

Synchronization in Coupled Complex Systems

Jürgen Kurths^{1 2 3}



POTSDAM INSTITUTE FOR
CLIMATE IMPACT RESEARCH



- ¹Potsdam Institute for Climate Impact Research,
RD Transdisciplinary Concepts and Methods
- ² Inst. of Physics, Humboldt University Berlin
- ³ University of Aberdeen, King's College



[http://www.pik-potsdam.de/members/kurths/
juergen.kurths@pik-potsdam.de](http://www.pik-potsdam.de/members/kurths/juergen.kurths@pik-potsdam.de)

Outline

- Introduction
- Complex synchronization in simple geometry
 - Phase-coherent complex systems
 - Applications
- Synchronization in non-phase coherent systems: phase and/vs. generalized synchronization
 - Concepts of curvature and recurrence
 - Applications
- Conclusions

Synchronization

Greek origin:

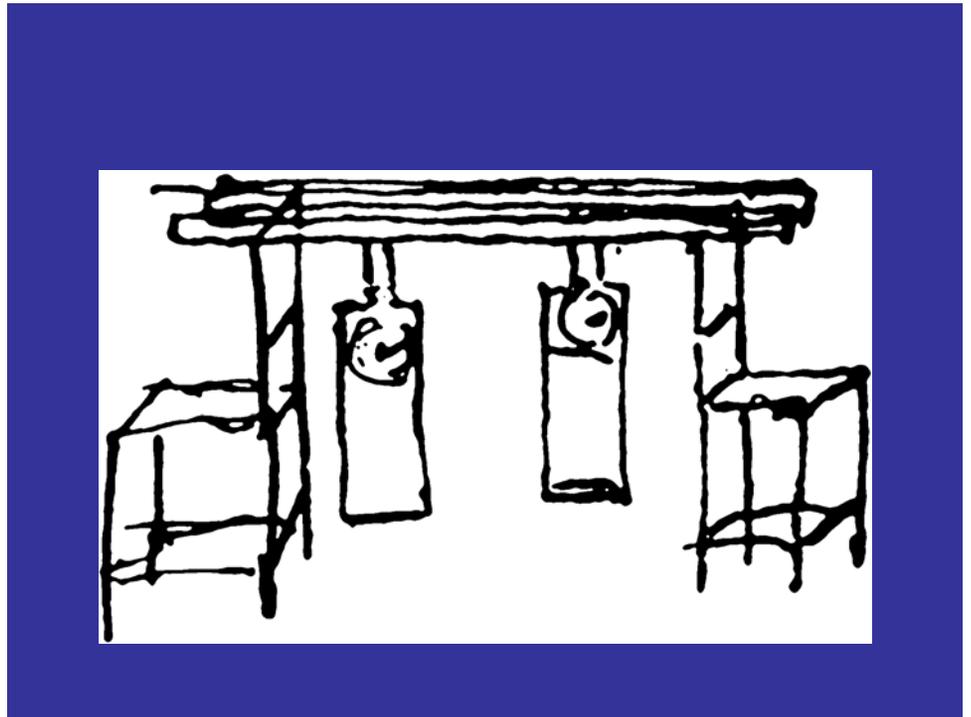
Σύγχρονος – sharing a common
property in time

Nonlinear Sciences

Start in 1665 by Christiaan Huygens:

Discovery of **phase synchronization**,
called **sympathy**

Huygens'-Experiment



Pendulum Clocks

- Christiaan Huygens:

Pendulum clocks hanging at the same wooden beam (half-timber house)

It is quite worth noting that when we suspended **two clocks** so constructed from two hooks imbedded in the **same wooden beam**, the motions of each pendulum in opposite swings were so much in agreement that they never receded the last bit from each other...Further, if this agreement was **disturbed by some interference**, it reestablished itself in a short time...after a careful examination I finally found that the cause of this is **due to the motion of the beam**, even though this is hardly perceptible (Huygens, 1673)

