Hypernetworks
in the
Science of Complex Systems
Part III
Examples

1st PhD School on “Mathematical Modelling of Complex Systems”. 18-29 July 2011, Patras, Greece
Type I Dynamics and Type II Dynamics

Type I dynamics

• relatively fast
• relatively low investment – ‘normal’ business
  includes movement and activity of agents

Type II dynamics

• relatively slow
• relatively expensive – infrastructure change
A Q-analysis of the Patras Student Hypernetwork Data

Jeffrey Johnson, Patras PhD School & The Open University, 24th July 2011

[1] Introduction

On 21st July 2011 a group of fifty two students and teachers at the Patras PhD School were asked to rate the first forty six of the following things according to how much they liked them, 0 meaning not at all and 6 or 7 meaning very much. The last eight were contributed by member of the group and also rated.

The list is rather heterogeneous and the data rather poor, since some people used a 0-6 scale while a few used a 0-7 scale. Also the last eight items were not all rated by all participants. Furthermore some participants complained that rating some items would be ambiguous with the possibility, for example, of liking very much to listen to one kind of music and hating to listen to another. This illustrates a common problem that complex systems data are often incomplete and inconsistent.
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Incomplete and inconsistent
<table>
<thead>
<tr>
<th></th>
<th>1. food</th>
<th>19. snow</th>
<th>37. psychology</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. wine</td>
<td>20. sunshine</td>
<td>38. sociology</td>
<td></td>
</tr>
<tr>
<td>3. smoking</td>
<td>21. wind</td>
<td>39. political science</td>
<td></td>
</tr>
<tr>
<td>4. reading science</td>
<td>22. holidays</td>
<td>40. economics</td>
<td></td>
</tr>
<tr>
<td>5. writing science</td>
<td>23. computer games</td>
<td>41. history</td>
<td></td>
</tr>
<tr>
<td>6. talking science</td>
<td>24. facebook-networking</td>
<td>42. geography</td>
<td></td>
</tr>
<tr>
<td>7. parties</td>
<td>25. email</td>
<td>43. literature</td>
<td></td>
</tr>
<tr>
<td>8. football</td>
<td>26. bankers</td>
<td>44. art</td>
<td></td>
</tr>
<tr>
<td>9. sport</td>
<td>27. children</td>
<td>45. statistics</td>
<td></td>
</tr>
<tr>
<td>10. gardening</td>
<td>28. pets</td>
<td>46. mathematics</td>
<td></td>
</tr>
<tr>
<td>11. watch TV</td>
<td>29. chat_friends</td>
<td>47. sleeping</td>
<td></td>
</tr>
<tr>
<td>12. listen music</td>
<td>30. be_with_family</td>
<td>48. travelling</td>
<td></td>
</tr>
<tr>
<td>13. play music-sing</td>
<td>31. teamwork</td>
<td>49. complexity science</td>
<td></td>
</tr>
<tr>
<td>14. read novels</td>
<td>32. administration</td>
<td>50. dreaming-daydreaming</td>
<td></td>
</tr>
<tr>
<td>15. cooking</td>
<td>33. organise_events</td>
<td>51. give_presentations</td>
<td></td>
</tr>
<tr>
<td>16. walk-hike</td>
<td>34. physics</td>
<td>52. philosophy</td>
<td></td>
</tr>
<tr>
<td>17. swimming</td>
<td>35. chemistry</td>
<td>53. democracy</td>
<td></td>
</tr>
<tr>
<td>18. rain</td>
<td>36. biology</td>
<td>54. meditation</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. The things people rated according to their likes and dislikes.
What can be done with messy data like this? At the level of the whole population, one possibility is to plot the responses as shown in Figure 1. Unremarkably, from this it can be concluded that for this group of people, the majority like sunshine and snow and the majority dislike wind and rain. This could be done at the population level for everything listed. This would show that no-one disliked liked food, 48% liked wine while 26% disliked it, 88% disliked smoking while 12% liked it, and so on.

Figure 1. Show and sunshine are preferred to rain and wind.
[2] A Q-analysis of the Student Hypernetwork

Recall that a relational simplex is a set of vertices combined by an n-ary relation. For example, a person $p_i$ might have a simplex $\sigma = \langle \text{food, wine, chat\_friends, be\_with\_family;} R, \text{questionnaire}\rangle$ where these four vertices are combined by them being positive answers to the questionnaire, as shown in Figure 2(a). A person with such a simplex is likely to enjoy dinner parties with their family and friends. In its simplest form, a hypernetwork is any set of relational simplices.

