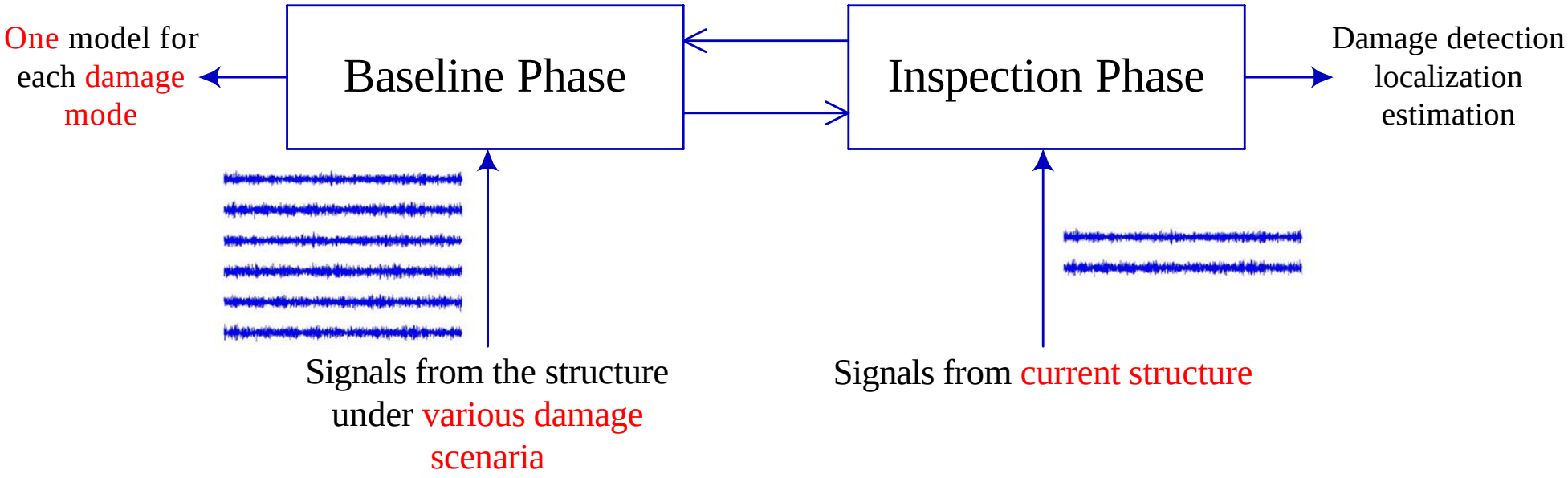
$a_{i,j}, b_{i,j}$: AR/X coefficients of projection
$G_1(\boldsymbol{k}), \ldots, G_p(\boldsymbol{k})$: basis functions (Chebyshev or other families)

Model identification

✔ Parameter estimation based on a Weighted Least Squares (WLS) scheme.

✔ Model structure selection based on Genetic Algorithms (GAs).

The VFP–ARX Model Based Method



A VFP-ARX model is estimated for representing each damage mode.

$\mathcal{M}^A(\boldsymbol{k})$: damage mode A
$\mathcal{M}^B(\boldsymbol{k})$: damage mode B
$\mathcal{M}^C(\boldsymbol{k})$: damage mode C

Current signals are used for estimating $\boldsymbol{k}$ for each damage mode model:

$\rightsquigarrow \quad \boldsymbol{k}^A$
$\rightsquigarrow \quad \boldsymbol{k}^B$ ✓ *correct* damage mode selected via model validation
$\rightsquigarrow \quad \boldsymbol{k}^C$

Baseline Phase:

A VFP–ARX model is identified for each damage mode (covers all mode locations and magnitudes).

Inspection Phase: Damage diagnosis

Damage detection: Is the current structure healthy? $(k^1 \stackrel{?}{=} 0)$

$$\widehat{\boldsymbol{k}} \stackrel{\Delta}{=} \arg\min_{\boldsymbol{k}} \sum_{t=1}^{N} e^2[t] \,, \qquad \sigma_e^2(\widehat{\boldsymbol{k}}) = \frac{1}{N} \sum_{t=1}^{N} e^2[t, \widehat{\boldsymbol{k}}]$$

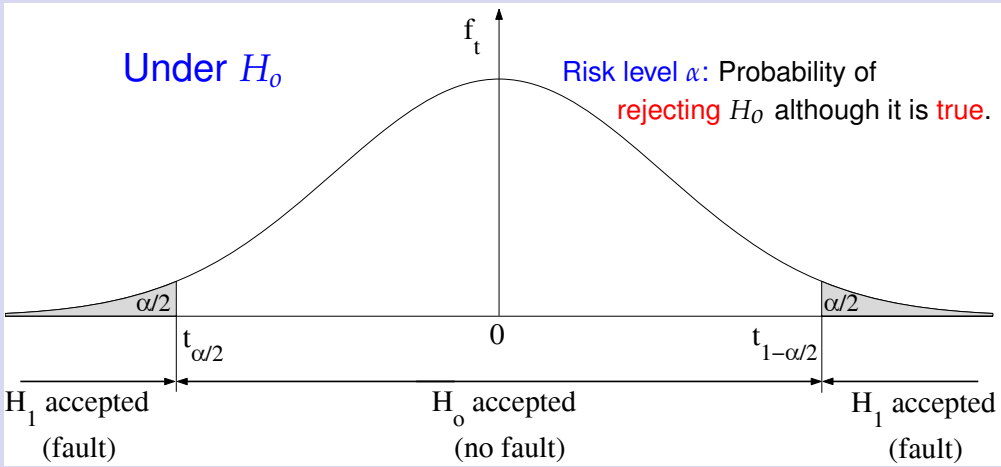
H_o : $k^1 = 0$ (null hypothesis – healthy structure)

H_1 : $k^1 \neq 0$ (alternative hypothesis – damaged structure)

Under H_o:

$$t = \frac{\widehat{k}^1}{\widehat{\sigma}_{k^1}} \sim t(N-2)$$

$\widehat{\sigma}_{k^1}$: standard deviation of $\widehat{k}^1$

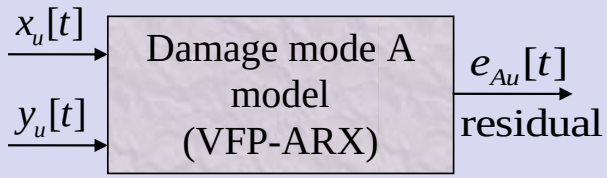


Inspection Phase: Damage diagnosis (cont'd)

Damage mode identification: Which of the damage mode models is currently valid?

For each damage mode, say mode A, set up the hypothesis testing problem:

$$\begin{array}{lcll} H_o & : & \rho[i]=0 & i=1,2,\ldots,h \quad \text{(damage} \in \text{mode A)} \\ H_1 & : & \rho[i]\neq 0 & \text{for some } i \quad \text{(damage} \notin \text{mode A)} \end{array}$$



where $\rho[i]$: normalized ACF of residuals

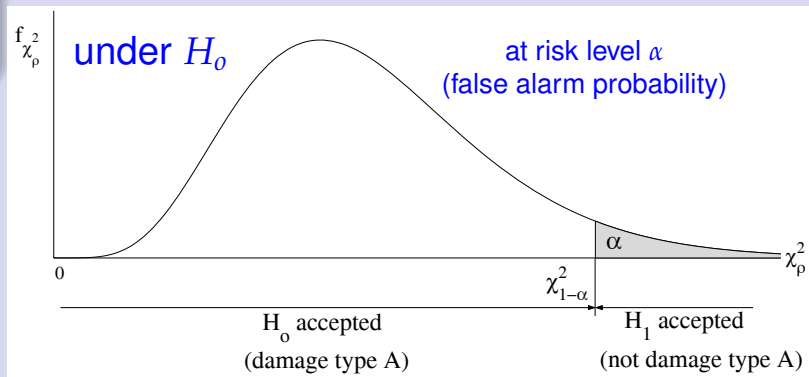
$$\widehat{\mathcal{Q}} = N\cdot(N+2)\cdot\sum_{i=1}^{h}(N-i)^{-1}\cdot\widehat{\rho}[i]^2$$

$H_o : \mathcal{Q}=0$ H_o accepted – damage mode A is valid
$H_1 : \mathcal{Q}>0$ H_1 accepted – damage mode A is not valid

Under H_o:

$$\widehat{\mathcal{Q}} \sim \chi^2(h-2)$$

$\chi^2(h-2)$: χ^2 dist. with $h-2$ dof

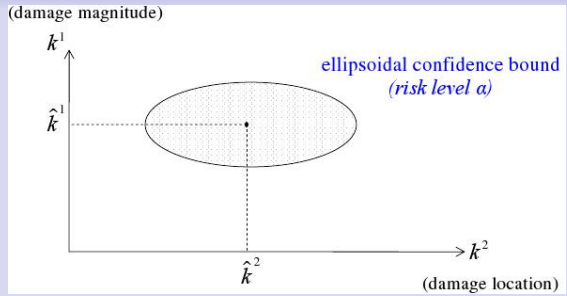


Damage localization & magnitude estimation: k interval estimate (bivariate confidence bounds)

Once damage mode is known $\Longrightarrow$ $\widehat{k}, \widehat{\Sigma}_k$ available

$$(\widehat{k}-k)^T{\Sigma_k}^{-1}(\widehat{k}-k)\sim\chi^2(2)$$

thus $(\widehat{k}-k)^T{\Sigma_k}^{-1}(\widehat{k}-k)\leq\chi^2_{1-\alpha}$ with prob. $=1-\alpha$



Application: Damage detection, localization & magnitude estimation in a scale aircraft skeleton structure

Structure

Skeleton dimensions:
fuselage: $1500 \times 150 \times 50$ mm
wings: $2000 \times 100 \times 10$ mm
wing tips: $400 \times 100 \times 10$ mm
vert. stab.: $400 \times 100 \times 10$ mm

Skeleton weight ~ 50 kg

Equipment

Exciter: Modal 50A, MB Dynamics Inc.

