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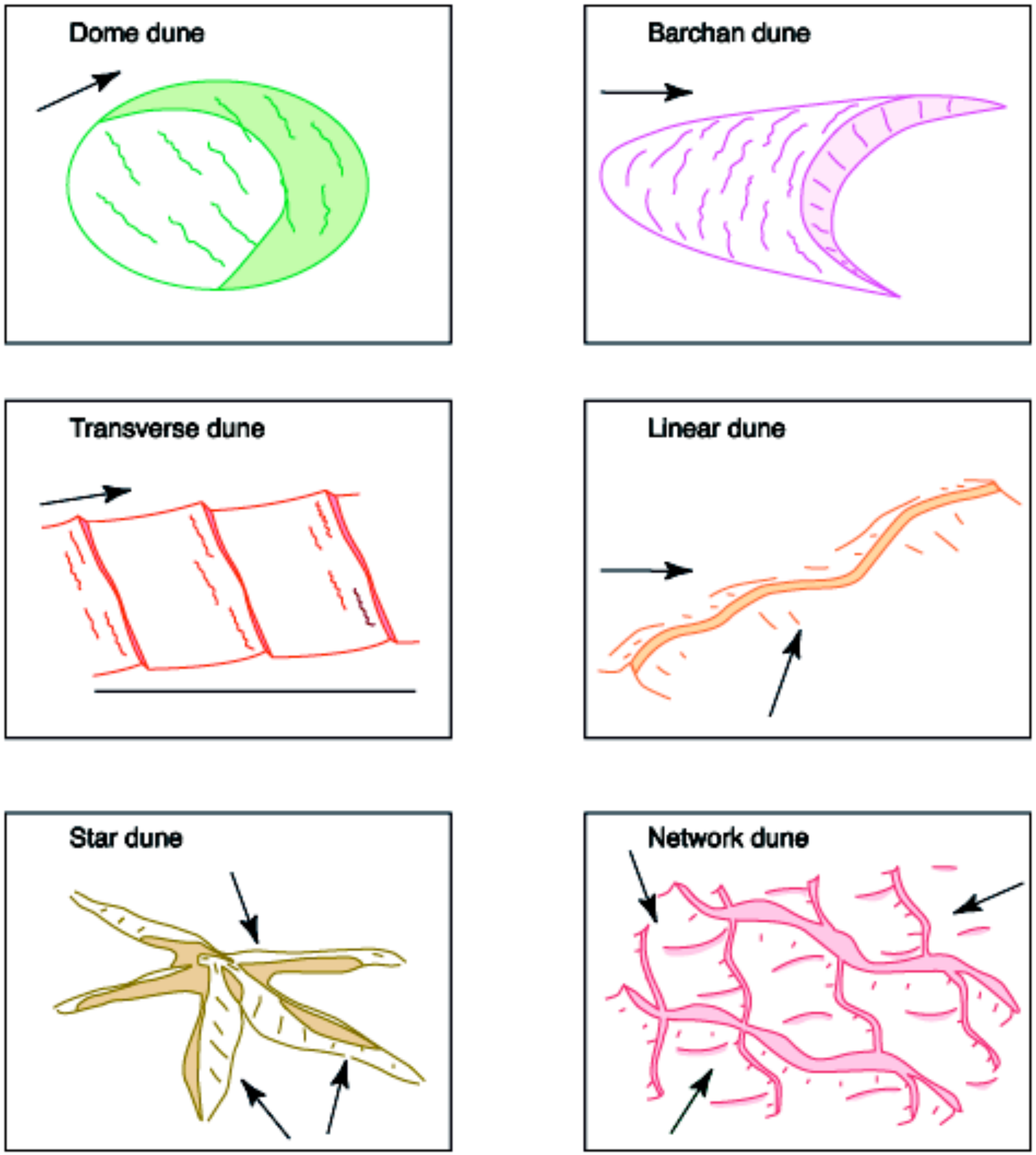