Two simplices are defined to be $q$-near if they share a $q$-dimensional face. For example the simplices in Figure 2(b) share the 1-dimensional face $\langle \text{economics, statistics}\rangle$ and they are 1-near.

Figure 2. Relational simplices (a) predisposed to social meals and (b) to discussing economic trends
The \( q \)-nearness relation is reflexive and symmetric and its transitive closure is an equivalence relation that partitions a hypernetwork into \textit{\( q \)-connected components}. The \textit{hub} of a component is defined to be the intersection of all its simplices. There are two major types of component, those that have an intersection and form star-like configurations (Figure 3(a)) and those that make chains which have no non-trivial hub (Figure 3(b)).

A \textit{Q-analysis} of a hypernetwork is a listing of all its \( q \)-connected components for all values of \( q \). The Q-analysis for the student’s like relation is given in Table 2.

![Diagram](image.png)

(a) simplices connected through a 2-hub  
(b) a hub-free chain of 1-connected simplices

\textbf{Figure 3. Two major classes of \( q \)-connected components}

1st PhD School on “Mathematical Modelling of Complex Systems”. 18-29 July 2011, Patras, Greece
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<table>
<thead>
<tr>
<th>Number of Students</th>
<th>Item liked</th>
<th>Number of Students</th>
<th>Item liked</th>
</tr>
</thead>
<tbody>
<tr>
<td>37</td>
<td>reading, science</td>
<td>16</td>
<td>email</td>
</tr>
<tr>
<td>33</td>
<td>talking, science</td>
<td>16</td>
<td>history</td>
</tr>
<tr>
<td>33</td>
<td>physics</td>
<td>16</td>
<td>literature</td>
</tr>
<tr>
<td>33</td>
<td>mathematics</td>
<td>15</td>
<td>pets</td>
</tr>
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<td>30</td>
<td>sunshine</td>
<td>15</td>
<td>read novels</td>
</tr>
<tr>
<td>29</td>
<td>travelling</td>
<td>15</td>
<td>statistics</td>
</tr>
<tr>
<td>28</td>
<td>listen music</td>
<td>14</td>
<td>cooking</td>
</tr>
<tr>
<td>28</td>
<td>democracy</td>
<td>14</td>
<td>dreaming-daydreaming</td>
</tr>
<tr>
<td>27</td>
<td>complexity science</td>
<td>13</td>
<td>teamwork</td>
</tr>
<tr>
<td>26</td>
<td>holidays</td>
<td>13</td>
<td>rain</td>
</tr>
<tr>
<td>26</td>
<td>be with family</td>
<td>12</td>
<td>sociology</td>
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<tr>
<td>26</td>
<td>writing, science</td>
<td>12</td>
<td>give presentations</td>
</tr>
<tr>
<td>26</td>
<td>walk-hike</td>
<td>12</td>
<td>meditation</td>
</tr>
<tr>
<td>25</td>
<td>chat, friends</td>
<td>11</td>
<td>wind</td>
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<td>25</td>
<td>children</td>
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<td>football</td>
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<td>psychology</td>
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<td>computer games</td>
</tr>
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<td>play, music-sing</td>
<td>8</td>
<td>political science</td>
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<tr>
<td>17</td>
<td>philosophy</td>
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<td></td>
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</tbody>
</table>

Table 3. Number of items liked by students
This hypernetwork is characterised by a dominant component that emerges at $q = 30$ with two students S02 and S26 sharing 31 items. This component grows steadily as $q$ decreases, as shown in Figure 4.