Amplifier: Labworks PA-138-1

Signal conditioner: PCB F482A20 8 ch.

Accelerometers:
PCB ICP 352A10, Piezotronics Inc.

Force sensor:
PCB 288D01 impedance head

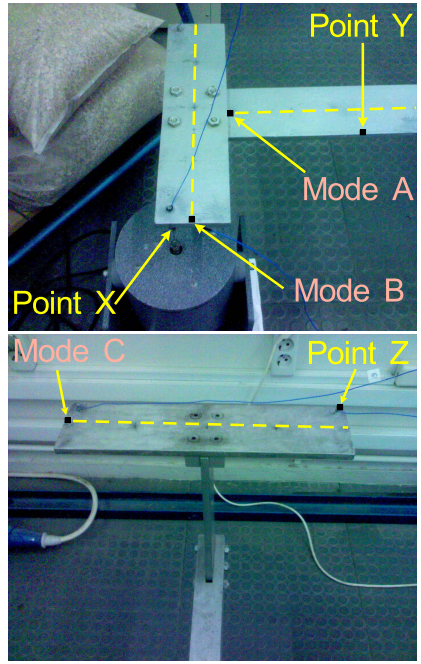
DAQ: Siglab model 20-40

Measurements

Force: point X
Vibration: points Y, Z

Damage representation

F_X^k : damage representation
X : damage mode
k^1 : damage magnitude (g)
k^2 : damage location (cm)



Damage test cases

I. *Healthy structure*

Damage Mode A (right wing)	Damage Mode B (right wing–tip)	Damage Mode C (horiz. stabilizer)
II. $F^A_{8.132,80}$	V. $F^B_{8.132,0}$	VII. $F^C_{12.19,10}$
III. $F^A_{32.528,0}$	VI. $F^B_{16.25,5}$	VIII. $F^C_{81.32,30}$
IV. $F^A_{40.66,75}$		

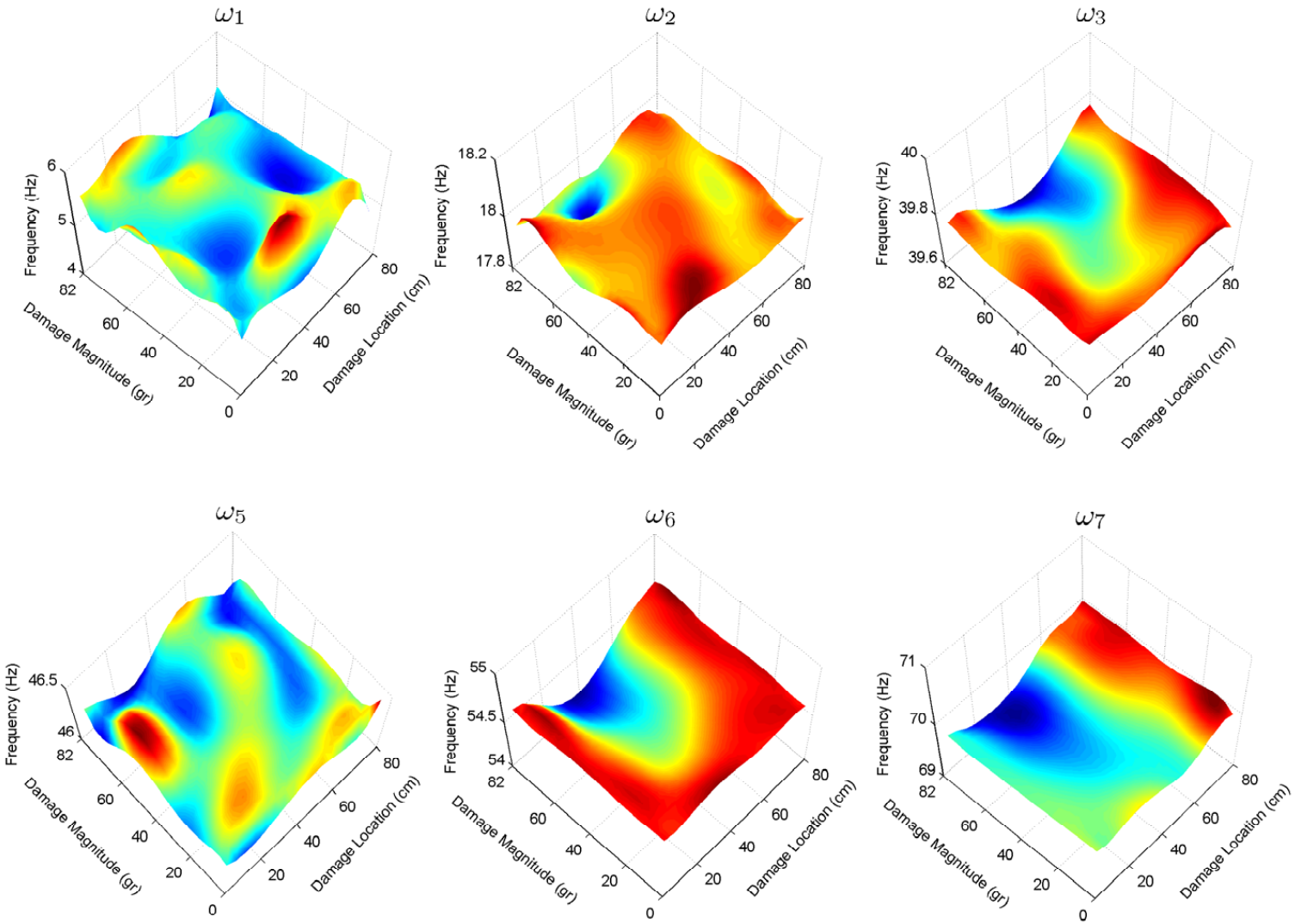
IX. $F_{40.66,10}$ **Unmodelled** – 40.66 g at 10 cm (left wing–tip)

Sampling frequency: $f_S = 256$ Hz, BWD: $4 - 90$ Hz

Signal length: $N = 1500$ samples $(5.86\,s)$

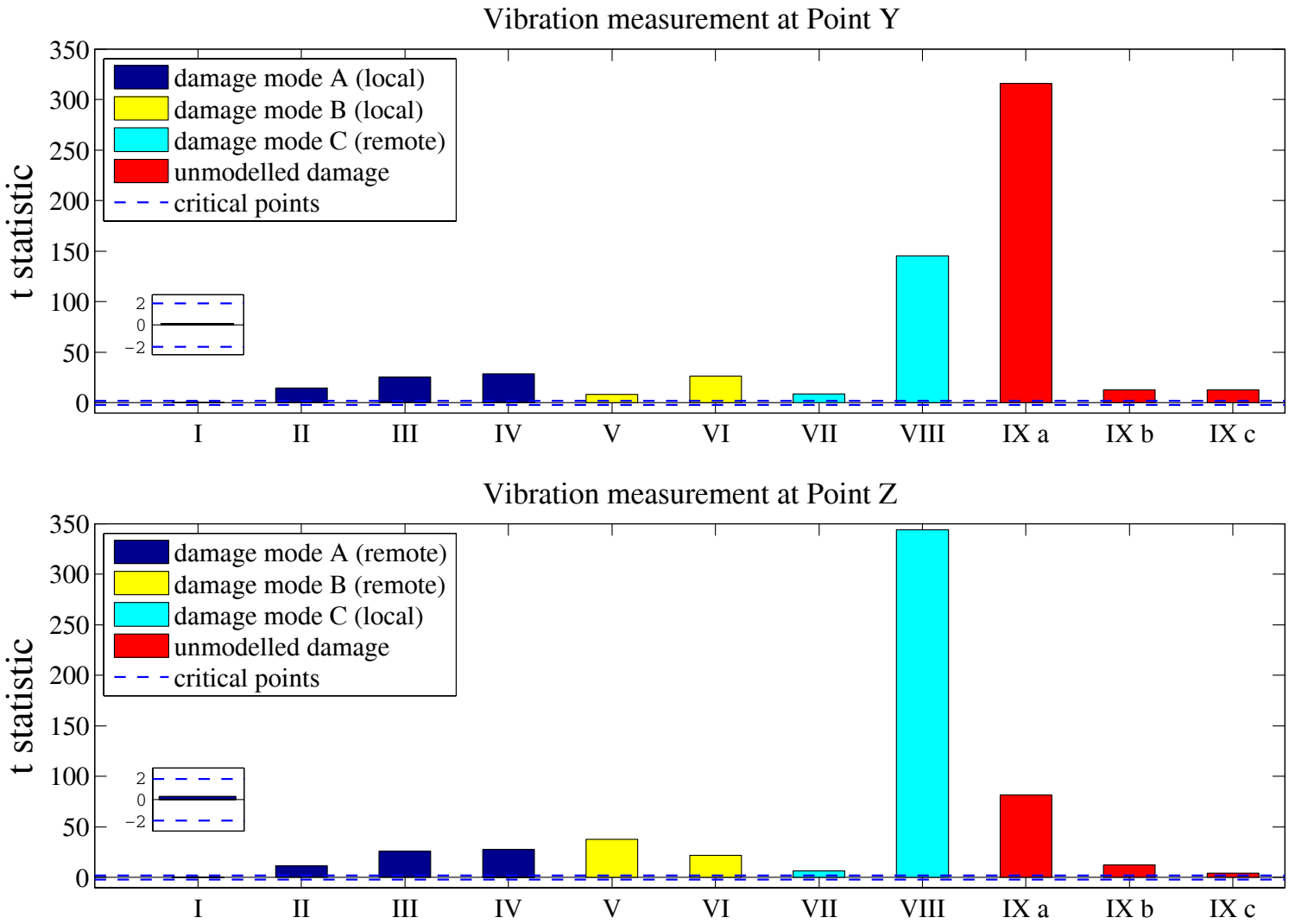
Training experiments: 99 for damage mode A, 55 for damage modes B, C

