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Global System Dynamics

- Overview

- Examples of complex systems to help policy-makers

- Open questions

Global

- physically global
- involves interconnected topics such as economics, social behaviour, environment, etc.

Systems

- involve a number of interconnected discrete entities
- with nonlinear interactions which depends on the network topology

Dynamics

- complex systems that show emergent properties
- effects may be interesting but also damaging

Interesting emergent behaviour

Floating objects which congregate

Flocks of birds

Shoals of fish

Ant colonies

Patterns in biology

Social networking sites

Clouds

Birds - http://vimeo.com/18813015

Fish - http://www.youtube.com/watch?v=clgHEhziUxU&feature=related

Ants – http://www.youtube.com/watch?v=g7VhvoMFn34&feature=related

We imagine that Natural Phenomena can be

- Understood
- Described
- Quantified
- Predicted
- and even, controlled

But what about Humans? Can they be

- Understood
- Described
- Quantified
- Predicted
- and even, controlled

Undesirable emergent behaviour

Systemic failure (Fukishima)

Disease spread

Burglaries

Terrorism

Stock market bubbles and crashes

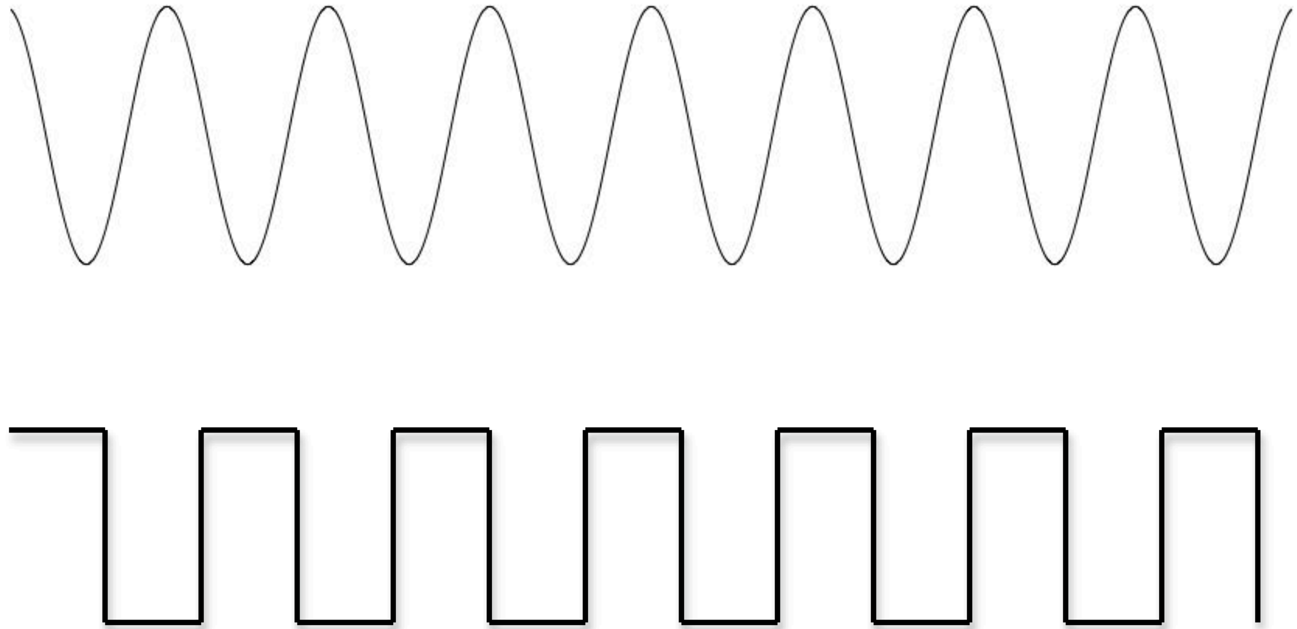
…..

What modelling techniques are there?

- Specific phenomenon can be modelled with a set of equations
- Set of ODEs for synchronisation
- PDEs for pattern such as waves
- Networks analysis to understand scaling laws
- Simulations: cellular automata (CA) and agent based models (ABM) including game theory
- Probabilistic models