Figure 4. The emergence of the dominant component in the students’ likes structure.
<table>
<thead>
<tr>
<th>Student</th>
<th>Dimension</th>
<th>(q)-\text{shared}</th>
<th>eccentricity</th>
<th>Student</th>
<th>Dimension</th>
<th>(q)-\text{shared}</th>
<th>eccentricity</th>
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<td>30</td>
<td>0.24</td>
<td>S41</td>
<td>40</td>
<td>30</td>
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<td>S01</td>
<td>25</td>
<td>19</td>
<td>0.23</td>
</tr>
<tr>
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<td>37</td>
<td>30</td>
<td>0.18</td>
<td>S11</td>
<td>21</td>
<td>16</td>
<td>0.23</td>
</tr>
<tr>
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<td>35</td>
<td>31</td>
<td>0.11</td>
<td>S02</td>
<td>40</td>
<td>31</td>
<td>0.22</td>
</tr>
<tr>
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<td>28</td>
<td>0.17</td>
<td>S33</td>
<td>37</td>
<td>30</td>
<td>0.18</td>
</tr>
<tr>
<td>S08</td>
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<td>0.18</td>
<td>S08</td>
<td>33</td>
<td>27</td>
<td>0.18</td>
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<tr>
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<td>14</td>
<td>0.17</td>
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<td>0.07</td>
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<td>0.14</td>
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<td>14</td>
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<td>S38</td>
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<td>11</td>
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<td>S32</td>
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<td>S31</td>
<td>13</td>
<td>12</td>
<td>0.07</td>
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<td>0.07</td>
<td>S03</td>
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<td>20</td>
<td>0.05</td>
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<td>18</td>
<td>17</td>
<td>0.05</td>
</tr>
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<td>S38</td>
<td>12</td>
<td>11</td>
<td>0.08</td>
<td>S16</td>
<td>18</td>
<td>17</td>
<td>0.05</td>
</tr>
<tr>
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<td>0</td>
<td>S23</td>
<td>8</td>
<td>8</td>
<td>0</td>
</tr>
</tbody>
</table>

(a) sorted by dimension
(b) sorted by eccentricity

Table 4. Student dimensions, largest value of \(q\)-\text{shared} and eccentricities.
[3] Filtering Out Low Information Items

As Table 3 shows, many items are liked by the majority of the students. For example 42 students like reading science, 33 like talking about science, 33 like physics and 33 like mathematics. This is not very surprising for a PhD school on mathematics and complexity! Nor is it surprising that 30 students like sunshine and 29 like travelling.

As an experiment these low information items were filtered out from the data, with only those liked by 21 or less students included in the Q-analysis. It would have been possible for the students to cluster differently but the pattern of high connectivity again emerged. The following Galois pairs were detected in the components of this Q-analysis.

\[ q_2 = 18 \]
\[ \langle S02 \text{ S33} \rangle < \text{play music-sing, snow, computer games, biology, dreaming-daydreaming, food, cooking, swimming, email, pets, teamwork, chemistry, psychology, sociology, political science, economics, give presentations, philosophy, meditation} \rangle \]

\[ q_2 = 14 \]
\[ \langle S02 \text{ S33 S41} \rangle < \text{play music-sing, snow, biology, food, cooking, swimming, email, pets, teamwork, chemistry, psychology, sociology, philosophy} \rangle \]

\[ q_2 = 12 \]
\[ \langle S02 \text{ S33 S41 S08} \rangle < \text{play music-sing, biology, food, cooking, swimming, pets, psychology, philosophy} \rangle \]

\[ q_2 = 11 \]
\[ \langle S02 \text{ S08 S14 S26 S33 S41 S42} \rangle < \text{food 7, psychology 7} \rangle \]
[5] Conclusions

The questionnaire used to collect the date had typical imperfections and the data collected were messy and imperfect.

Despite this the analysis suggests that the students at the school are highly connected in terms of the things they like, and this is consistent with them being selected to attend the school.

... what if?
Example: robot soccer
Example: robot soccer
Example: robot soccer
Example: robot soccer
Example: robot soccer

‘scoring a goal’
Example: robot soccer

‘scoring a goal’

‘the old 1-2 move’
Example: robot soccer

‘The old 1-2’ as a trajectory in multidimensional space
Example: robot soccer

‘The old 1-2’ as a trajectory in multidimensional space
Example: robot soccer

‘The old 1-2’ as a trajectory in multidimensional space
Example: robot soccer

‘The old 1-2’ as a trajectory in multidimensional space
Example: robot soccer

‘The old 1-2’ as a polyhedral trajectory in multidimensional space
Example: robot soccer

formation of a polyhedron is a structural event

Goal scoring event

Passing ball event

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Fig. 6. The hypergraph edge \( \{w_1, w_2, b_1\} \) configured into two different structures.