Aircraft Structure's Natural Frequencies ($\mathcal{M}_Y^A(k)$ model)



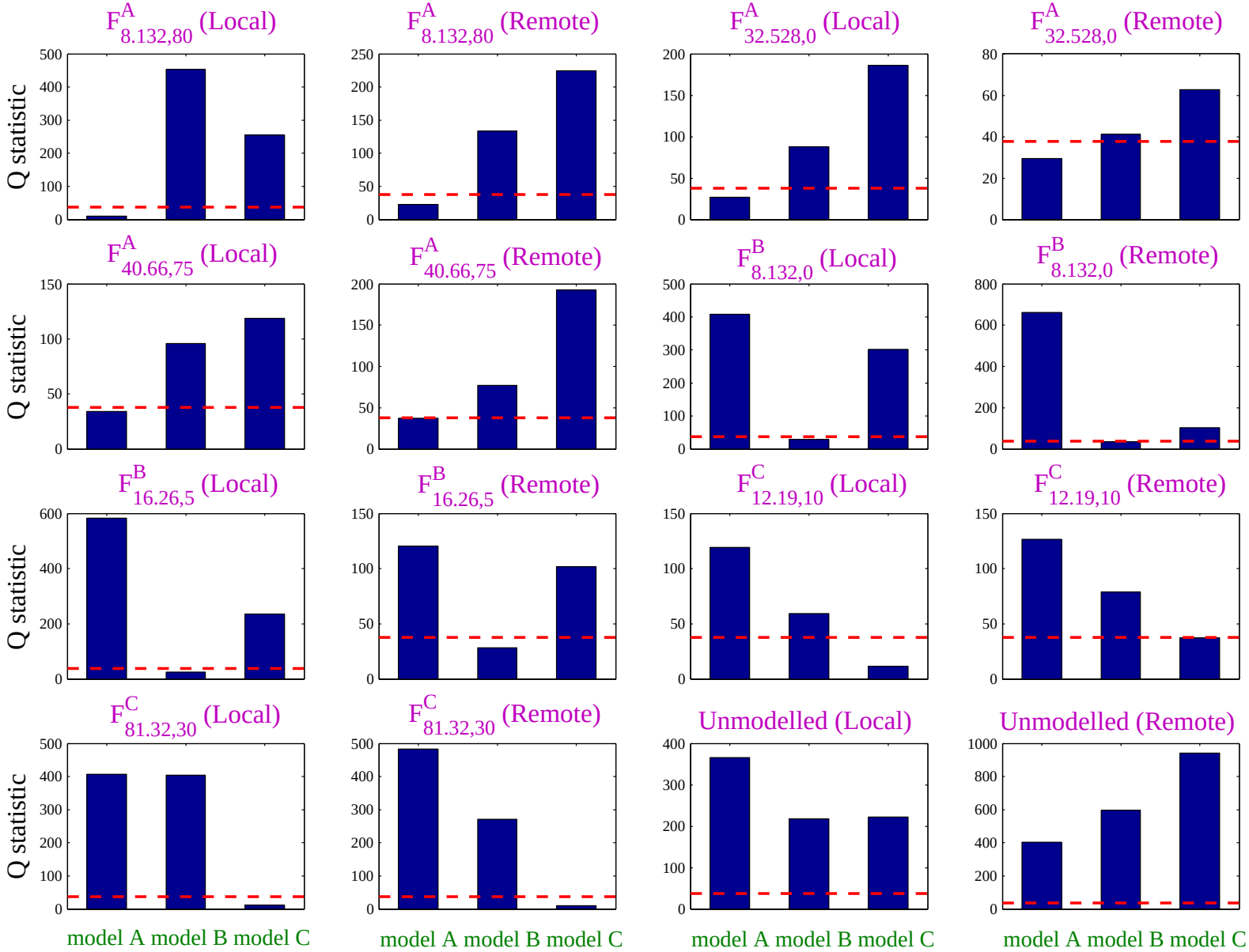
Aircraft structure's natural frequencies (Hz) versus damage magnitude (g) and damage location (cm) based on the VFP–ARX$(48,48)_{30}$ model ($\mathcal{M}_Y^A(k)$).

Damage detection results



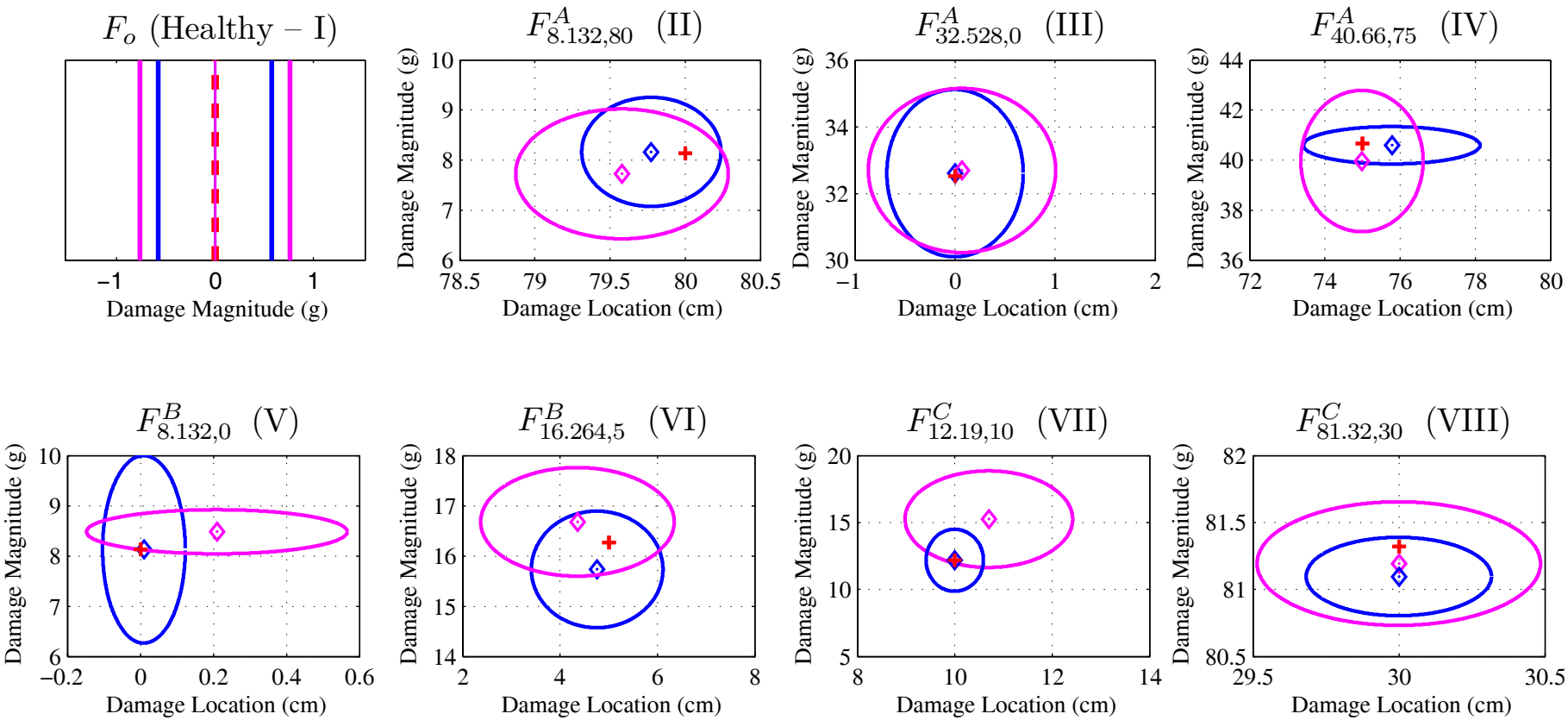
Damage detection results for both vibration measurement positions (Points Y and Z): t statistic (bars) and the critical points (- - -) at the $\alpha = 0.05$ level. Damage is detected if the t statistic is higher than the critical point.

Damage mode identification results



Damage identification results for both vibration measurement positions (Points Y an Z): Q statistic (bars) and the critical point (- - -) at the $\alpha = 0.05$ level $(h = 25)$. A damage mode is identified as current if Q is lower than the critical point.

Damage localization and magnitude estimation results



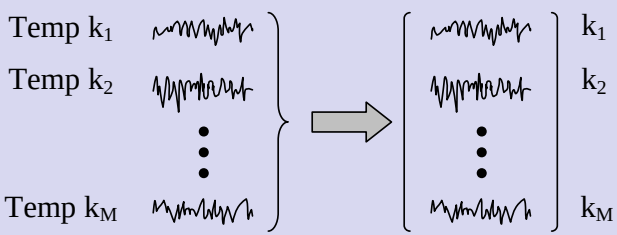
Damage detection, and damage localization and magnitude interval estimation results for the considered damage test cases (the correct damage indicated above the plot).

Blue ellipses: "local" sensor. Magenta ellipses: "remote" sensor [+: true values, ◇: point estimates].