PDEs that produce waves and patterns

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$



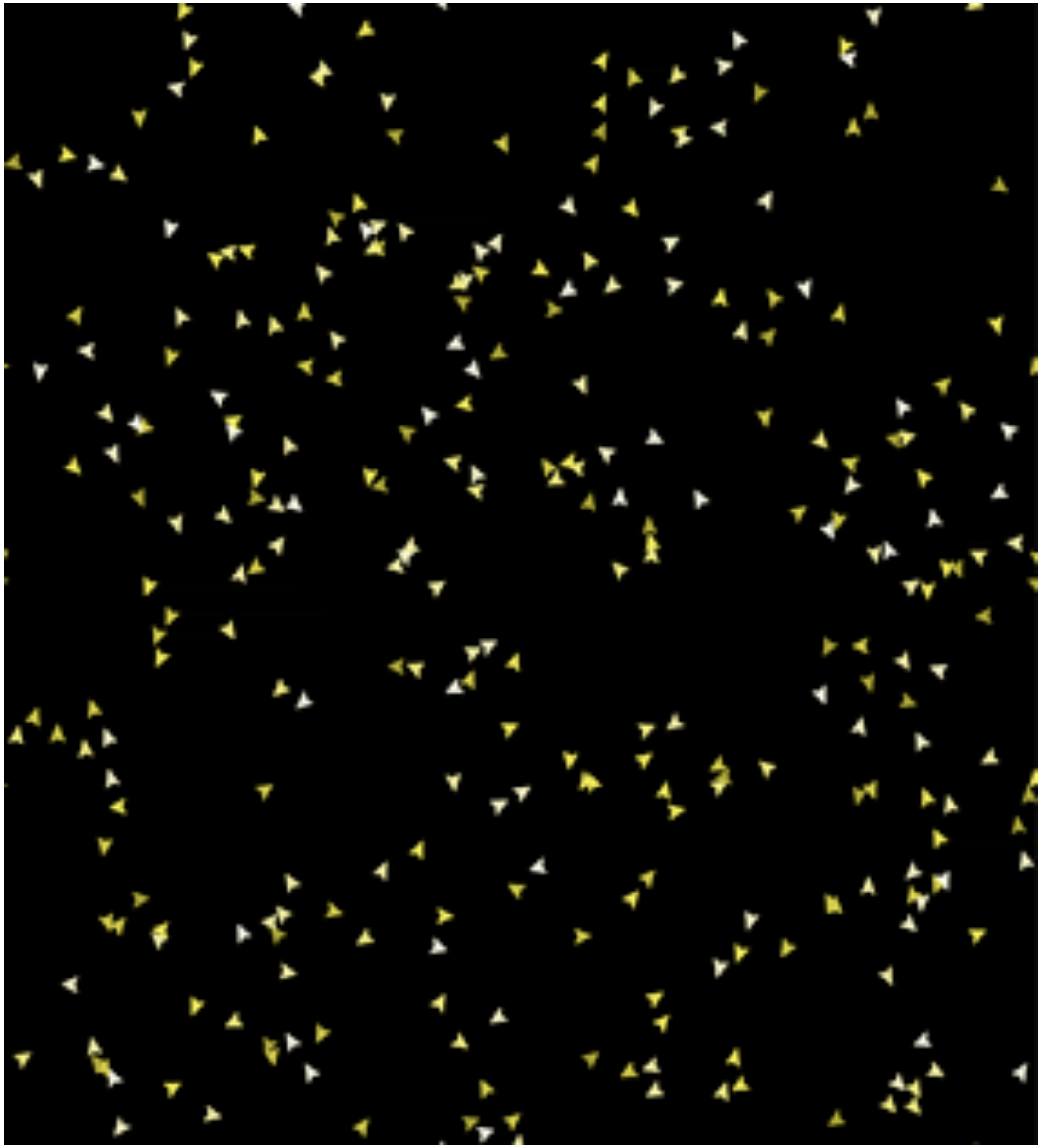
Model of birds flocking

<u>Rules</u>

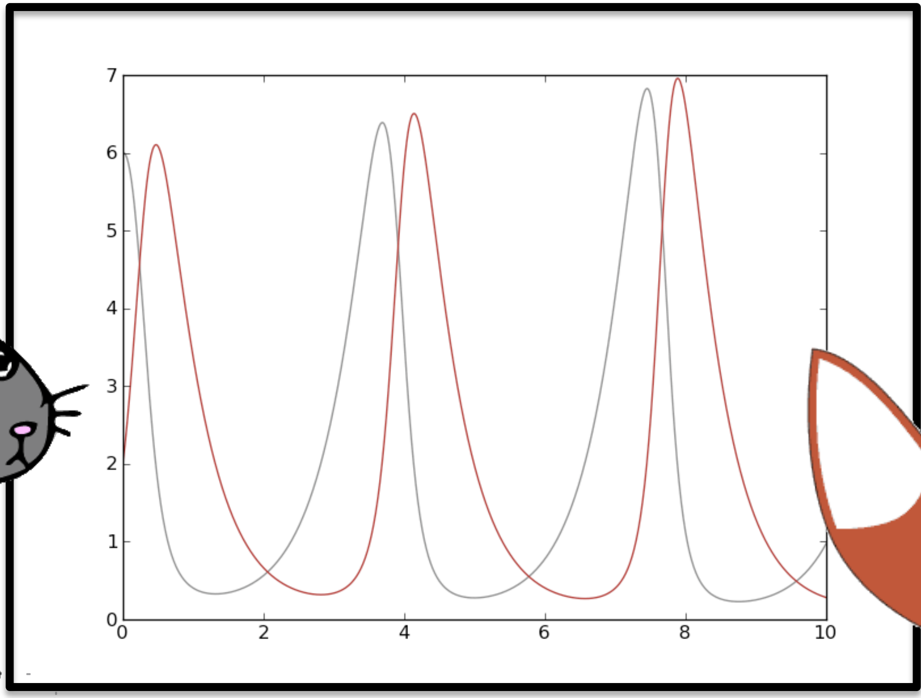
• Separation
 – avoid collisions with other birds

