Abstract

It is proved that the greatest zero $\lambda_N(a)$ of the Laguerre polynomials $L_N^{(a)}(x)$ of degree N satisfies the differential inequalities

$$\lambda'_N(a) > 1, \quad a > -1$$

$$\lambda'_N(a) < \frac{\lambda_N(a)}{2(1+a)} + \frac{1}{2}, \quad a > -1$$

while all the zeros $\lambda_k(a)$, $k = 1, 2, ..., N$, satisfy

$$\lambda'_k(a) > 1 - \left(\frac{N}{N+a}\right)^{1/2}, \quad a \geq 0.$$

The same method applied to the largest positive zero $k_{[\frac{N}{2}]}(\lambda)$ of the Ultraspherical Polynomials gives the differential inequality

$$-\frac{k'_{[\frac{N}{2}]}(\lambda)}{k_{[\frac{N}{2}]}(\lambda)} < \frac{1}{2(1+\lambda)}, \quad \lambda \geq \frac{1}{2}.$$