6.2 An FP–VAR model based method for damage detection under various temperatures *(Hios & Fassois 2009)*

The Functionally Pooled VAR (FP–VAR) model class: **"Global model"** identification

Data pooling



The FP–VAR$(n)_{[p]}$ model: modelling of the healthy structure under various temps

$$\underbrace{\boldsymbol{y}_k[t]}_{\text{response at temp } k} + \sum_{i=1}^{n} \underbrace{\boldsymbol{A}_i(k)}_{\text{AR par depends on } k} \cdot \boldsymbol{y}_k[t-i] = \underbrace{\boldsymbol{e}_k[t,\boldsymbol{\theta}]}_{\text{residual depends on } k}$$

$$E\{\boldsymbol{e}_k[t,\boldsymbol{\theta}]\,\boldsymbol{e}_\ell^T[t-\tau,\boldsymbol{\theta}]\} = \boldsymbol{\Sigma}_{k,\ell}(\boldsymbol{\theta})\cdot\delta[\tau] \quad \forall\ k,\ell \quad \text{Covariance matrix}$$

$\delta[\tau]$: Kronecker delta

Functional dependencies of the AR pars on k:

$$\boldsymbol{A}_i(k) = \sum_{j=1}^{p} \boldsymbol{A}_{i,j}\ \underbrace{G_j(k)}_{\text{basis functions}}$$

$$\boldsymbol{\theta} = \text{vec}\left([\boldsymbol{A}_{1,1}\,\boldsymbol{A}_{1,2}\,...\,\boldsymbol{A}_{1,p}\vdots\ldots\vdots\boldsymbol{A}_{n,1}\,\boldsymbol{A}_{n,2}\,...\,\boldsymbol{A}_{n,p}\,]^T\right)$$

coefficients of projection

Application: Output–only damage detection in a composite beam

The Structure and the Setup

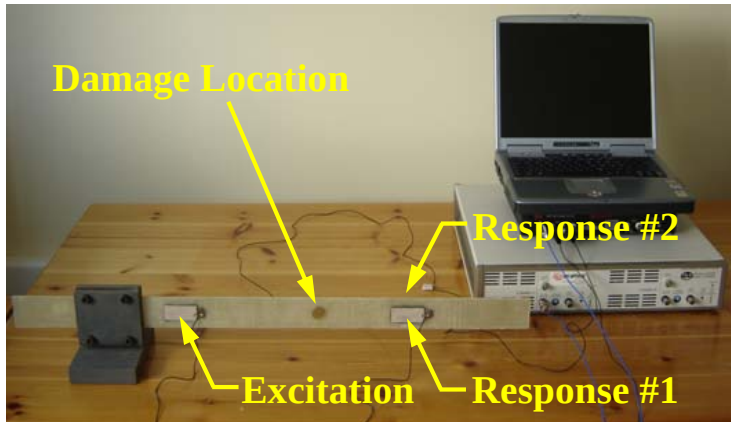
Beam: $600 \times 52.5 \times 3\ mm$ 3–ply glass–polyester composite, (fibers in longitudinal orientation)

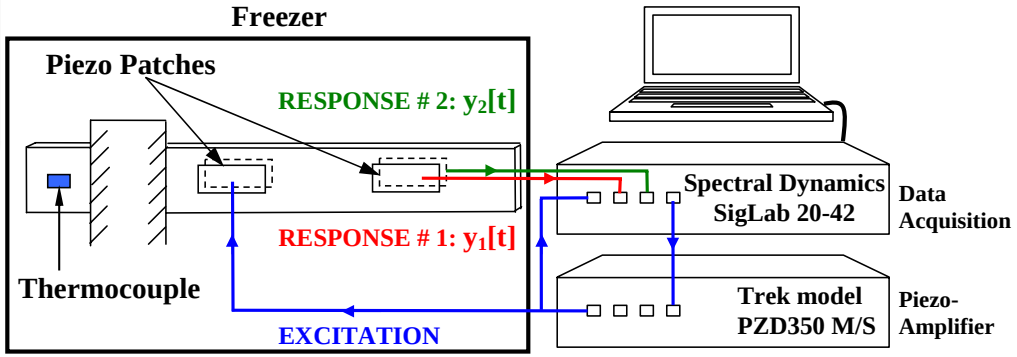
Actuators/Sensors: $50 \times 25 \times 1\ mm$ Ferroperm PZT–5A piezoceramic patches

Data Acquisition: Spectral Dynamics SigLab 20–42

Actuator Amplifier: Trek model PZD350 M/S

Temp measurements: K–type bead thermocouple





Data Acquisition

Temp range: $[-20, +20]^o C$

Signal bandwidth: $[0,\ 1000]\ Hz$

Sampling frequency: $f_s = 2\,560\ Hz$

Excitation: Gaussian white noise

Signal length: $N = 76\,800$ samples ($\approx 30\ s$)

Structural States

Healthy Structure: $M_1 = 259$ exps

Damaged Structure: 3 different damage types

Type A: single saw cut of depth 11 mm ($M_2 = 263$ exps)

Type B: single saw cut of depth 20 mm ($M_3 = 264$ exps)

Type C: single saw cut of depth 32 mm ($M_4 = 261$ exps)

The crack is located 430 mm from the clamped end.

Baseline Phase: FP–VAR$(n)_{[p]}$ identification of the healthy structure

Data length: $N = 5\,000$ samples ($\approx 1.95\ s$)

No. of exp: $M = 21$ ($\Theta = -20^o C, -18^o C, ..., -20^o C$) (nominal temps)

Sampling frequency: $f_s = 2\,560\ Hz$

FP–VAR model order specification

Fitted models: Pooled VAR(n) models with $n = 10, 11, ..., 100$

Innovations dependency: Groupwise Heteroscedastic

Parameter Estimation: WLS (QR implementation)

Model order selection criteria: BIC

Selected model order: $n = 72$

FP–VAR subspace dimensionality specification

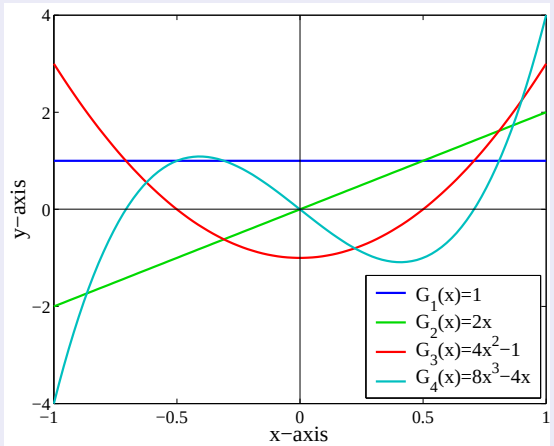
Fitted models: FP–VAR$(72)_{[p]}$ models with $p = 2, 3, ..., 10$

Func. subspaces: spanned by Chebyshev II polynomials

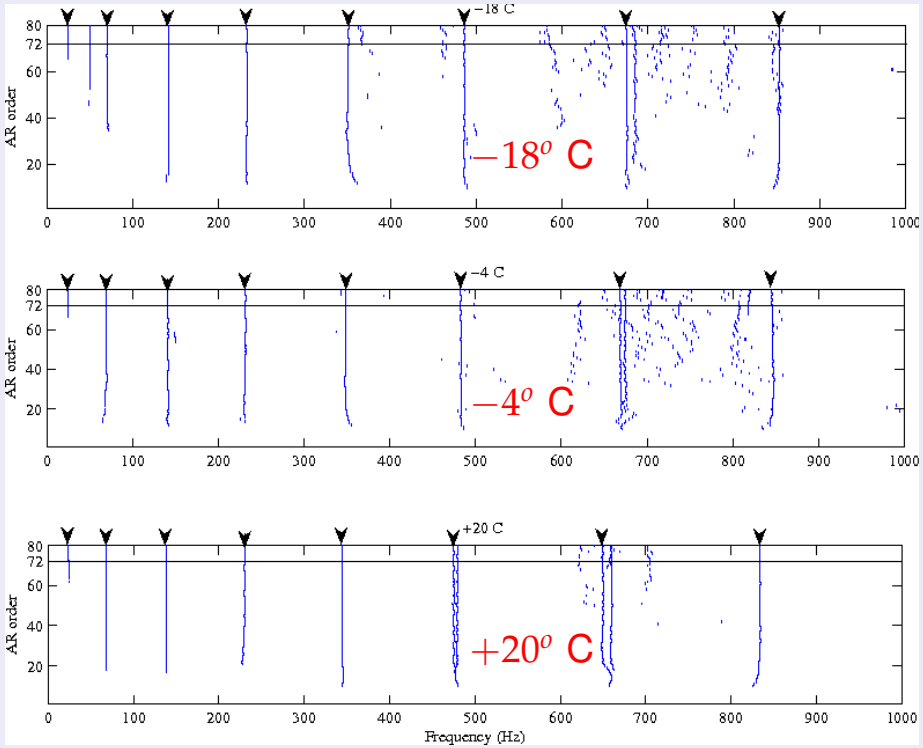
Parameter Estimation: WLS (QR implementation)

Model order selection criteria: BIC

Selected model: FP–VAR$(72)_{[6]}$



Chebyshev II polynomials in $[-1, 1]$.

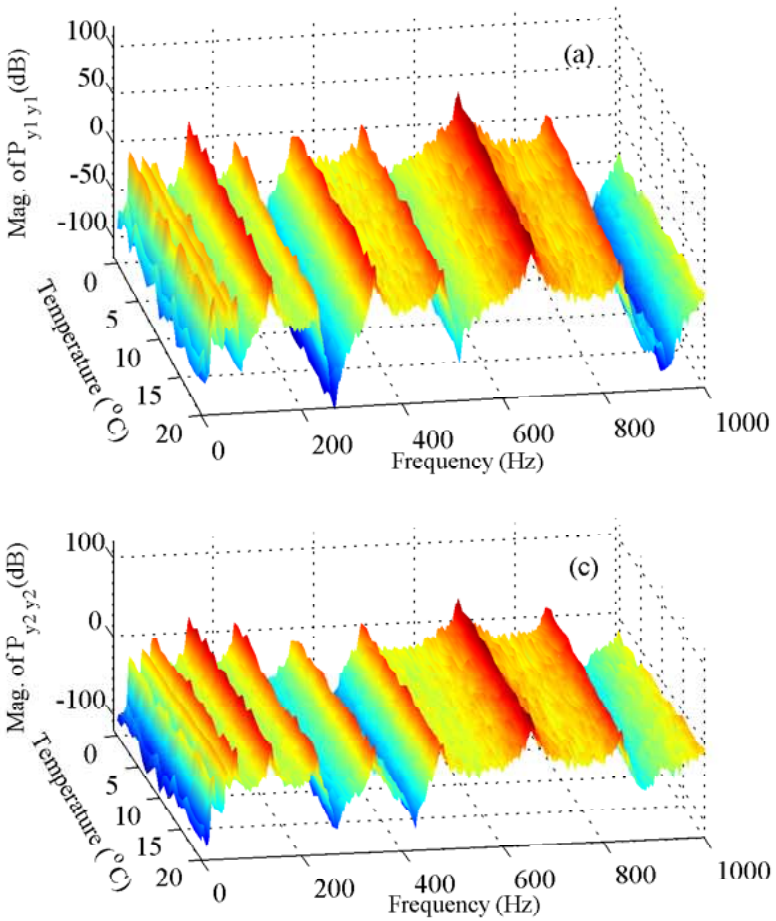


Pooled VAR$(n)_{[p]}$ stabilization plots (modes with $\zeta < 3\%$ are shown).