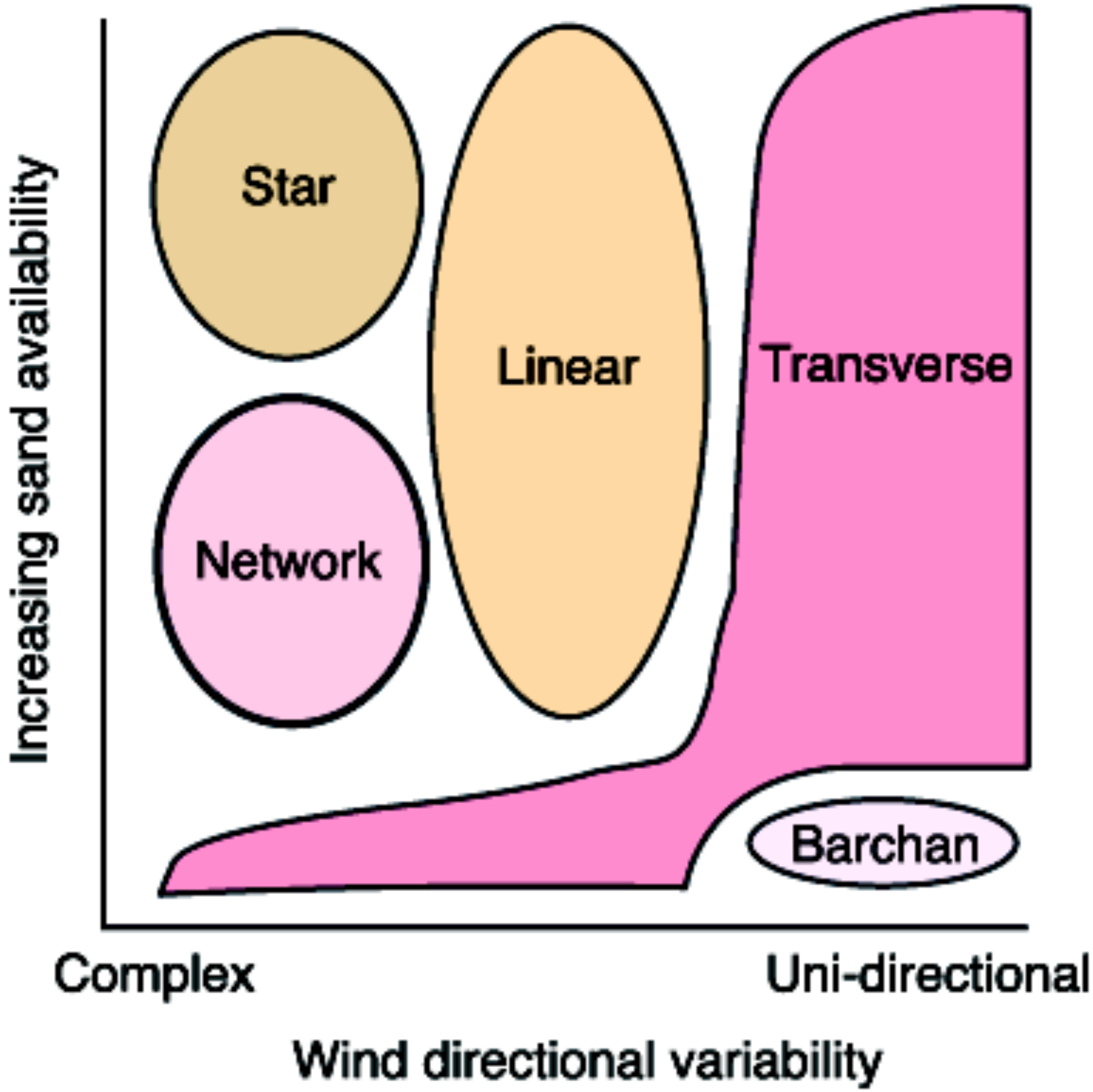
Who cares about sand movement?