(a) \( \langle w_1, w_2, b_1; R_1 \rangle \) \hspace{1cm} (b) \( \langle w_1, w_2, b_1; R_2 \rangle \) \hspace{1cm} (c) \text{base(defender’s dilemma)} = \text{base(XYZ)}

Fig. 7. The same set of players assembled in different ways forms a structure at Level \( N + 2 \).
Example: design
Example: designing the future - policy
The relation between 45 students and their answers to 20 maths questions
The first column of Table 1.1 shows that all students except two gave the answer \( C_1 \), making it highly likely that this is the correct answer. In general one would expect the majority response to be correct. For each question the highest responses were as follows:

<table>
<thead>
<tr>
<th>Answer</th>
<th>Students</th>
<th>Answer</th>
<th>Students</th>
<th>Answer</th>
<th>Students</th>
<th>Answer</th>
<th>Students</th>
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</thead>
<tbody>
<tr>
<td>( C_1 )</td>
<td>43</td>
<td>( F_6 )</td>
<td>24</td>
<td>( F_{11} )</td>
<td>37</td>
<td>( D_{16} )</td>
<td>31</td>
</tr>
<tr>
<td>( B_2 )</td>
<td>32</td>
<td>( C_7 )</td>
<td>40</td>
<td>( C_{12} )</td>
<td>41</td>
<td>( G_{17} )</td>
<td>42</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>45</td>
<td>( D_8 )</td>
<td>26</td>
<td>( B_{13} )</td>
<td>35</td>
<td>( D_{18} )</td>
<td>33</td>
</tr>
<tr>
<td>( G_4 )</td>
<td>34</td>
<td>( E_9 )</td>
<td>36</td>
<td>( D_{14} )</td>
<td>30</td>
<td>( C_{19} )</td>
<td>30</td>
</tr>
<tr>
<td>( C_5 )</td>
<td>45</td>
<td>( A_{10} )</td>
<td>34</td>
<td>( F_{15} )</td>
<td>26</td>
<td>( F_{20} )</td>
<td>36</td>
</tr>
</tbody>
</table>

Table 1.1. The most popular answers selected by students.

At first sight these are the correct answers, since in all cases more than half the students gave these responses. For most of the questions the students overwhelmingly agree, but for some the agreement is not so clear. For example \( F_6 \) was selected by 24 students (53\%), \( D_8 \) and \( F_{15} \) were selected by 26 students (58\%). How certain can one be sure that the most popular answers are really correct?
Of particular interest is that 21 students give the answer $E_6$ (47%) $q_6$, compared to 24 (53%) for $F_6$. Are the majority correct in this case?

A Q-analysis of the hypernetwork has the component \{s_4, s_{16}, s_{19}, s_{32}, s_{42}, s_{43}\} at $q = 19$, meaning that each of these students has given exactly the same answers to all twenty questions, i.e. the simplex $\sigma = \langle C_1, B_2, A_3, G_4, C_5, E_6, C_7, D_8, E_9, A_{10}, F_{11}, C_{12}, B_{13}, D_{14}, F_{15}, D_{16}, G_{17}, D_{18}, C_{19}, F_{20} \rangle$. Its vertices are exactly the same as the list of most frequently occurring answers given in Table 1.1, with the exception of $E_6$, instead of $F_6$. Have these six students answered $q_6$ incorrectly?

To investigate the lower level connectivities, students $s_4, s_{16}, s_{19}, s_{32}, s_{42},$ and $s_{43}$ were removed from the system, and the Q-analysis rerun. Again one large component emerges, this time at $q = 17$, with students $s_1, s_3, s_6, s_8, s_{11}, s_{13}, s_{20}, s_{25}, s_{27}, s_{29}, s_{31}, s_{34},$ and $s_{36}$. 8 of these students favour $E_6$ (62%) while five favour $F_6$ (38%). Combined with the previous six, this means that 14 of the most highly connected students favour $E_6$ (74%) while 5 favour $F_6$ (26%).

Thus although $F_6$ is the most popular answer for $q_6$, the most highly connected students overwhelmingly prefer $E_6$. Assuming that the most highly connected students will be the best, this is a very strong indication that $E_6$ is the correct answer.
Of particular interest is that 21 students give the answer $E_6$ (47%) $q_6$, compared to 24 (53%) for $F_6$. Are the majority correct in this case?

A Q-analysis of the hypernetwork has the component $\{s_4, s_{16}, s_{19}, s_{32}, s_{42}, s_{43}\}$ at $q = 19$, meaning that each of these students has given exactly the same answers to all twenty questions, i.e. the simplex $\sigma = \{C_1, B_2, A_3, G_4, C_5, E_6, C_7, D_8, E_9, A_{10}, F_{11}, C_{12}, B_{13}, D_{14}, F_{15}, D_{16}, G_{17}, D_{18}, C_{19}, F_{20}\}$. Its vertices are exactly the same as the list of most frequently occurring answers given in Table 1.1, with the exception of $E_6$, instead of $F_6$. Have these six students answered $q_6$ incorrectly?