Selected model: FP–VAR$(72)_6$

Welch based approach

FP–VAR$(72)_6$–based approach



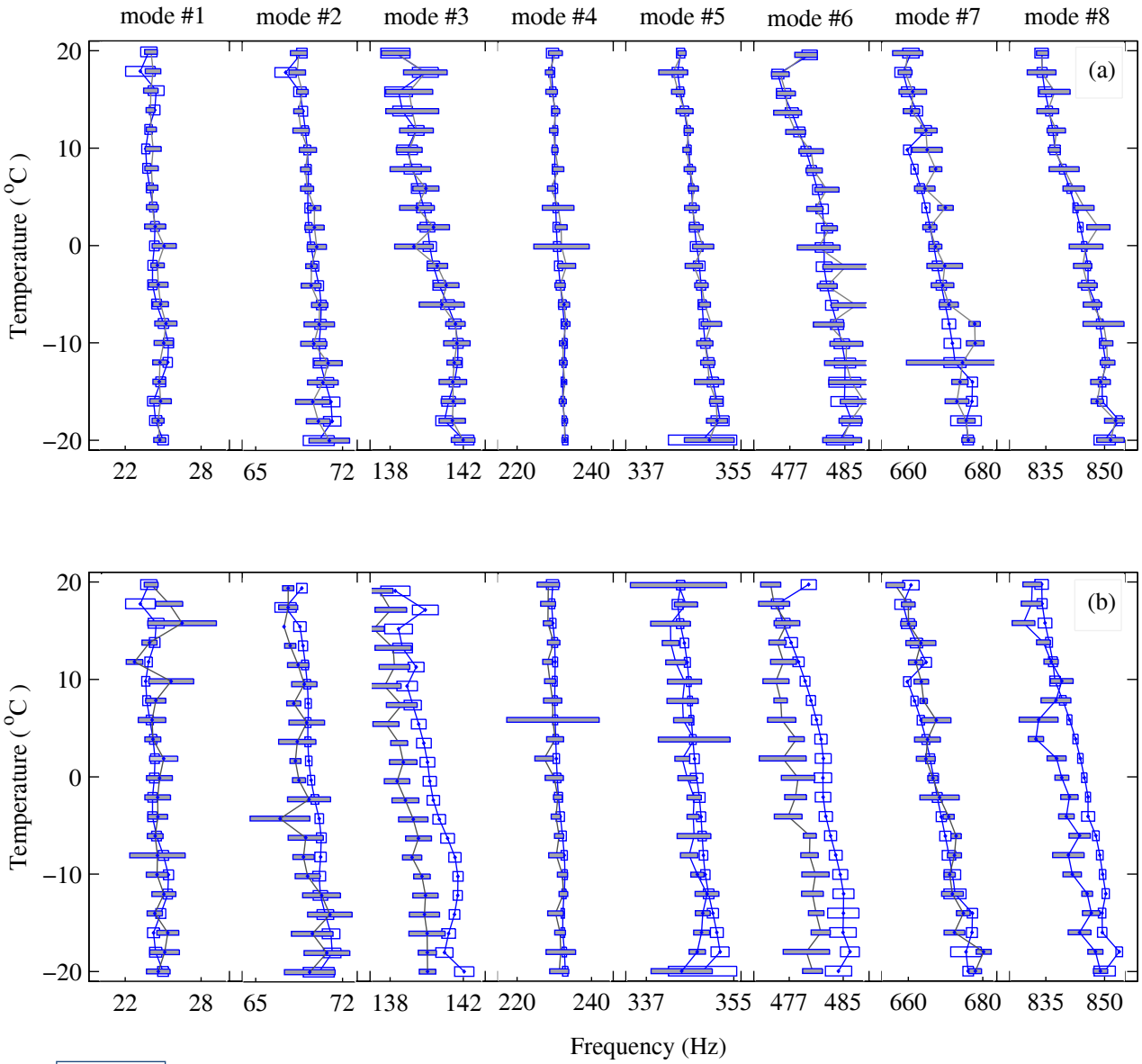
Response #1 : $y_1[t]$

Response #2 : $y_2[t]$

Power spectral densities (PSDs) for the healthy beam versus frequency and temp ($M = 21$ nominal temps): (a),(c) Welch–based estimates; (b),(d) FP–VAR$(72)_6$–based estimates.

Natural frequencies with interval estimates

FP–VAR$(72)_6$



VAR(72)

Remarks:

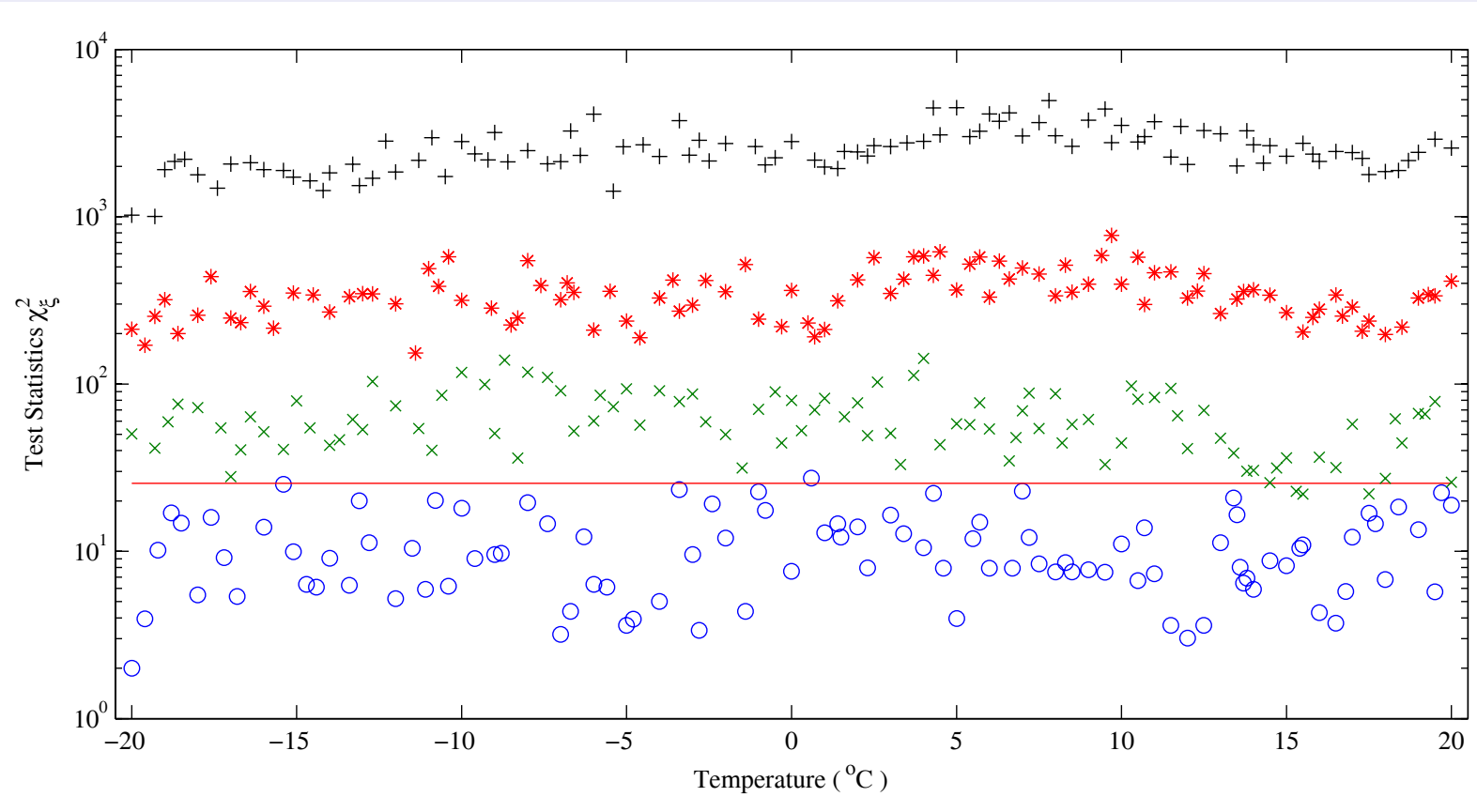
- √ Good agreement b/w healthy FP–VAR$(72)_6$ and VAR(72) estimates.
- √ Frequencies decrease with increasing temp.
- √ Damage is more evident in higher modes.