Dome dune
Barchan dune
Transverse dune
Linear dune
Star dune
Network dune


Star
Network
Linear
Transverse
Barchan
Increasing sand availability
Complex
Uni-directional
Wind directional variability

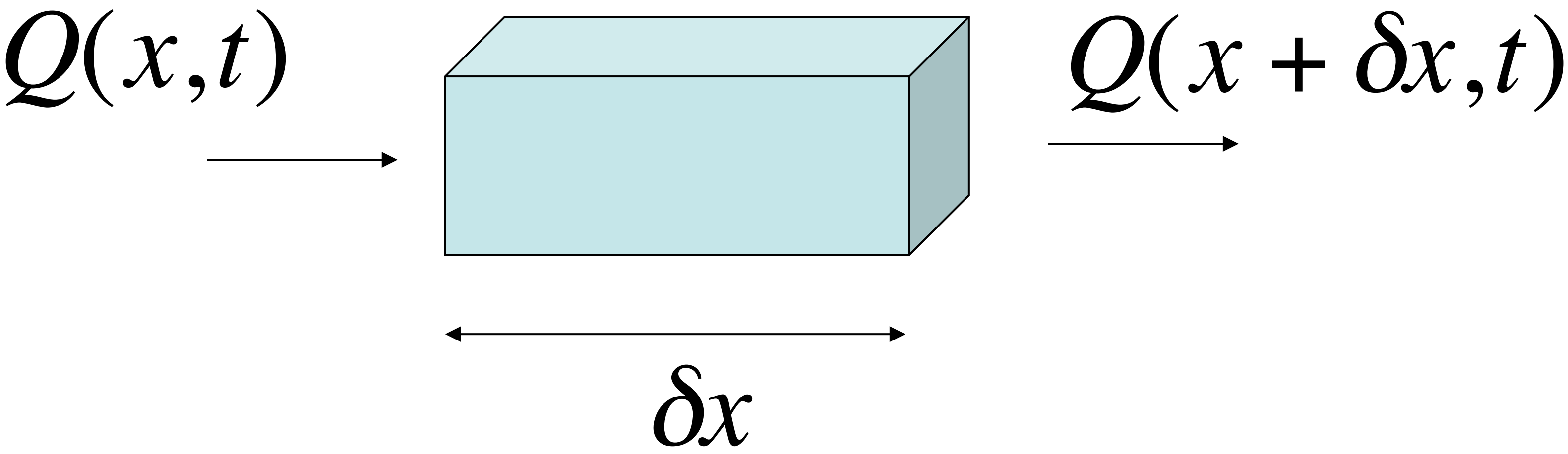
Can we Model sand dune formation and morphology?

Mathematics of Modelling

- Temporal?
  - then use ordinary differential equations
  - or discrete time iterative maps
- Spatial Temporal (combined)?
  - then we normally use partial differential equations (PDE's) to consider rates of change of variables or gradients

Spatial temporal modelling of sediment transport

Consider the flux - defined as the amount that flows through a unit area per unit time



If we are lucky!

We arrive at a PDE
or system of PDE’s

However ....

