Abstract

A differential equation for any positive zero $\varrho(\nu)$ of the function $\alpha J_\nu(z) + \gamma z J'_\nu(z)$ is found, where J_ν is the Bessel function of the first kind of order $\nu > -1$, J'_ν is the derivative of J_ν and α, γ are real numbers. It is proved that:

(i) The function $\varrho(\nu)/(1+\nu)$ decreases with $\nu > -$ in the case $\alpha \geq 1$, and the function $\varrho(\nu)/(\alpha+\nu)$ decreases with $\nu > -\alpha$ in the case $\alpha < 1$.

(ii) The zeros of the function $\alpha J_\nu + z J'_{nu}(z)$ increase with $\nu > -1$ in the case $\alpha \geq 1$ and with $\nu > -\alpha$ in the case $\alpha < 1$. The first result leads to a number of lower and upper bounds for the zeros of the function $\alpha J_{nu}(z) + z J'_\nu(z)$ which complete and improve previously known bounds. The second result improves a well-known result.