Modern Example: Mechanics

London's Millenium Bridge

- pedestrian bridge
- 325 m steel bridge over the Themse
- Connects city near St. Paul's Cathedral with Tate Modern Gallery

Big opening event in 2000 -- movie

Bridge Opening

- Unstable modes always there
- Mostly only in vertical direction considered
- Here: extremely strong unstable lateral Mode – If there are sufficient many people on the bridge we are beyond a threshold and synchronization sets in
(Kuramoto-Synchronizations-Transition, book of Kuramoto in 1984)

Supplemental tuned mass dampers to reduce the oscillations



GERB Schwingungsisolierungen GmbH, Berlin/Essen

Examples: Sociology, Biology, Acoustics, Mechanics

- Hand clapping (common rhythm)
- Ensemble of doves (wings in synchrony)
- Mexican wave
- Menstruation (e.g. female students living in one room in a dormitory)
- Organ pipes standing side by side – quenching or playing in unison (Lord Rayleigh, 19th century)
- Fireflies in south east Asia (Kämpfer, 17th century)
- Crickets and frogs in South India

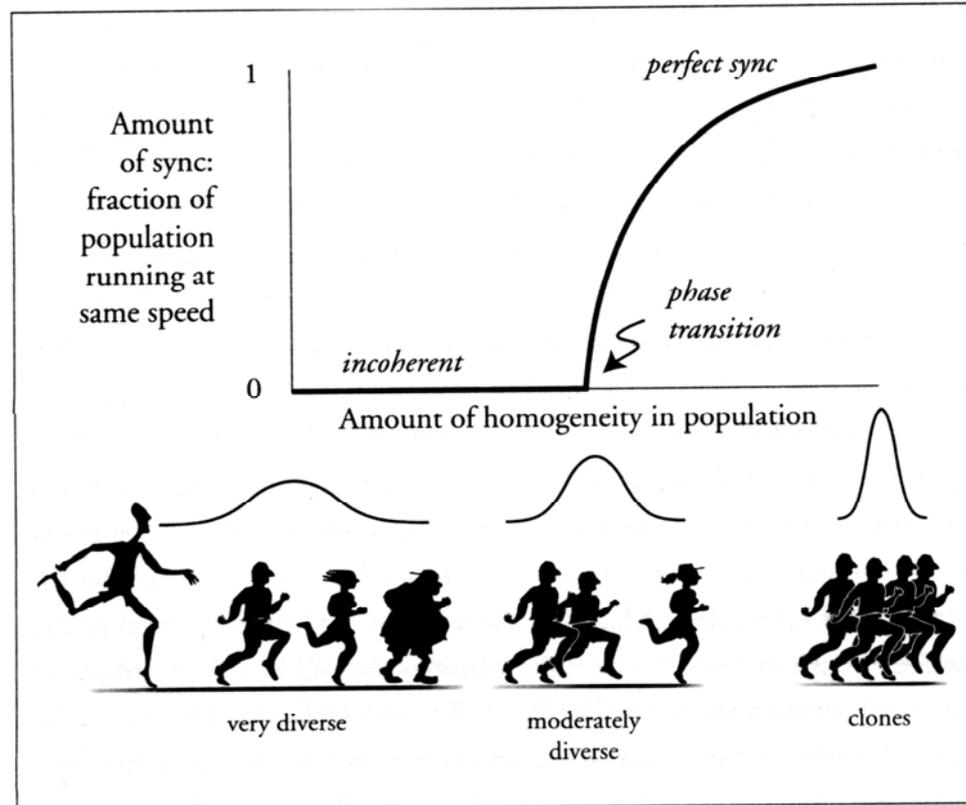
Synchronization in Physiology

- Neurons firing in synchrony –
 - i) positive: necessary for any action
 - ii) negative: wide-spread synchrony
 - ➔ epilepsy, Parkinson
- Breathing controls/synchronizes heart rhythm (Yoga)
- Cells in intestine, pacemaker cells (heart), insulin-secreting cells (pancreas), posture control, eyes
- Violation/Strengthening of synchrony
 - ➔ **cause of dynamical diseases**

Necessary Conditions for Synchronization

- **Two Oscillators** (or more; best: self-sustaining)
- **Coupling:** Master – Slave, or mutually coupled
- **Starting:** (slightly) different systems
(initial conditions, internal frequencies)
- **Goal:** becoming identical in a main property or sharing some important behaviour due to forcing or interaction
(becoming identical, adjusting their phases...)