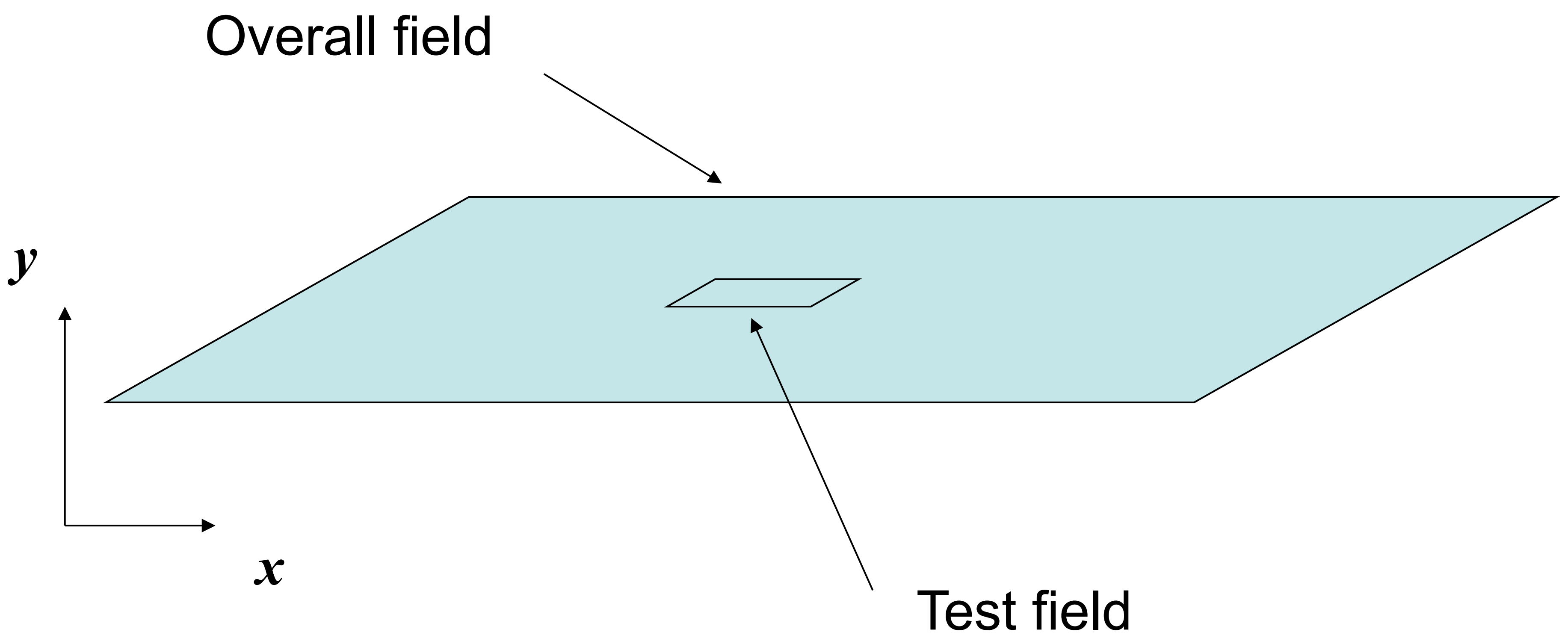
- The PDE is likely to be nonlinear with feedback between different aspects
- We may not be able to prove a solution exists
- Even if we can prove a solution exists, we may not be able to actually find one
- If we can find a solution we may not be able to prove that it is unique - it usually isn’t
- Even if we can solve it for some cases, we may not be able to find ***the*** solution that matches the boundary conditions
- Probably have to solve it numerically and even then, it is hard to geometrically illustrate the solution
- But .... if we succeed, results provide **quantitative** as well as **qualitative** information

Discrete Approach

- We use computer simulations only ..... we do not even write down the differential equations
- We divide the space into a finite number of cells based on a regular grid. Each cell is centred on a site (i,j) in our (x,y) position plane, i.e. space is discrete
- The variables in each cell have only a finite number of states, i.e. state is discrete
- Time is also discrete
- The state in a cell at any time is a function of the state of that cell, and the neighbouring cells (usually 8), at the previous time step (updating)