: natural frequency interval estimate (healthy)

: natural frequency interval estimate (damage type A)

Inspection Phase: Damage detection results (q contains **natural frequencies**)

$259 + 263 + 264 + 261$ experiments



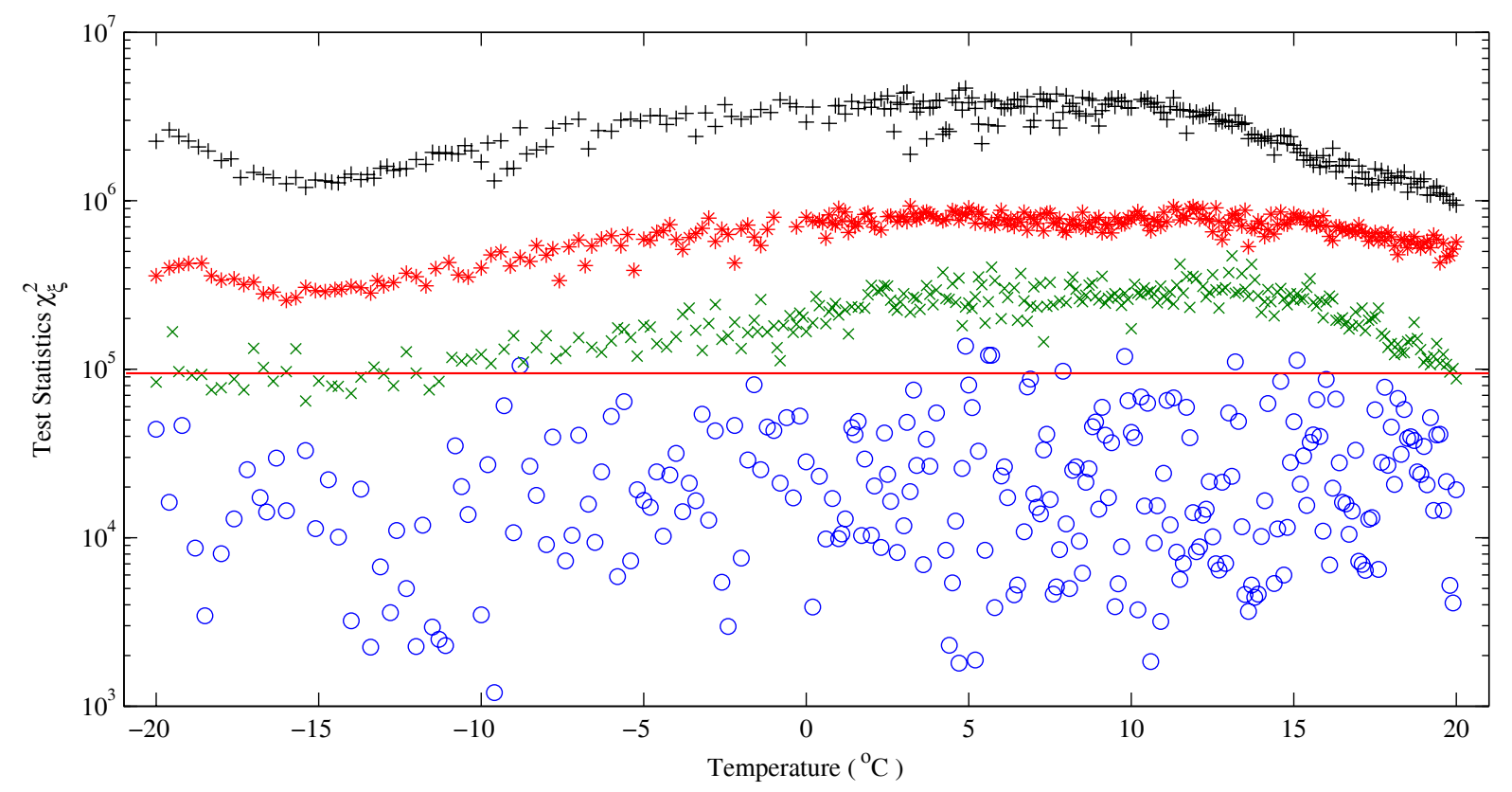
○: healthy ×: type A (11 mm) ∗: type B (20 mm) +: type C (32 mm) $(\alpha = 10^{-3})$

Remarks:

- ✔ Effective damage detection.
- ✔ **1/259** false alarm and **4/788** missed damage cases.

Inspection Phase: Damage detection results (q contains the **model parameters**)

$259 + 263 + 264 + 261$ experiments



○: healthy ×: type A (11 mm) ∗: type B (20 mm) +: type C (32 mm)

Remarks:

- ✔ Temperature trends evident.
- ✔ The adoption of a constant threshold leads to:
  - → 7/259 false alarms
  - → 17/788 missed damage cases

6.3 An FS–VTAR model based method for damage detection in time–varying structures *(Spiridonakos & Fassois 2009)*

The **F**unctional **S**eries **V**ector **TAR** (**FS–VTAR**) model class *(Poulimenos & Fassois 2006, Sato et al. 2007, Spiridonakos & Fassois 2009)*

$$\boldsymbol{x}[t] + \sum_{i=1}^{n_a} \boldsymbol{A}_i[t] \cdot \boldsymbol{x}[t-i] = \boldsymbol{e}[t], \qquad \boldsymbol{e}[t] \sim \text{NID}(\boldsymbol{0}, \boldsymbol{\Sigma}[t])$$

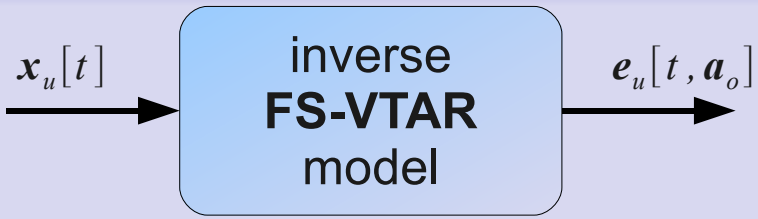
AR subspace dimensionality; AR basis functions; AR coeficients of projection; AR basis function indices

$$A_i[t]\left\{a^i_{\ell,m}[t]\right\} : a^i_{\ell,m}[t] = \sum_{j=1}^{p_a} a^{i,j}_{\ell,m} \cdot G_{b_a(j)}[t]$$

Σ subspace dimensionality; Σ basis functions; Σ coeficients of projection; Σ basis function indices

$$\boldsymbol{\Sigma}[t]\left\{s_{\ell,m}[t]\right\} : s_{\ell,m}[t] = \sum_{j=1}^{p_s} s^j_{\ell,m} \cdot G_{b_s(j)}[t]$$

Damage detection



Healthy structure

Hypothesis testing problem

H_o : $\boldsymbol{\theta} = \boldsymbol{\theta}_o$ healthy structure

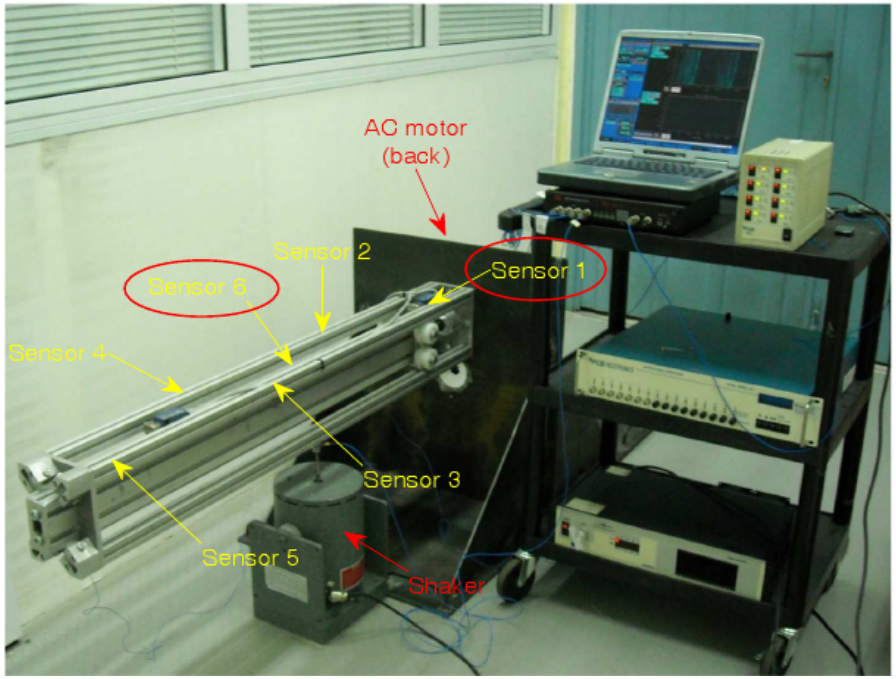
H_1 : $\boldsymbol{\theta} \neq \boldsymbol{\theta}_o$ damaged structure $\qquad \boldsymbol{\theta} = [\boldsymbol{a}^T, \boldsymbol{s}^T]^T$

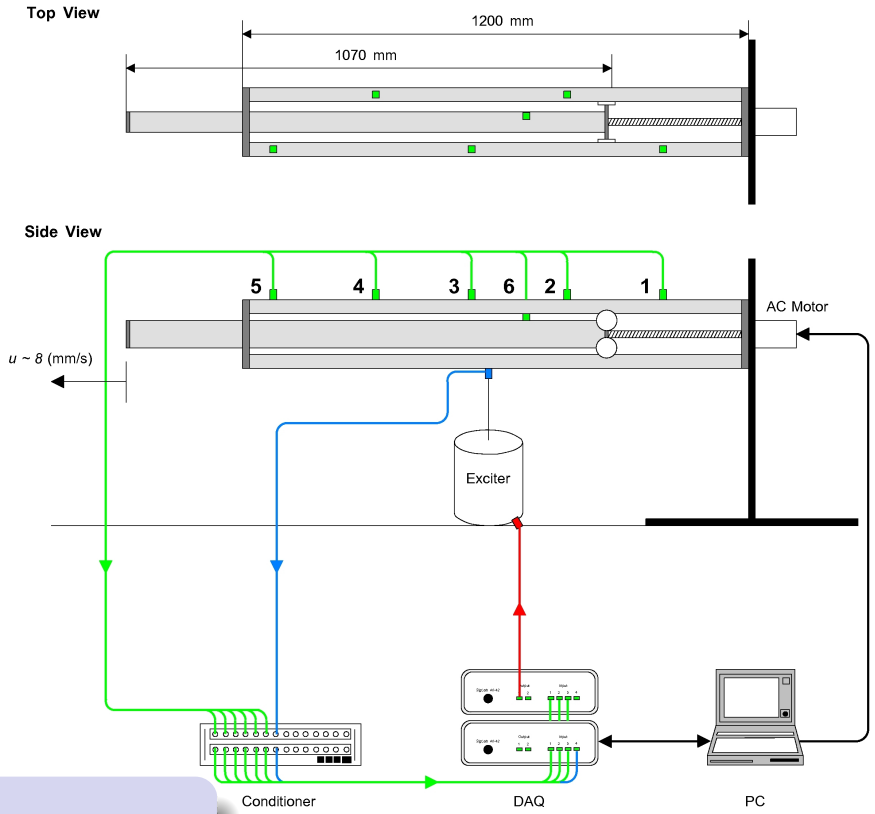
Is $\ln \mathcal{L}(x_u^N | \boldsymbol{a}_o, \boldsymbol{s}_o) \geq \ell$? $\qquad \ell$: selected threshold

yes → structure is healthy

no → structure is damaged

Application: Output–only damage detection in a time–varying arm structure





Structural states

Healthy: 8 experiments

Damages: Two damage types considered, 8 experiments each

◇ Random excitation via an electromechanical shaker (MB dynamics Modal 50A).