• Alignment
– direct towards local centre of mass

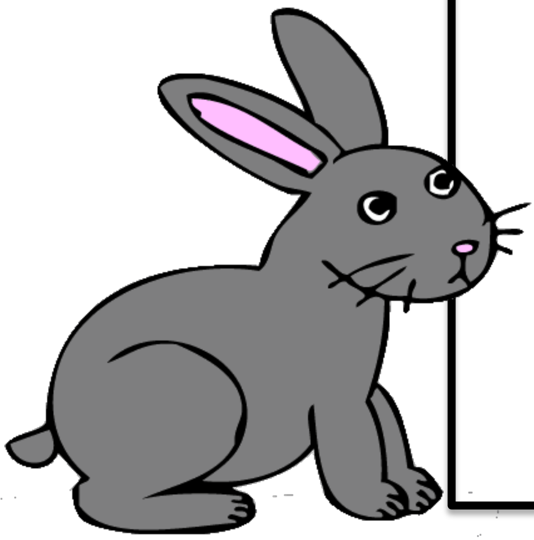
• Cohesion
– align with local average velocity



The predator prey model: top down



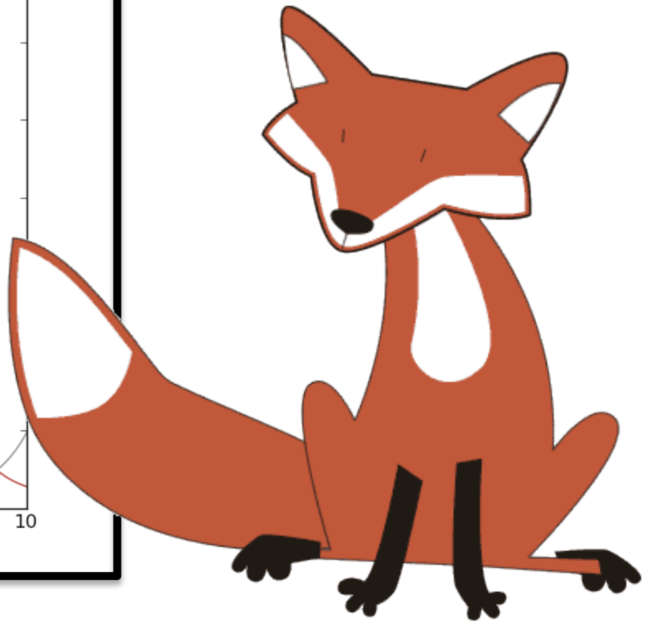
Rabbits:



- In the absence of foxes, the per capita rabbit growth rate is constant.
- In the presence of foxes, the per capita rabbit growth rate will fall linearly as a function of the fox population.

$$\frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}t} = a - bF$$

Foxes:

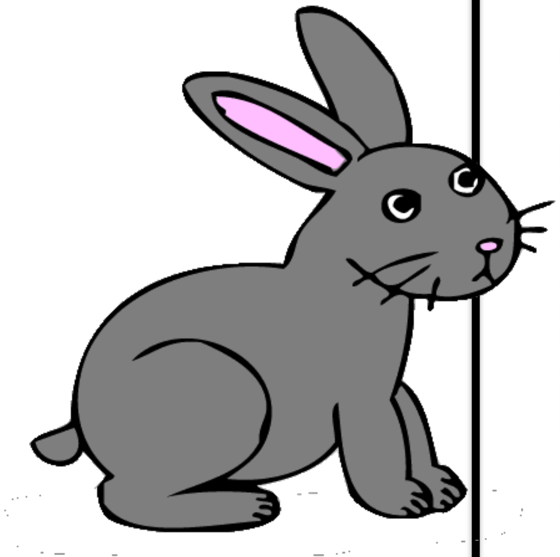


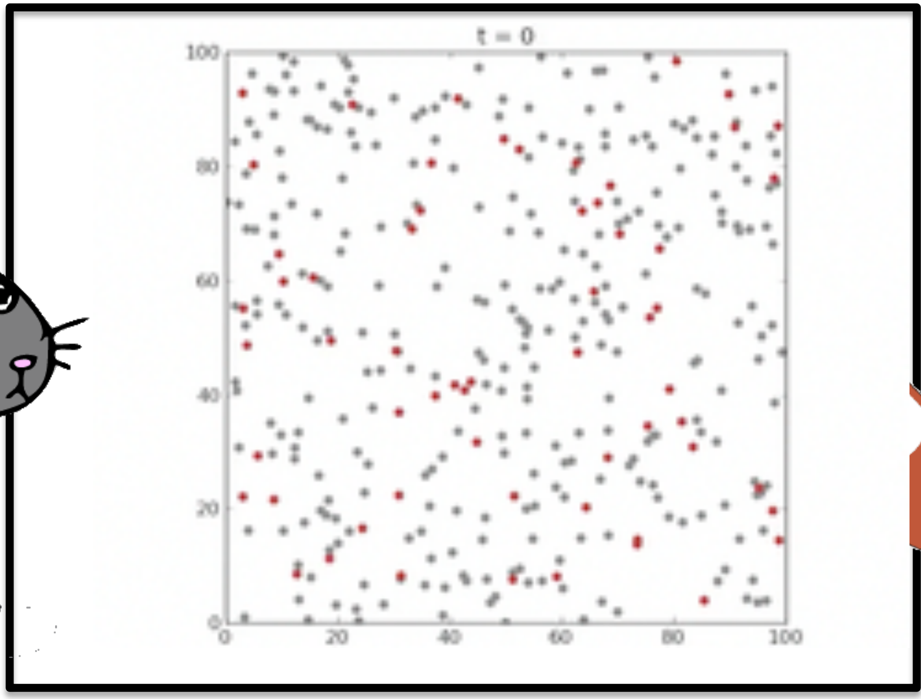
- In the absence of rabbits, the per capita fox growth rate is constant and negative.
- In the presence of rabbits, the per capita growth rate of foxes increases linearly as a function of the rabbit population.

$$\frac{1}{F}\frac{\partial F}{\partial t} = cR - d$$

The predator prey model: bottom up

Rabbits:





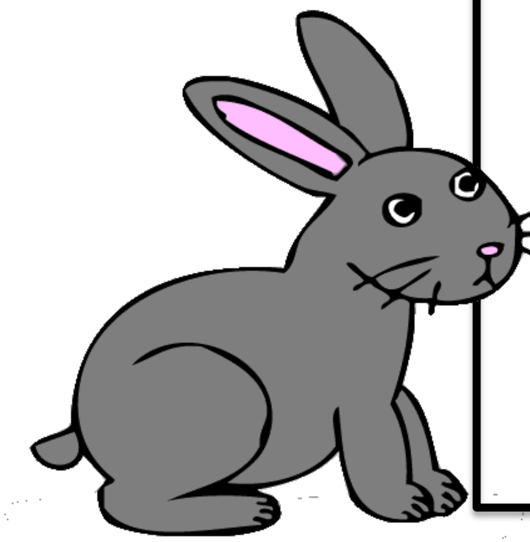
Foxes:

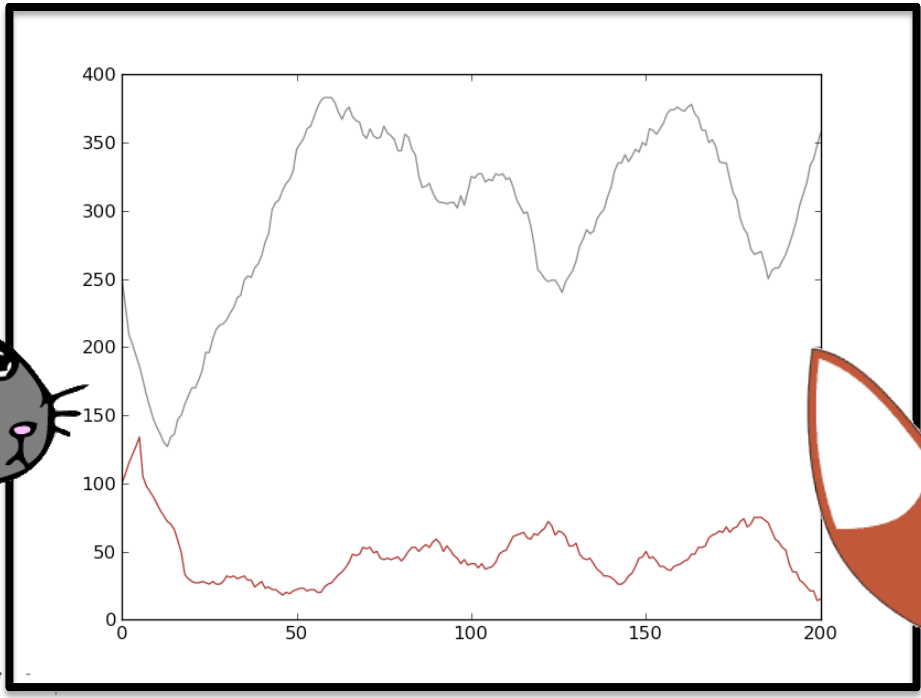
- Die if they meet a fox
- Breed if they meet a rabbit
- Die after a certain number of time steps

- Die if they don't meet enough rabbits
- Breed if they meet a fox only if they've eaten recently
- Die after a certain number of time steps

The predator prey model: bottom up

Rabbits:





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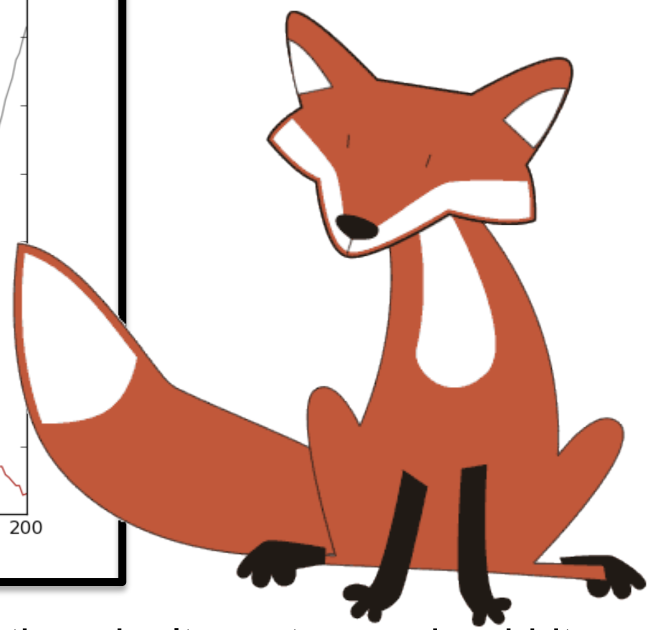

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Complex systems: examples

- The retail model of Harris and Wilson (1978)
- The Short et al model of residential burglary (2008)
- Sand Dune morphology (SR Bishop and H Momiji)

The retail model: top down

$\{X_i\}$ Money spent by residents of area *i*

$\{Z_j\}$ The floor space of centre *j*

$\{Y_{ij}\}$ Money spent in centre *j* by residents of *i*

$\{c_{ij}\}$ The distance between *i* and *j*

$\{b_j\}$ The benefit of shopping in *j* ($b_j = \log Z_j$)

Constraints

$$\boxed{\sum_j Y_{ij} = X_i}$$

$$\boxed{\sum_{ij} Y_{ij} c_{ij} = C}$$

$$\boxed{\sum_{ij} Y_{ij} \log Z_j = B}$$

The probability of a given flow configuration:

$$W(\{Y_{ij}\}) = \frac{Y!}{\prod_{ij} Y_{ij}!}$$

Taking logs:

$$\log W(\{Y_{ij}\}) = \log Y! - \sum_{ij} \log Y_{ij}!$$

Using Stirling's approximation:

$$\log W(\{Y_{ij}\}) = \log Y! - \sum_{ij} Y_{ij} \log Y_{ij} - Y_{ij}$$

The retail model

Total spending from residential area *i*

Benefit associated with centre *j*

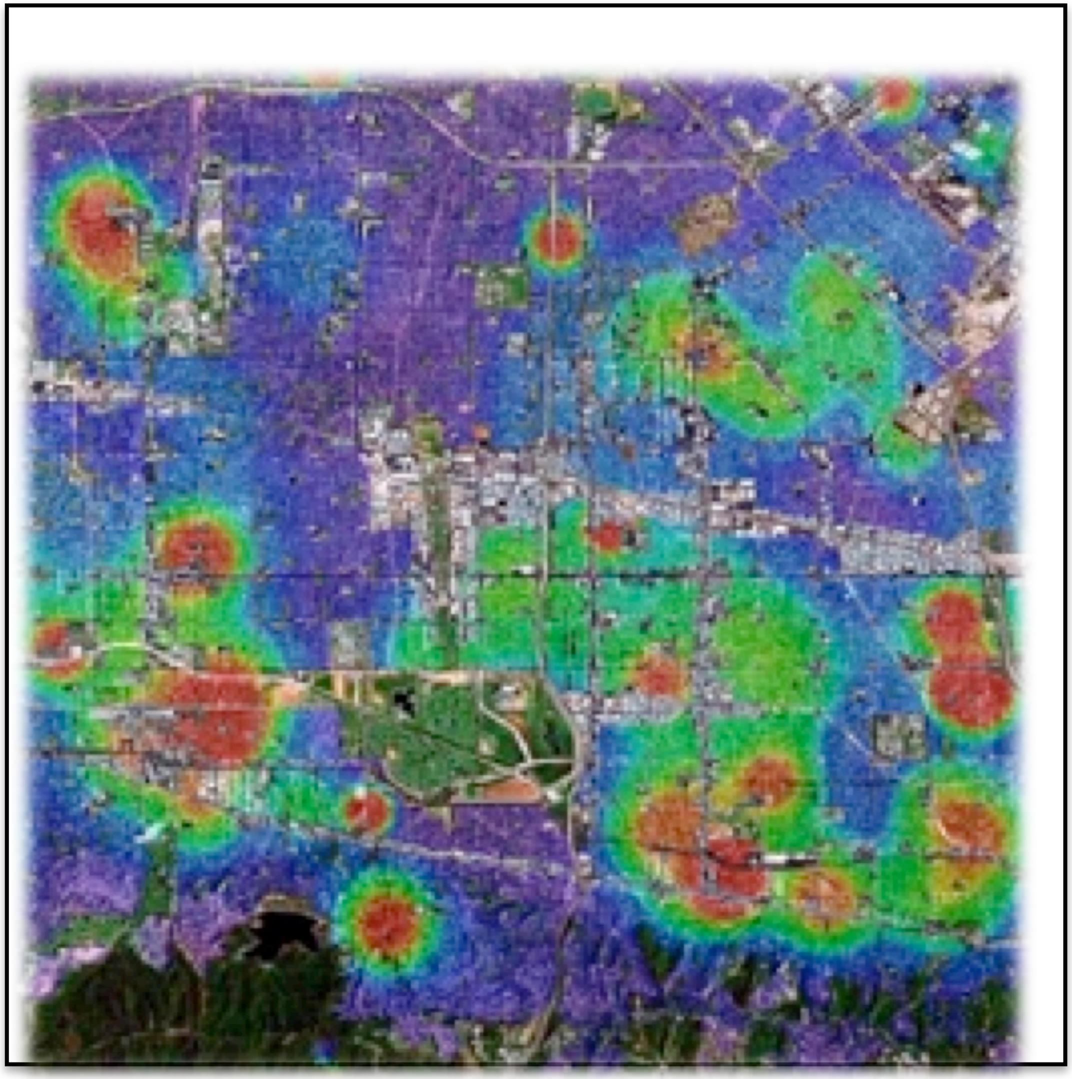
Impedance associated with centre *j*

$$Y_{ij} = X_i \frac{\exp\left(\alpha \log Z_j - \beta c_{ij}\right)}{\sum_k \exp\left(\alpha \log Z_k - \beta c_{ik}\right)}$$

Most likely set of flows between *i* and *j*

Relative attractiveness of shopping centre *j* http://vimeo.com/10050422

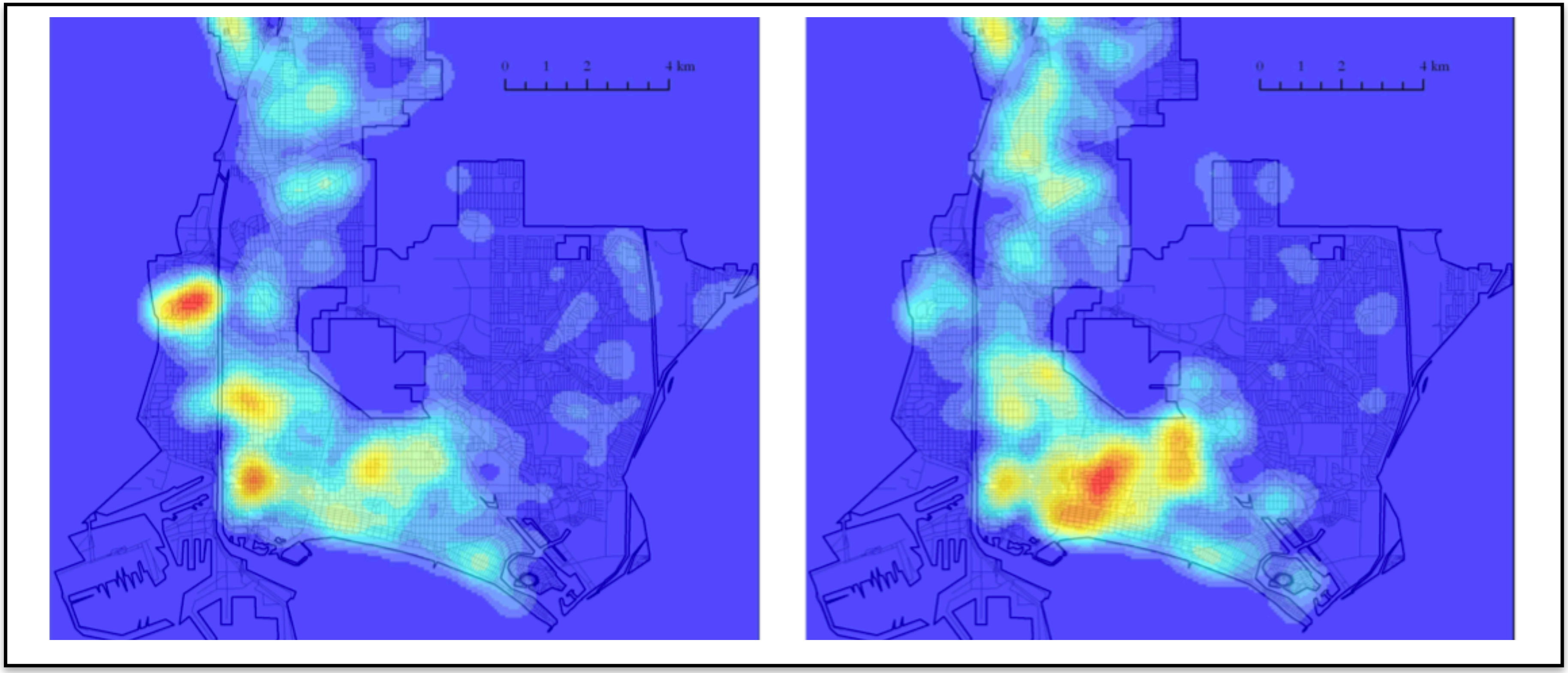
The Short et al. model of residential burglary