Synchronization in Populations



Types of Synchronization in Complex Processes

- **phase synchronization**

phase difference bounded, a zero Lyapunov exponent becomes negative (phase-coherent)

(Rosenblum, Pikovsky, Kurths, 1996)

- **generalized synchronization**

a positive Lyapunov exponent becomes negative, amplitudes and phases interrelated

(Rulkov, Sushchik, Tsimring, Abarbanel, 1995)

- **complete synchronization** (Fujisaka, Yamada 1983)

Phase Synchronization in Complex Systems

Most systems not simply periodic

→ Synchronization in complex (non-periodic) systems

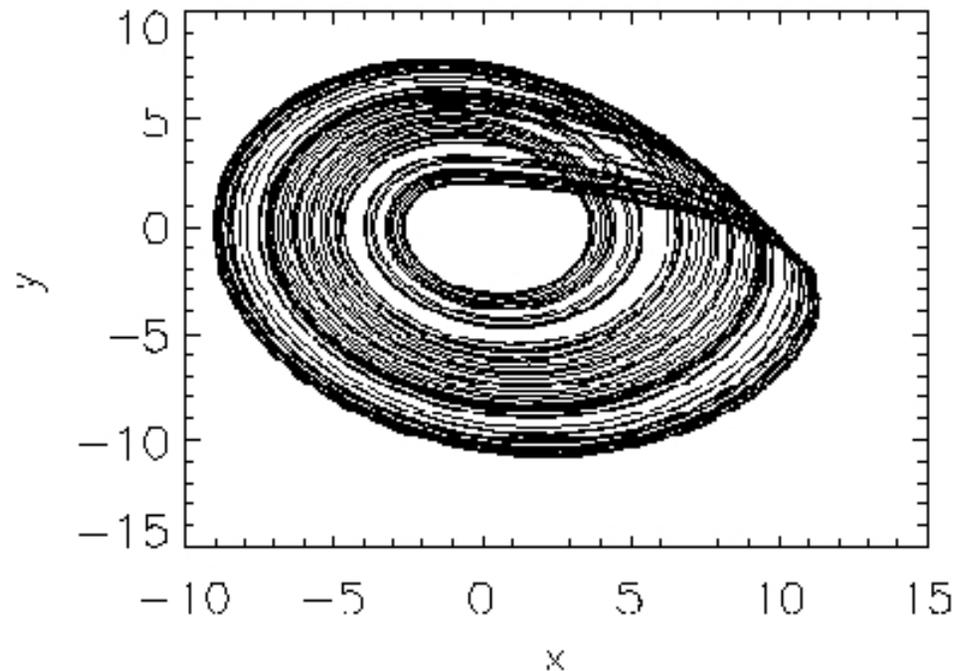
Interest in **Phase Synchronization**

How to retrieve a phase in complex dynamics?

Phase Definitions in Coherent Systems

Rössler Oscillator – 2D Projection

Phase-coherent (projection looks like a smeared limit cycle, low diffusion of phase dynamics)



Phase dynamics in periodic systems

- Linear increase of the phase

$$\varphi(t) = t \omega$$

$\omega = 2 \Pi / T$ – frequency of the periodic dynamics

T – period length

➔ $\varphi(t)$ increases 2Π per period

$$d \varphi(t) / d t = \omega$$

Phase Definitions

Analytic Signal Representation (Hilbert Transform)

$$\psi(t) = s(t) + j\tilde{s}(t) = A(t)e^{j\phi(t)},$$

$$\tilde{s}(t) = \frac{1}{\pi} \text{P.V.} \int_{-\infty}^{\infty} \frac{s(\tau)}{t - \tau} d\tau$$