◇ Strain signals measured via strain gauges (PCB 740B02).

◇ Sampling frequency: 128 Hz, Bandwidth: $0 - 50$ Hz.

◇ Signal length: 13 288 samples (103.81 sec).

Extendable arm structure:

Frame:

$1200_{(\mathrm{L})} \times 125_{(\mathrm{W})} \times 170_{(\mathrm{H})}$ mm

Beam:

$1070_{(\mathrm{L})} \times 45_{(\mathrm{W})} \times 90_{(\mathrm{H})}$ mm

The damage types

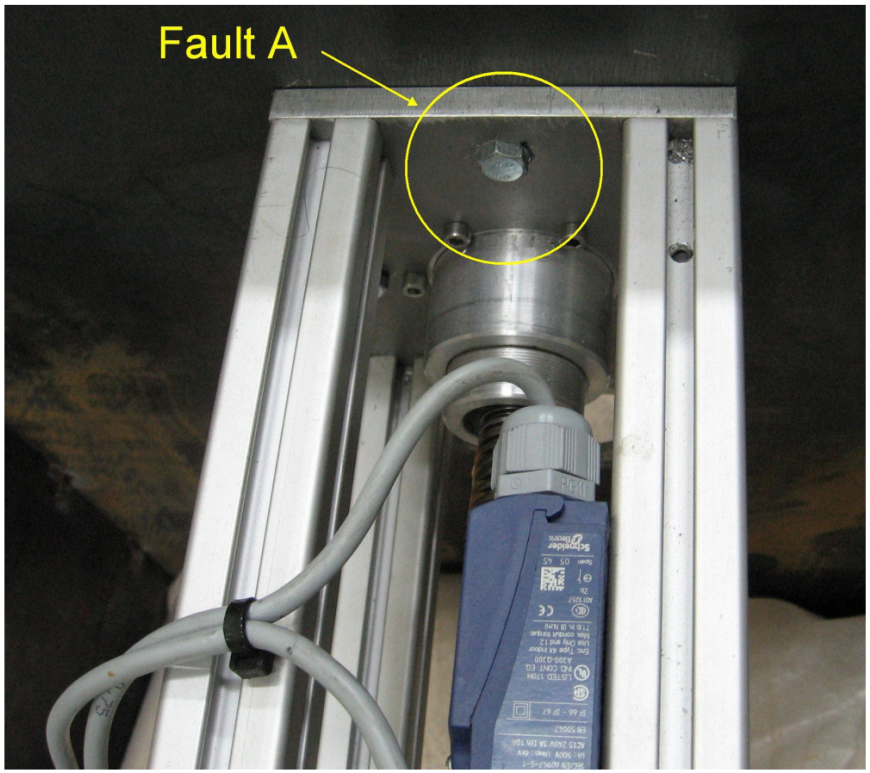
Damage type A: 8 experiments

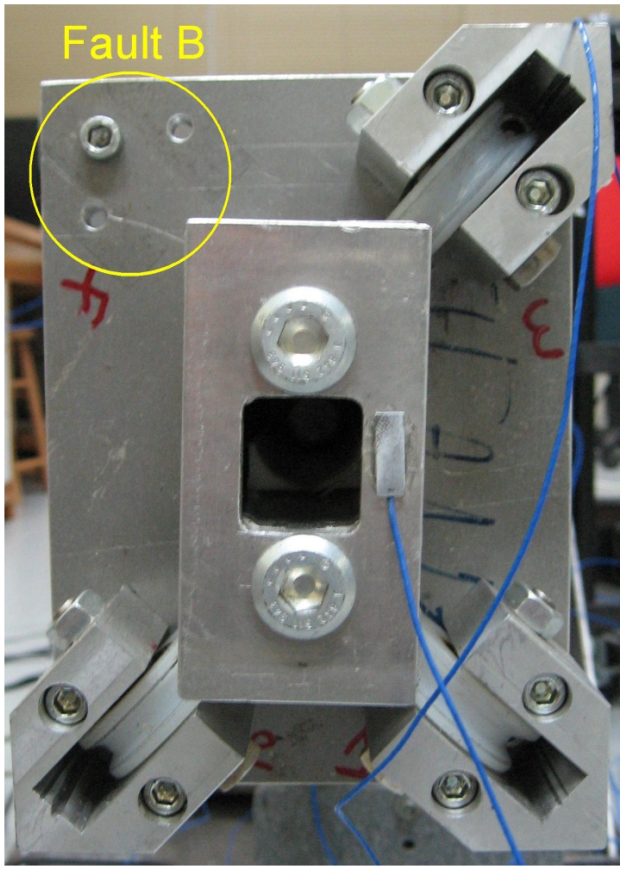
Loosening one of the three bolts at the fixed end of the structure (from 25 to 5 Nm).

Damage type B: 8 expreriments

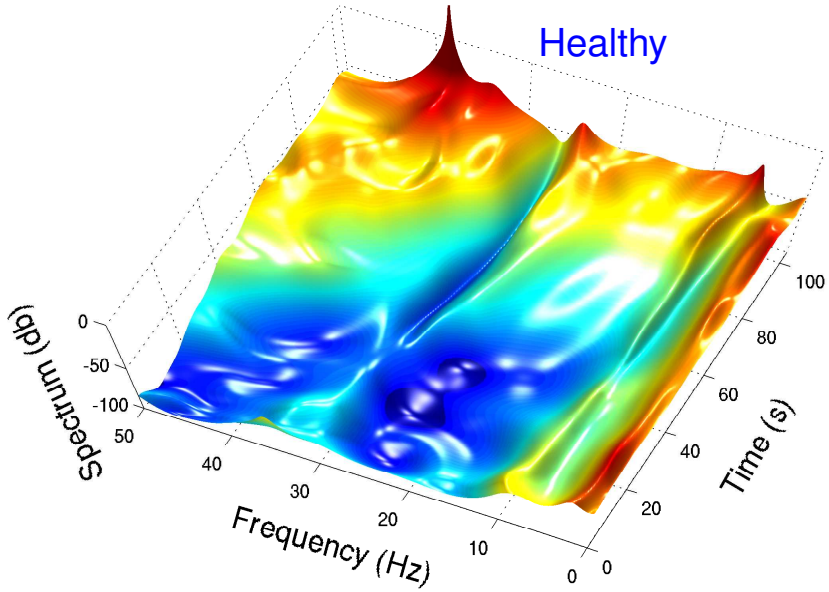
Removing one of the rolling guides at the free end of the structure.

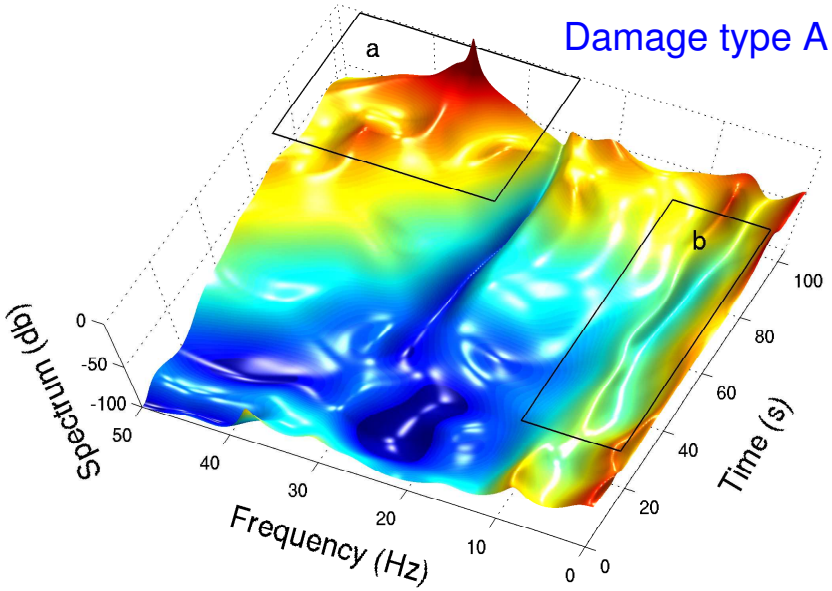
$$\left(\frac{m}{M} = \frac{0.195 \text{ kg}}{16.2 \text{ kg}} = 1.2\%\right)$$