Crime modelling

- Crime clusters in time and space, and preferentially occurs in certain locations
- Understanding this behaviour has clear implications from the perspective of policing and urban planning
- Existing models are mainly qualitative or statistical
- Similarities with various other social phenomena

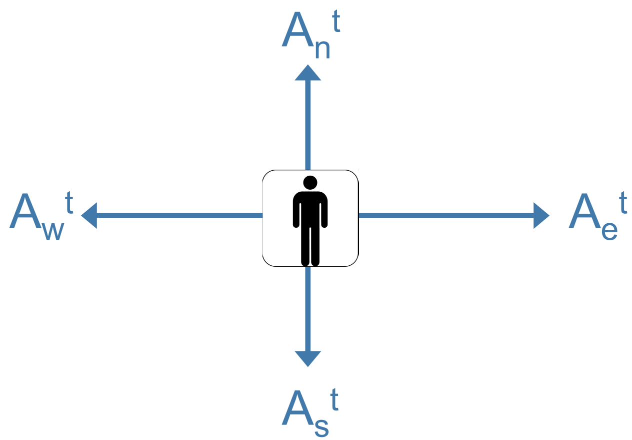
The Short et al model of residential burglary



Dynamic changes in residential burglary hotspots for two consecutive three-month periods beginning June 2001 in Long Beach, CA

Agent behaviour

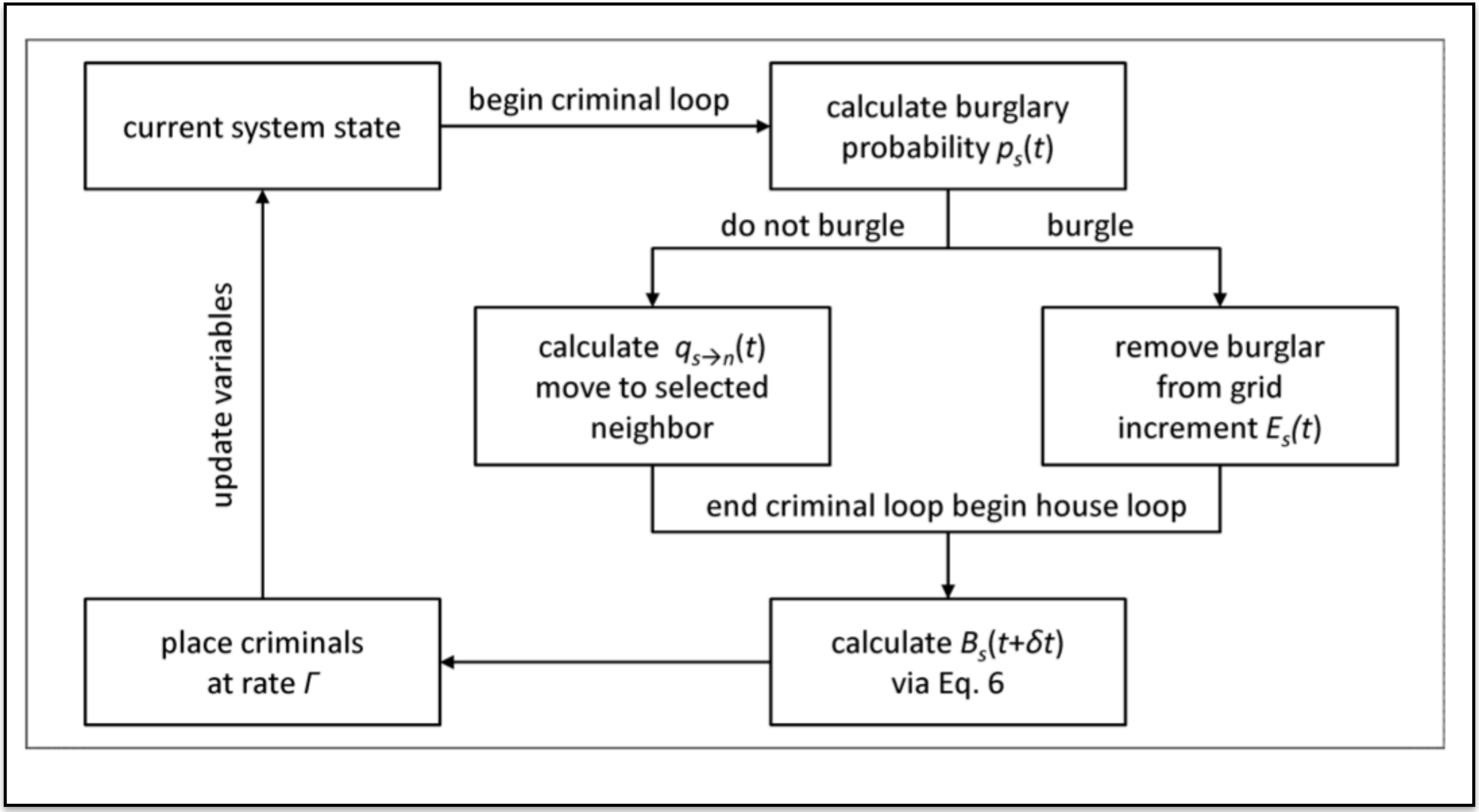
Agents are generated at nodes with a certain rate, and move to one of their von Neumann neighbours at each time-step, choosing according to attractiveness