Direct phase

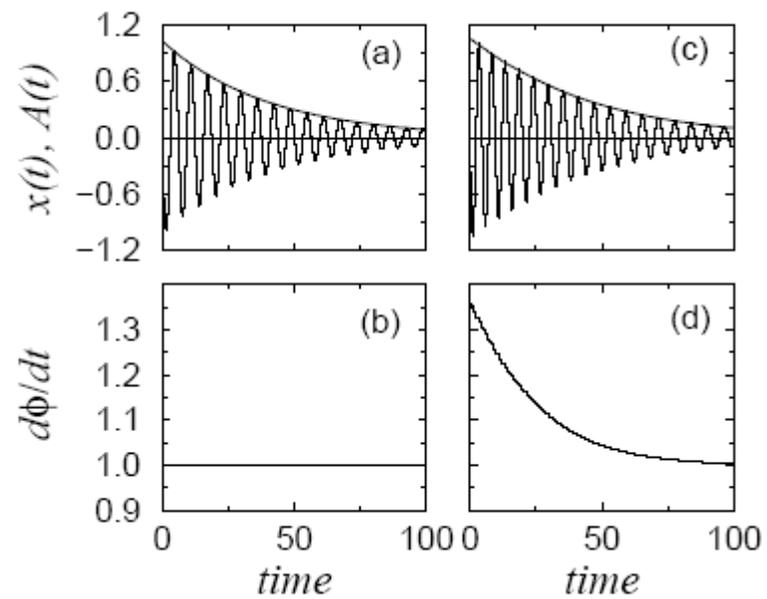
$$\phi(t) = \arctan(y(t)/x(t))$$

Phase from Poincare' plot

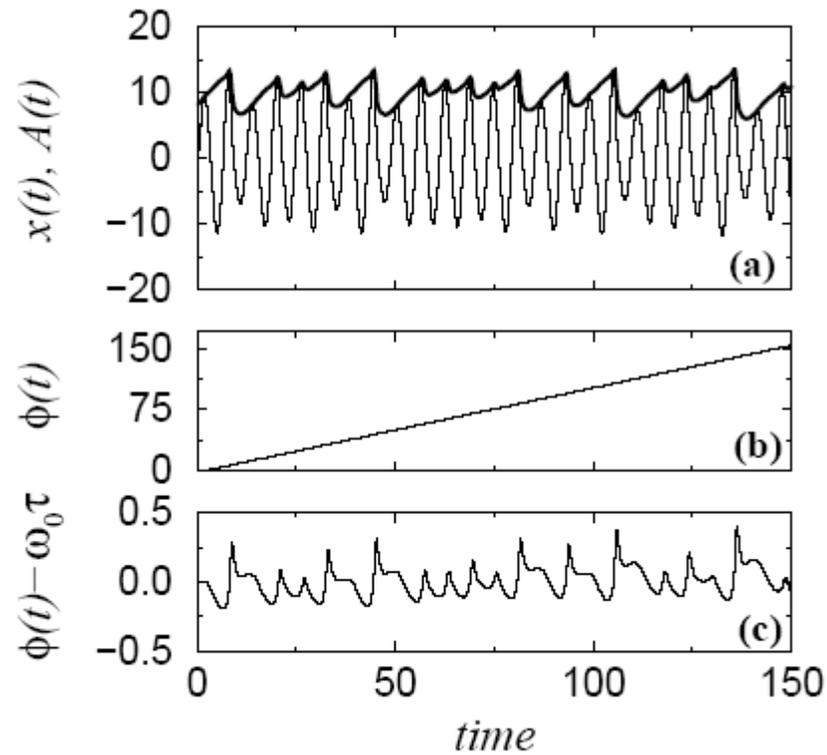
$$\phi(t) = 2\pi k + 2\pi \frac{t - \tau_k}{\tau_{k+1} - \tau_k} \quad (\tau_k < t < \tau_{k+1})$$