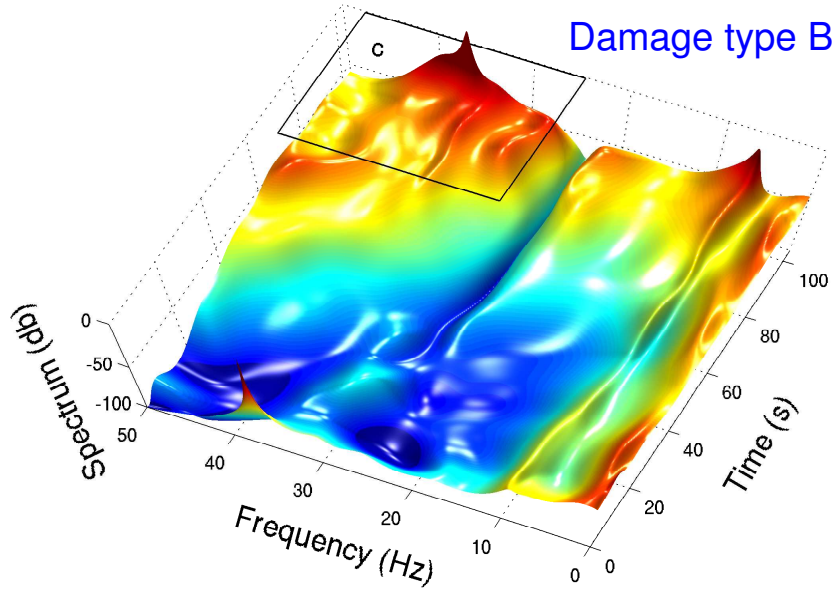




FS–VTAR(15)$_{[9,9]}$ based "frozen–time" PSD estimates ($S_{2,2}[\omega, t]$) for arbitrary selected experiments (WLS method; 5 iterations)

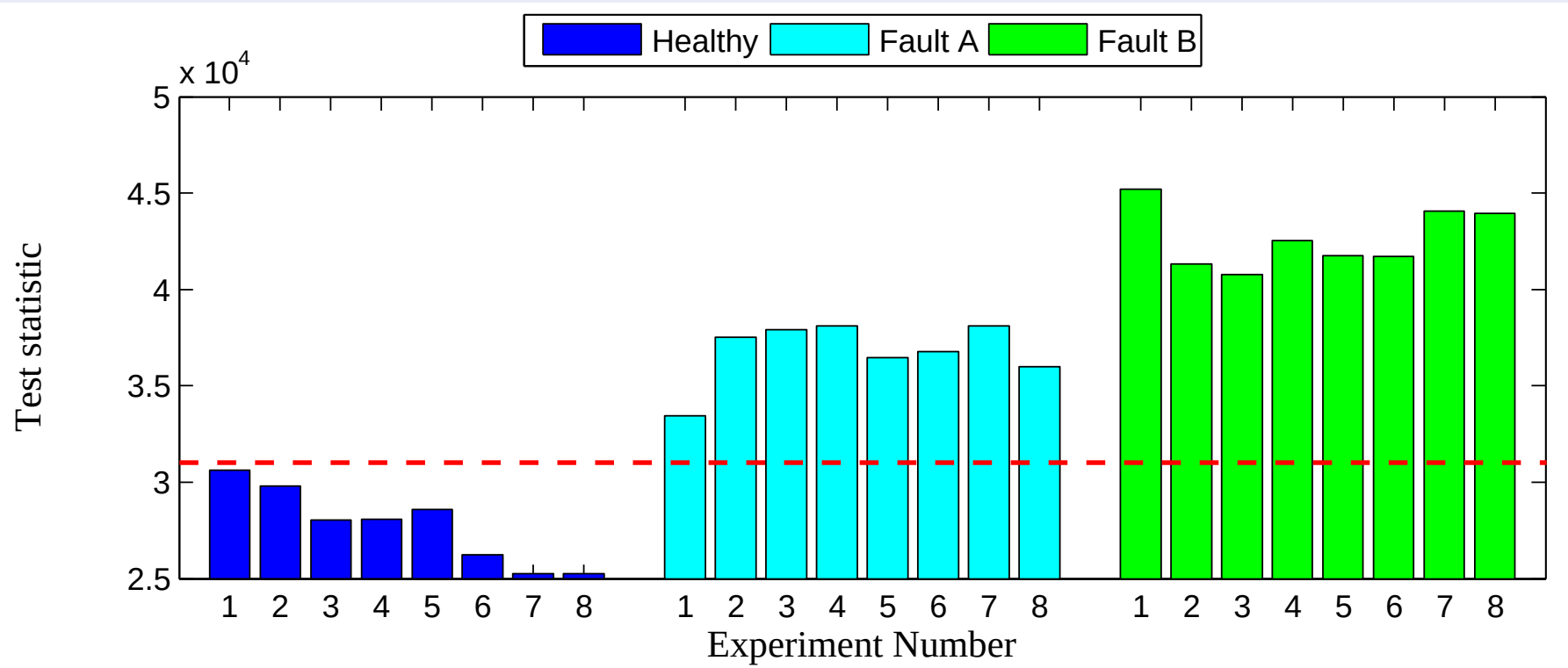






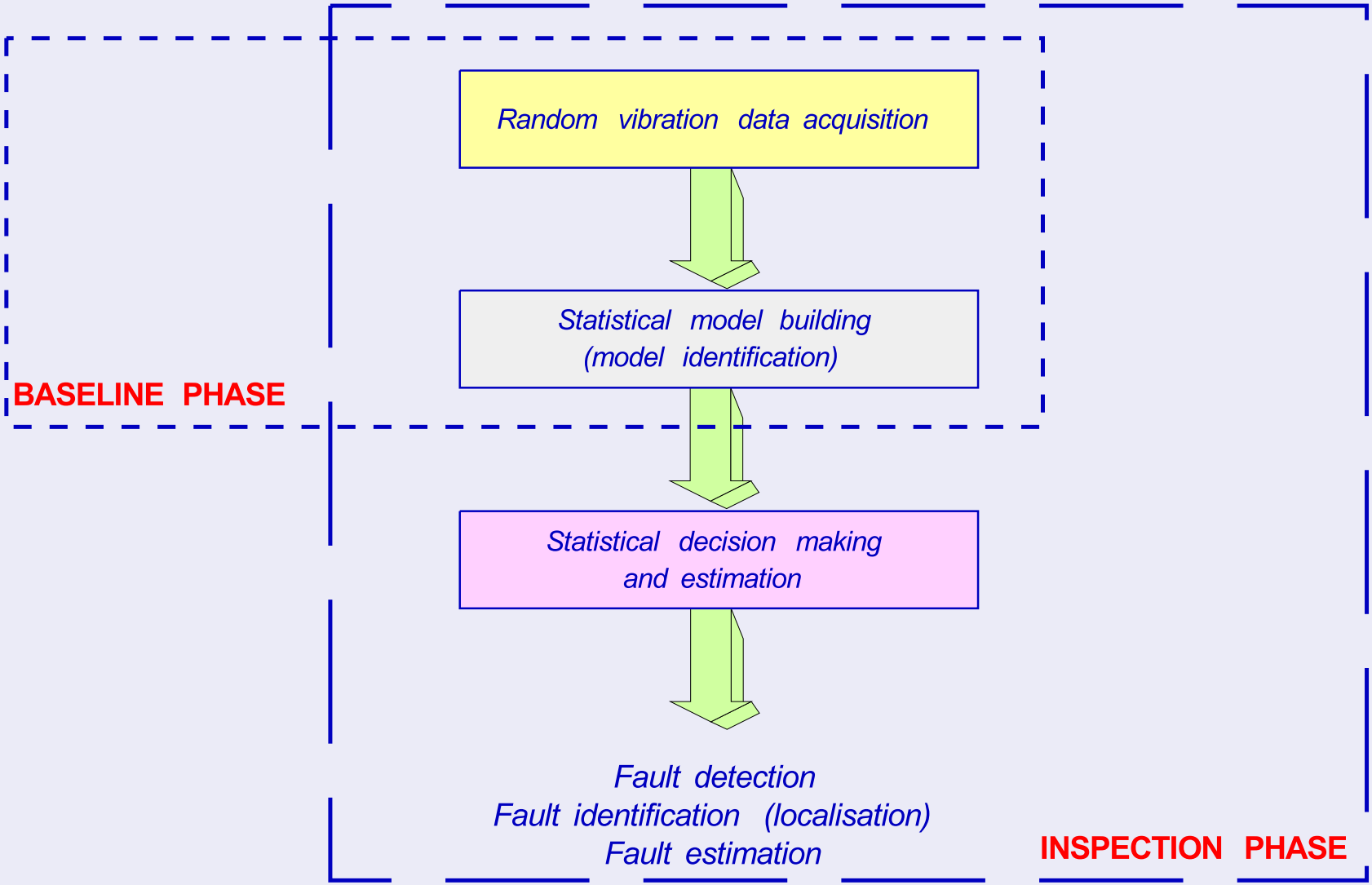
Damage detection results

Statistic test results (threshold $\ell^* = 3.1 \times 10^4$)



7. Conclusions

The main elements and phases of statistical time series methods for SHM:



Statistical time series SHM methods are:

inverse type
vibration based

non–parametric
OR
parametric

output–only
OR
input–output

Although a certain level of **maturity** has been achieved, **further developments** are needed:

- Comparative assessments to reveal limitations.
- Focus on the level 2, 3, 4 subproblems.
- Improved model to better describe uncertainties.
- Methods for complex, time–varying, non–linear structures.
- Effective multivariate methods to handle large volumes of data.
- Automated methods with minimal user expertise is a challenge.
- Effective SHM under varying operating & environmental conditions is a challenge.

Further reading

Fassois, S.D. and Sakellariou, J.S. 2007, "Time Series Methods for Fault Detection and Identification in Vibrating Structures", *Journal of the Royal Society*, Series A, Vol. 365, pp. 411-448. doi:10.1098/rsta.2006.1929

Sakellariou, J.S. and S.D. Fassois 2008, "Fault Detection and Identification in an Aircraft Skeleton Structure via a Stochastic Functional Model Based Method", *Mechanical Systems and Signal Processing*, Vol. 22(3), pp. 557-573. http://dx.doi.org/10.1016/j.ymssp.2007.09.002

Fassois S.D., Sakellariou, J.S. 2009, Statistical Time Series Methods for SHM, *in Encyclopedia of Structural Health Monitoring*, Boller, C., Chang, F. and Fujino, Y. (eds.), John Wiley and Sons Ltd, Chichester, UK, pp 443-472.

Further **information, papers** and **presentations** may be downloaded from our **web site**:

www.smsa.upatras.gr