To investigate the lower level connectivities, students $s_4, s_{16}, s_{19}, s_{32}, s_{42},$ and $s_{43}$ were removed from the system, and the Q-analysis rerun. Again one large component emerges, this time at $q = 17$, with students $s_1, s_3, s_6, s_8, s_{11}, s_{13}, s_{20}, s_{25}, s_{27}, s_{29}, s_{31}, s_{34},$ and $s_{36}$. 8 of these students favour $E_6$ (62%) while five favour $F_6$ (38%). Combined with the previous six, this means that 14 of the most highly connected students favour $E_6$ (74%) while 5 favour $F_6$ (26%).

Thus although $F_6$ is the most popular answer for $q_6$, the most highly connected students overwhelmingly prefer $E_6$. Assuming that the most highly connected students will be the best, this is a very strong indication that $E_6$ is the correct answer.
Example: Recognising cells in noisy images

Figure 1.4 an microscope image of a zebra fish. Advanced laser microscopy produces sequences of three-dimensional images of the organism *in vivo* through time, allowing scientists to study the cells as they divide and plot the development of the organism. This requires machine vision to detect the cells and match them through the sequence of images over a period of hours. The images are noisy and have variable contrast and illumination making it hard to segment them accurately into polygons contains the cells.

Fig. 1.4 An *in-vivo* microscope image of a zebra fish embryo.
Example: Recognising cells in noisy images

(a) The neighbours of $p$   (b) $p'$ is the darkest neighbour of $p$  (c) a light-to-dark path

Fig. 1.5  The darkest neighbour relations

(a) a cell in the noisy image   (b) the max-gradient graph   (c) the segmented image

Fig. 1.6  Oriented links in image segmentation
Example: Recognising cells in noisy images

Fig. 1.7 A digraph-based segmentation of the zebra fish image
Example

Multilevel dynamics of greenhouse gas reduction.

UK carbon emission to cuts - 26% by 2020 80% by 2050
Example

Multilevel dynamics of greenhouse gas reduction.

UK carbon emission to cuts - 26% by 2020 80% by 2050

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Example

Multilevel dynamics of greenhouse gas reduction.

UK carbon emission to cuts - 26% by 2020 80% by 2050
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Multilevel dynamics of greenhouse gas reduction.

UK carbon emission to cuts - 26% by 2020 80% by 2050

Tipping point - huge change in how people live
Example

Multilevel dynamics of greenhouse gas reduction.

UK carbon emission to cuts - 26% by 2020 80% by 2050

Huge change for individual human agent’s values & beliefs

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Huge change for individual human agent’s values & beliefs

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Agents, Climate Change & Systems of Systems of Systems

This is a system of systems of systems

Greenhouse gas emissions system

Land-use transportation subsystem
Building heating subsystem

Personal belief subsystem
Personal norm and values subsystems
Personal aspirations subsystem
Personal action subsystem

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Figure 2. discrete micro-dynamic, semi-continuous meso-dynamics and continuous macro-dynamics.

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Figure 2. discrete micro-dynamic, semi-continuous meso-dynamics and continuous macro-dynamic
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Government Policies

Macro-level (continuous dynamics)

Meso-levels (semi-continuous dynamics)

Agents

Micro-level (discrete dynamics)

Figure 2. discrete micro-dynamic, semi-continuous meso-dynamics and continuous macro-dynamic
Agents, Climate Change & Systems of Systems of Systems

Must apply bottom-up agent based models
Agents, Climate Change & Systems of Systems of Systems

Must apply bottom-up agent based models

Create a synthetic micropopulation
- used for TRANSIMS transportation model
- agents choose where they live & their travel

Add data on beliefs, values, aspirations
- can model how people change behaviour
Agents, Climate Change & Systems of Systems of Systems

Model agent interactions on the hypernetworks

How people change their beliefs, values, aspirations

Figure 2. discrete micro-dynamic, semi-continuous meso-dynamics and continuous macro-dynamic
Agents, Climate Change & Systems of Systems of Systems

Model agent interactions on the hypernetworks

How people change their beliefs, values, aspirations

How people change their behaviour

Figure 2. discrete micro-dynamic, semi-continuous meso-dynamics and continuous macro-dynamic
Agents, Climate Change & Systems of Systems of Systems

Model agent interactions on the hypernetworks

How people change their beliefs, values, aspirations.