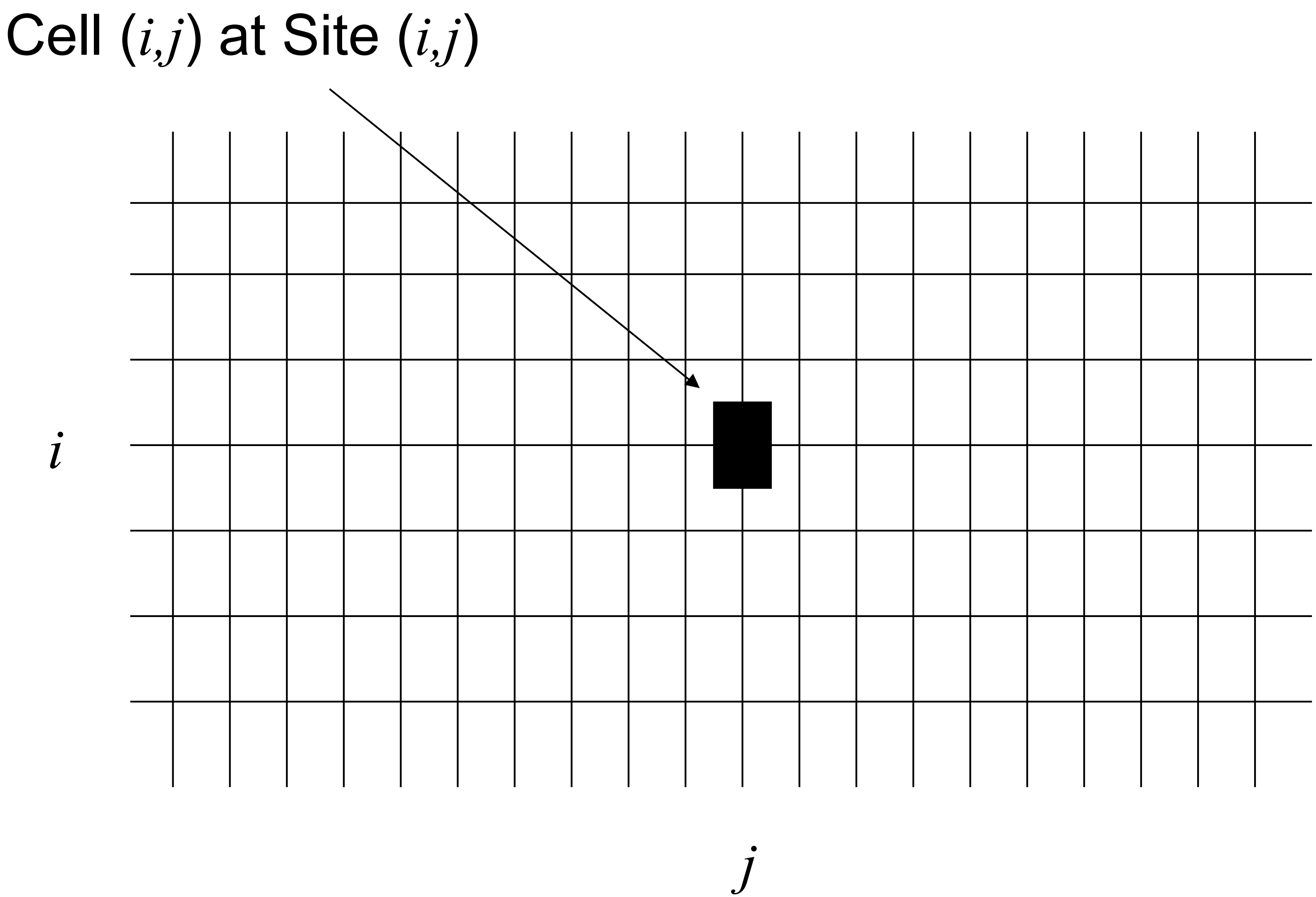
Since the rules for each (and every) cell is the same, we use the term **Cellular Automaton (CA)** to describe this type of model or, referring to the cells, we also use the plural of the term **Cellular Automata**

Setting up the Test Field

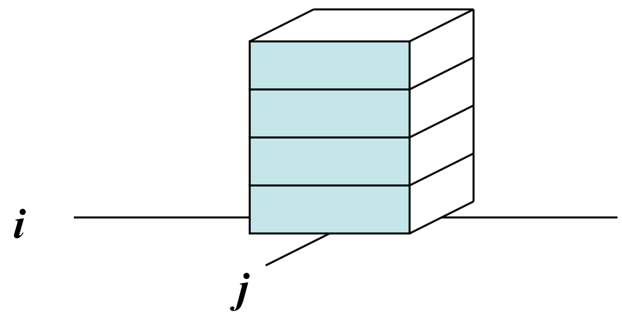


Test Field

(plan view)



We envisage blocks, or slabs, of sediment piled up at the site (i, j)



- The size of the slabs is fixed for an individual simulation but can be varied (which will introduce parameters into the model)
- With the size of slab fixed, the height of the sediment at (i,j) depends on the number of slabs
- Each of the slabs is made up of the same material so that the field in homogeneous (so the same rules apply at each site)

