Abstract

Let $\varrho(\nu)$ be a positive zero of the ordinary Bessel function $J_\nu(z)$ of order ν. It is shown that for every ν in the interval $-1 < \nu < \infty$ the function $\varrho(\nu)$ satisfies the differential equation $\varrho'(\nu) = \varrho(\nu)\cdot(L_\nu(x(\nu), x(\nu))$ where L_ν is the diagonal operator $L_\nu e_n = \dfrac{1}{n+\nu} e_n$ on an abstract separable Hilbert space H with the orthonormal basis e_n, $n = 1, 2, ...$ and $x(\nu)$ is a normalizes element in H. A basic result which follows easily from this equation is that the differential inequality $\varrho'(\nu) > 1$ holds for every ν in the interval $-1 < \nu < \infty$. This inequality proves that the function $\varrho(\nu) - \nu$ is a strictly increasing function in the interval $-1 < \nu < \infty$ and unifies a number of simple zeros of $J_\nu(z)$. Also from the above equation it follows easily that the function $(1+\nu)^{-1} \cdot \varrho(\nu)$ is a strictly increasing function in the interval $-1 < \nu < \infty$ for every positive zero of $J_\nu(z)$. This generalizes a previous result.