Expect rapid changes – tipping points?

Figure 2. discrete micro-dynamic, semi-continuous meso-dynamics and continuous macro-dynamic

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Agents, Climate Change & Systems of Systems of Systems

Figure 2. discrete micro-dynamic, semi-continuous meso-dynamics and continuous macro-dynamic

Government Policy

macro-level
(continuous dynamics)

meso-levels
(semi-continuous dynamics)

Agents

micro-level
(discrete dynamics)
Grand Challenge: Automating Ontology Formation

Can a machine abstract appropriate intermediate word within changing environments?

Appropriate = ‘useful’ = survival

Work with Valerie Rose
Grand Challenge: Automating Ontology Formation
Grand Challenge: Automating Ontology Formation

Learn to recognise things like this please

OK – you’re the boss (my environment)

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Grand Challenge: Automating Ontology Formation

I want the system to discriminate smiley and frownie face
I want the system to discriminate smiley and frownie face
Grand Challenge: Automating Ontology Formation

- Level $N+3$: Shapes
- Level $N+2$: Polygons
- Level $N+1$: Runs
- Level $N$: Pixels

I want the system to discriminate smiley and frownie face

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Grand Challenge: Automating Ontology Formation

Level N+3
Shapes

Level N+2
Polygons
Q-analysis – emergent classes of polyhedra

Level N+1
Runs

Level N
Pixels

I want the system to discriminate smiley and frownie face

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Grand Challenge: Automating Ontology Formation

Level $N+3$  
Shapes  

Level $N+2$  
Polygons  
Q-analysis – emergent classes of polyhedra

Level $N+1$  
Runs

Level $N$  
Pixels

I want the system to discriminate smiley and frownie face

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Grand Challenge: Automating Ontology Formation

- **Level N+3**: Shapes
  - 😊 = 😞
  - 😊 =/= 😞

- **Level N+2**: Polygons
  - Q-analysis – emergent classes of polyhedra

- **Level N+1**: Runs

- **Level N**: Pixels

I want the system to discriminate smiley and frownie face

---

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Grand Challenge: Automating Ontology Formation

I want the system to discriminate round and square faces
5. Machine vision and the intermediate word problem

It is our objective: it is desired to create a system able to generate its own vocabulary between the highest level of ‘the system’ and the lowest level of ‘the soup’. To research this we have used simple graphical objects such as hand-drawn squares, circles and other shapes. Figure 6 shows a digitized drawing of a square. On the left its atomic parts, the pixels, are shown at, say, Level N. These can be aggregated into horizontal runs of pixels called runs at Level N+1. The runs can then aggregated into polygons at Level N+2.

![Diagram of pixels, runs, and polygons.](image)

(a) Pixels at Level N  
(b) Runs at Level N+1  
(c) Polygons at Level N+2

**Figure 6. From pixels to horizontal runs to polygons.**

This illustrates the process of *bottom-up aggregation* of objects from low level sensory primitives. In this case the aggregation depends on a human-created process rather than a machine-created process.
(a) Sixty objects in an image

(b) Sixteen pixel configurations

Figure 8. Sets of shapes and descriptors
**circles**

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<td>9</td>
<td>300</td>
</tr>
<tr>
<td>Polygon 43</td>
<td>1586</td>
<td>12</td>
<td>9</td>
<td>75</td>
<td>10</td>
<td>0</td>
<td>62</td>
<td>9</td>
<td>5</td>
<td>64</td>
<td>0</td>
<td>12</td>
<td>81</td>
<td>5</td>
<td>10</td>
<td>304</td>
</tr>
</tbody>
</table>
6. Q-connectivity