Rules for Sediment Movement

- A slab does not represent a single grain of sand but a portion of sediment
- However, the rules follow our knowledge of how individual sand particles behave. The science is incorporated by defining the rules such that they take into account the physics (or, later, biology)
- To define these rules we will introduce a series of processes each involving parameters
- We assume that the fluid flow has a tendency to transport sediment downstream

Processes During CA Modelling

- Erosion
  - (Shadow Zone)
- Transportation
- Deposition
- Avalanching
- Boundary continuity

Erosion

- A site is picked at random to be investigated, introducing a probabilistic element
- Since we have a flow direction, then we define a shadow zone on the lea-side formed by any bed-form. We define the shadow zone to make an angle of 15 degrees to the horizontal. If the site under investigation is within the shadow zone then the slab is not transported and left alone
- Otherwise the slab is transported downstream
- We might consider a time step $t = t_1$ to been completed when i x j sites have been investigated

Transportation

- A slab is transported downstream a fixed number of sites given a transport length, L
- The concept of transport here is a convolution of suspension, saltation and creep
- Determining L can be a complex calculation. L can be a function of waves, current, slope, bed-form, grain size, vegetation, etc. usually we take

  L = constant + other terms = L_0 +...
- Other terms include a speed-up factor for slabs near the top of a bed-form

Deposition

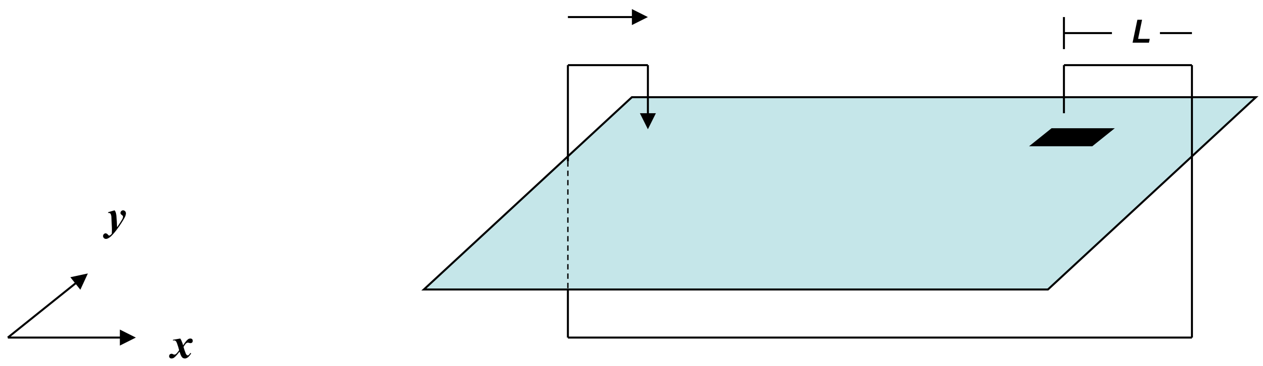
- A transported slab may be deposited after L sites or it may bounce.
- Here we introduce another probabilistic element. The probability, p, of a bounce is set at 0.4, unless there is no sediment present (hard ground) at the arrival site in which case the probability is higher. The bounce procedure is then repeated
- If the arrival site is within the shadow zone then it does not bounce, $p(0)$

Avalanching

- If the deposition of a slab causes the angle of repose for the bed-form to exceed 34 degrees then we implement an avalanching strategy.
- The slab will move towards a lower adjacent site following the steepest descent
- Avalanching is thus a form of erosion

Boundary Continuity

- Slabs can potentially leave the test field either due to the transport or avalanching processes
- A slab leaving the test field is re-inserted using a periodic boundary condition (in either the x or y direction)