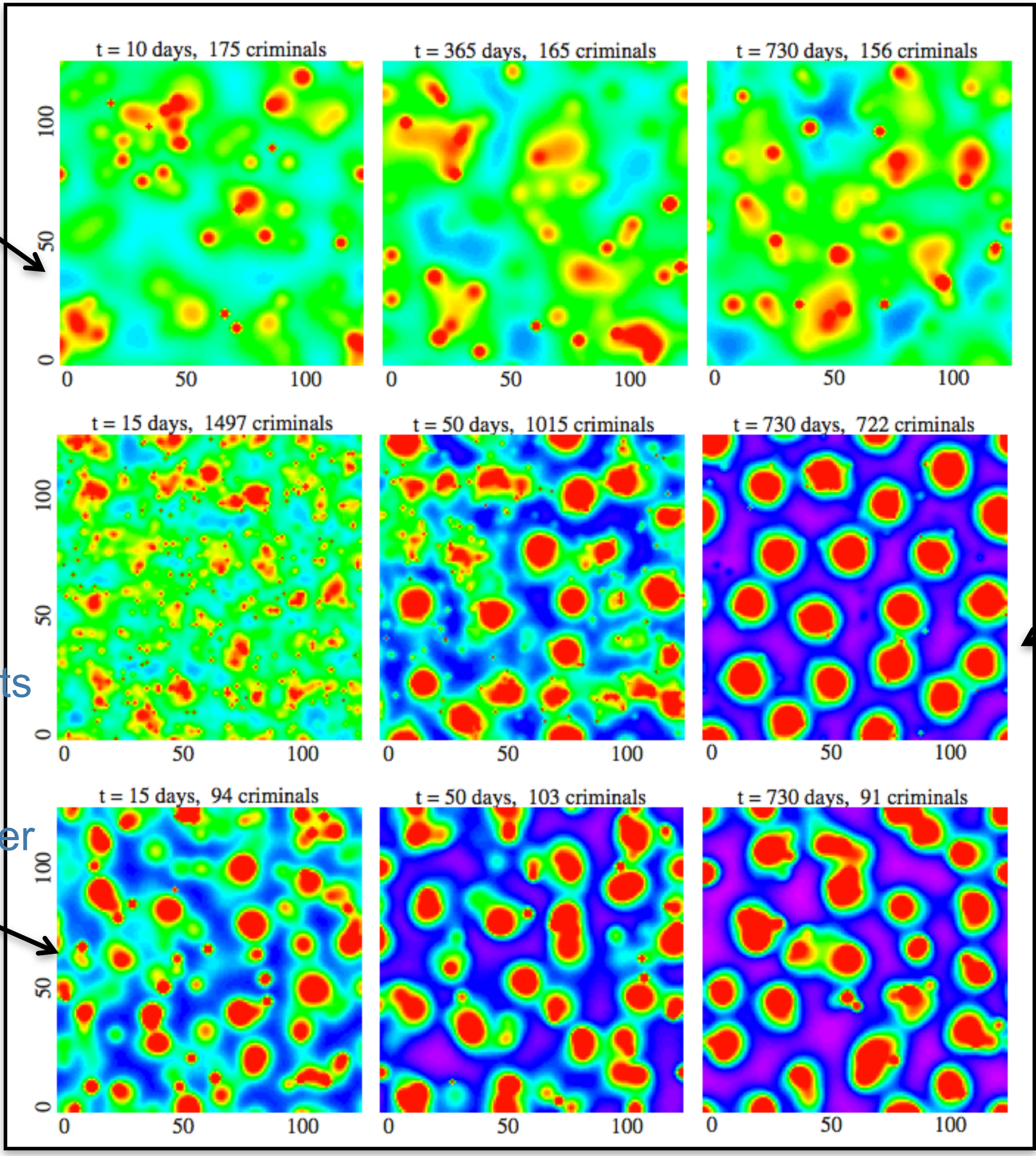
Every time an agent arrives at a new location, they then carry out a burglary with probability determined by the attractiveness

$$p_s^t = 1 - e^{-A_s^t \delta t}$$

Agent behaviour



Dynamic hotspots



Stationary hotspots

Dynamic hotspots which linger but display large deformations over time