Let \( \sigma(P6) \) represent the simplex \(< D0-3, D1-6, D2-6, D3-3, D4-7, D5-0, D6-2, D7-5, D8-6, D9-2, D10-0, D11-6, D12-3, D13-6, D14-7, D15-4 >\) and let \( \sigma(P7) \) represent the simplex \(< D0-3, D1-4, D2-5, D3-3, D4-5, D5-0, D6-2, D7-5, D8-5, D9-2, D10-0, D11-4, D12-3, D13-5, D14-5, D15-4 >\), where these are the intervals related to polygons 6 and 7 respectively. Then it can be seen that the polygons are connected by sharing nine vertices

\[
\begin{align*}
\sigma(P6) &= < D0-3 \ D1-6 \ D2-6 \ D3-3 \ d4-7 \ d5-0 \ d6-2 \ d7-5 \ d8-6 \ d9-2 \ d10-0 \ d11-6 \ d12-3 \ d13-6 \ d14-7 \ d15-4 > \\
\sigma(P7) &= < D0-3 \ D1-4 \ D2-5 \ D3-3 \ d4-5 \ d5-0 \ d6-2 \ d7-5 \ d8-5 \ d9-2 \ d10-0 \ d11-4 \ d12-3 \ d13-5 \ d14-5 \ d15-4 > 
\end{align*}
\]

In contrast polygons P6 and P39 share just three vertices and are less highly connected.

\[
\begin{align*}
\sigma(P6) &= < D0-3 \ D1-6 \ D2-6 \ D3-3 \ d4-7 \ d5-0 \ d6-2 \ d7-5 \ d8-6 \ d9-2 \ d10-0 \ d11-6 \ d12-3 \ d13-6 \ d14-7 \ d15-4 > \\
\sigma(P39) &= < D0-5 \ D1-3 \ D2-5 \ D3-6 \ d4-3 \ d5-0 \ d6-6 \ d7-5 \ d8-6 \ d9-6 \ d10-0 \ d11-3 \ d12-5 \ d13-6 \ d14-3 \ d15-7 > 
\end{align*}
\]

Figure 10. Polygons P6, P7, P35 and P36
These connectivity between the simplices reflect the shapes of the polygons they represent as shown in Figure 10.

The set of sixty descriptor simplices representing the polygons in Figure 8(a) forms what will be called a descriptor hypernetwork. Two simplices are $q$-connected in a hypernetwork if there is a chain of pairwise $q$-near simplices between them. For example $\sigma(P7)$ is 8-near $> 4$-near $\sigma(P6)$ which is 4-near $\sigma(P35)$ which is 7-near $> 4$-near $\sigma(P36)$, so $\sigma(P7)$ is 4-connected to $\sigma(P36)$. In contrast $\sigma(P7)$ 1-near to $\sigma(P36)$ two vertices because they share just two vertices. This shows that $q$-connectivity is a weaker connectivity than $q$-nearness.
The skyscraper diagram for the interval descriptor data.
The only thing we know for sure from the Q-analysis is that the components collapse into a single component at \( q = 7 \). The components at \( q = 8 \) are:

\[
\begin{align*}
&\text{C8-2 P4} \\
&\text{C8-3 P8} \\
&\text{C8-4 P10 P13} \\
&\text{C8-5 P17} \\
&\text{C8-6 P30 P31 P33 P34 P37 P32 P44 P59 P45 P48 P49 P36 P35 P53 P43 P47 P41 P38 P57 P39 P56} \\
&\text{C8-8 P42} \\
&\text{C8-9 P46} \\
&\text{C8-10 P50 P58} \\
&\text{C8-11 P51} \\
&\text{C8-12 P52 P54} \\
&\text{C8-13 P55}
\end{align*}
\]

Here there are two large components shown as C8-4 and C8-6. Suppose that the system makes the hypothesis that these components reflect the existence of two or distinct classes of objects. Then the classes are distinct at \( q = 8 \) but connected at \( q = 7 \). This means that there is one or more simplices which are 7-connected to one or more simplices in C8-4 and one or more simplices in C8-6, forming a bridge between them. Such simplices will be called *bridge simplices*.
(a) the simplex $\sigma$ make a bridges between the $q$-connected but $(q+1)$-disjoint components $C_{q-1}$ and $C_{q-2}$

(b) the $q$-near simplices $\sigma$ and $\sigma'$ belongs to $(q+1)$-disjoint components $C_{q-1}$ and $C_{q-1}$ and together they make a $q$-bridge.

A $q$-dimensional bridge simplex connects the $(q+1)$-disjoint components $C_{q-1}$ and $C_{q-2}$.
This process is based on the hypothesis that the descriptor simplices in C8-1 and C86 represent objects of different classes – the classes the machine is trying to discover. The bridge simplex we have discovered allows the least powerful descriptors to be identified and removed. For example below it can be seen that the simplex D5-0, D8-3, D10-0 and D13-3 are all vertices of the P16 in C8-1, P54 in C86 and the bridge simplex P53.