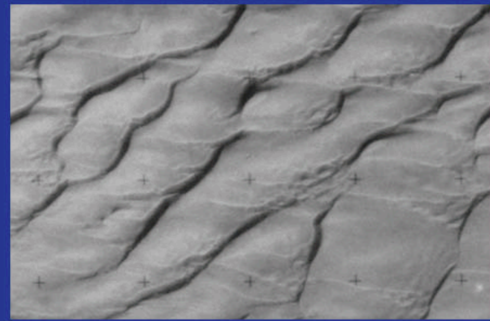


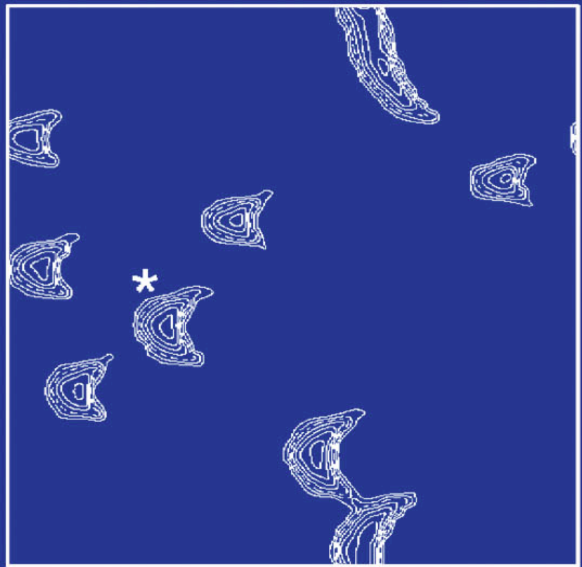
Cellular Automata Models

- Used successfully to model sand dune morphology
- Other areas of physical movement and pattern formation
- Some attempts for ecological growth

Amazing Success

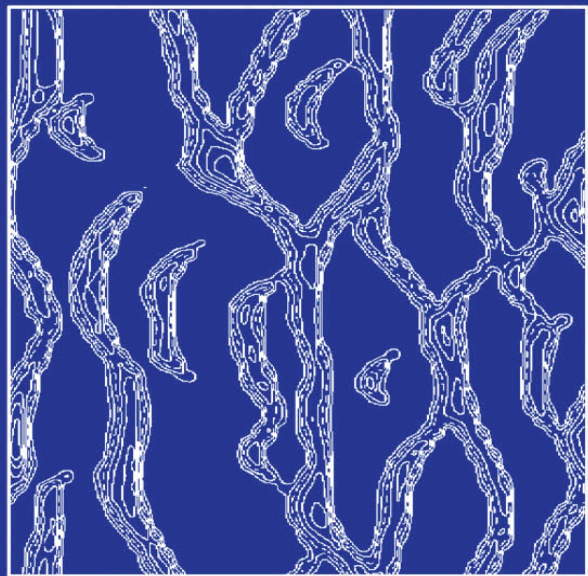
- Previous work on desert dunes by Hiroshi Momiji was able to capture different patterns of sand dunes using the same model but varying an inbuilt parameter.
- For sea-bed activity the situation is more complicated. We need to be able to capture qualitative features of bed-form found around the UK





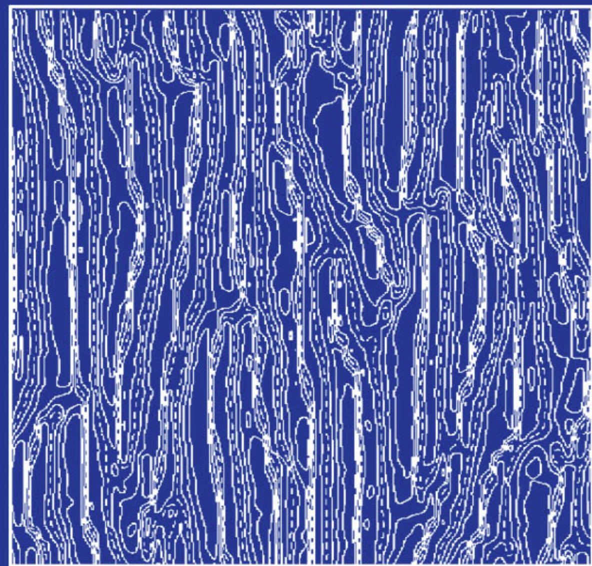
Barchan dunes

Increasing sand depth



Laterally connected Barchan dunes

Increasing sand depth



Closely spaced transverse dunes