<table>
<thead>
<tr>
<th></th>
<th>D0</th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>D4</th>
<th>D5</th>
<th>D6</th>
<th>D7</th>
<th>D8</th>
<th>D9</th>
<th>D10</th>
<th>D11</th>
<th>D12</th>
<th>D13</th>
<th>D14</th>
<th>D15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polygon 16</td>
<td>2</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>6</td>
<td>0</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>6</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>Polygon 53</td>
<td>9</td>
<td>3</td>
<td>2</td>
<td>9</td>
<td>3</td>
<td>0</td>
<td>9</td>
<td>2</td>
<td>3</td>
<td>9</td>
<td>0</td>
<td>3</td>
<td>9</td>
<td>3</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>Polygon 54</td>
<td>9</td>
<td>6</td>
<td>3</td>
<td>8</td>
<td>6</td>
<td>0</td>
<td>8</td>
<td>3</td>
<td>3</td>
<td>9</td>
<td>0</td>
<td>6</td>
<td>9</td>
<td>3</td>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

Thus the 3-dimensional simplex <D5-0, D8-3, D10-0, D13-3> is a face of them all and connects them all. Bringing our human intelligence to bear it is obvious that D5 and D10 are not useful descriptors because they take the same value of zero for almost all the polygons since the pixel configurations and are very rare in this kind of image. The columns fifth and tenth columns in Table 1 and Table 2 provide little or no useful information for discriminating the polygons and are redundant. In this case! In other cases these configuration might be very useful and since there is no prior knowledge, the usefulness or otherwise of a descriptor has to be determined in some way. Here it is suggested that having identified the descriptors as contributing nothing to the desired discrimination they should be removed. In other words this is a procedure for pruning the less useful descriptors from the initial set.
The skyscraper diagram for the interval descriptor data.

Figure 14. The revised Q-analysis with descriptors D5-0, D8-3, D10-0 and D13-3 removed
The skyscraper diagram for the interval descriptor data.

**Figure 14.** The revised Q-analysis with descriptors D5-0, D8-3, D10-0 and D13-3 removed

**Figure 15.** The revised Q-analysis with descriptors D2-5, D8-4, D1-5 and D11-5 removed
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**Figure 15.** The revised Q-analysis with descriptors D2-5, D8-4, D1-5 and D11-5 removed

**Figure 16.** The revised Q-analysis without the descriptors D1-4, D1-6, D4-5, D11-4, D11-6, D14-5 and D14-6
Figure 15. The revised Q-analysis with descriptors D2-5, D8-4, D1-5 and D11-5 removed

Figure 16. The revised Q-analysis without the descriptors D1-4, D1-6, D4-5, D11-4, D11-6, D14-5 and D14-6

Figure 17. The revised Q-analysis without the descriptors D0-4, D0-5, D1-8, D2-4, D8-6, D3-5, D7-4, D7-5, D11-8, D12-5 and D13-6
Figure 17. The revised Q-analysis without the descriptors D0-4, D0-5, D1-8, D2-4, D8-6, D3-5, D7-4, D7-5, D11-8, D12-5 and D13-6

Figure 18. The revised Q-analysis without the descriptors D1-7 and D11-7
Figure 17. The revised Q-analysis without the descriptors D0-4, D0-5, D1-8, D2-4, D8-6, D3-5, D7-4, D7-5, D11-8, D12-5 and D13-6

Figure 18. The revised Q-analysis without the descriptors D1-7 and D11-7
Figure 21. Three classes of object automatically discovered by the iterated Q-analysis method
Figure 22. A set of horizontal and vertical shapes with their width-height scatter diagram.
Suppose a shape has width \( w \) and height \( h \). Then it could be said to be related to descriptor \( D-w-h \). Suppose it were also related to all the descriptors \( D-(w+i)-(h+j) \) for some \textit{dilation value}, \( d \) with \( -d \leq i \leq d \) and \( -d \leq j \leq d \). Then, for example, if the dilation is 5 the shapes will be related to \( 121 = (5+1+5)^2 \) descriptor points.

Figure 23. Consecutive skyscraper diagrams separate the shapes
Conclusions

Robust methods for incomplete and inconsistent data
If you have n-ary relations you have hypernetworks
Hypernetwork connectivity constrains dynamics
Hypernetwork connectivity has its own dynamics
Hypernetworks can model many kinds of systems

The Grand Challenge of systems that develop their own vocabulary may be solved by Hypernetwork Q-analysis?