(Rosenblum, Pikovsky, Kurths, Phys. Rev. Lett., 1996)

Hilbert transform for periodic signals



Hilbert transform for chaotic signals



Phase for coherent chaotic oscillators

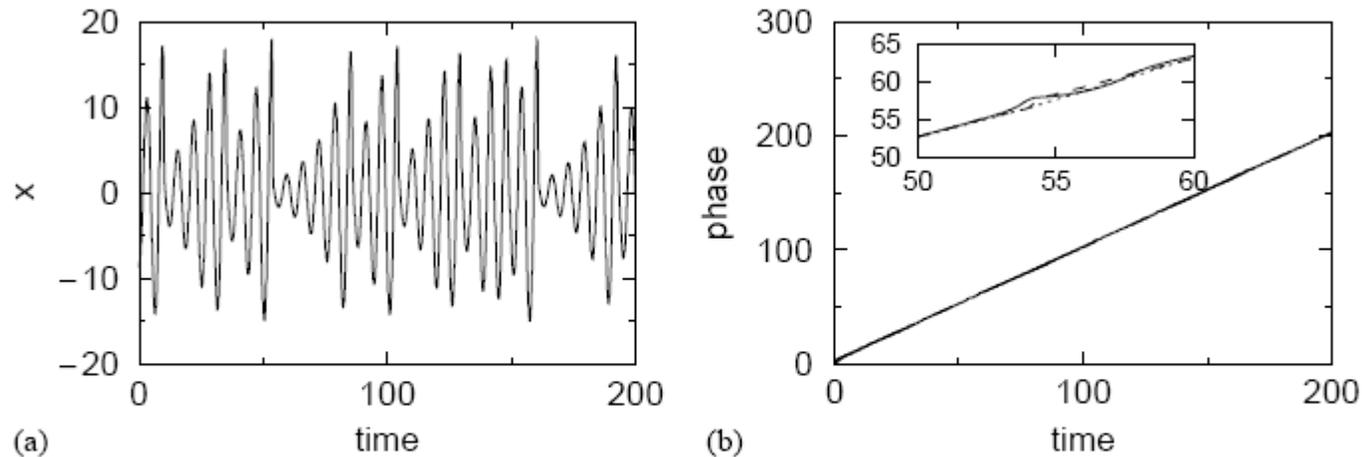


Fig. 3.3. (a) Chaotic signal $x(t)$ of the chaotic Rössler oscillator. (b) Phase of the chaotic signal. Solid line: phase of Eq. (3.5); dashed line: phase of Eq. (3.7); and dotted line: phase of Eq. (3.8).

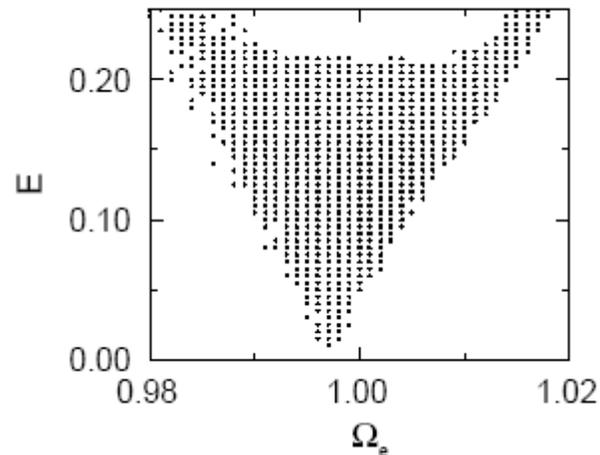
Phase dynamics and phase synchronization phenomena very similar in periodic and phase-coherent chaotic systems, e.g. one zero Lyapunov exponent becomes negative

Synchronization due to periodic driving

$$\dot{x} = -\omega y - z + E \sin(\Omega_e t)$$

$$\dot{y} = \omega x + ay,$$

$$\dot{z} = f + z(x - c)$$



Synchronization due to periodic driving

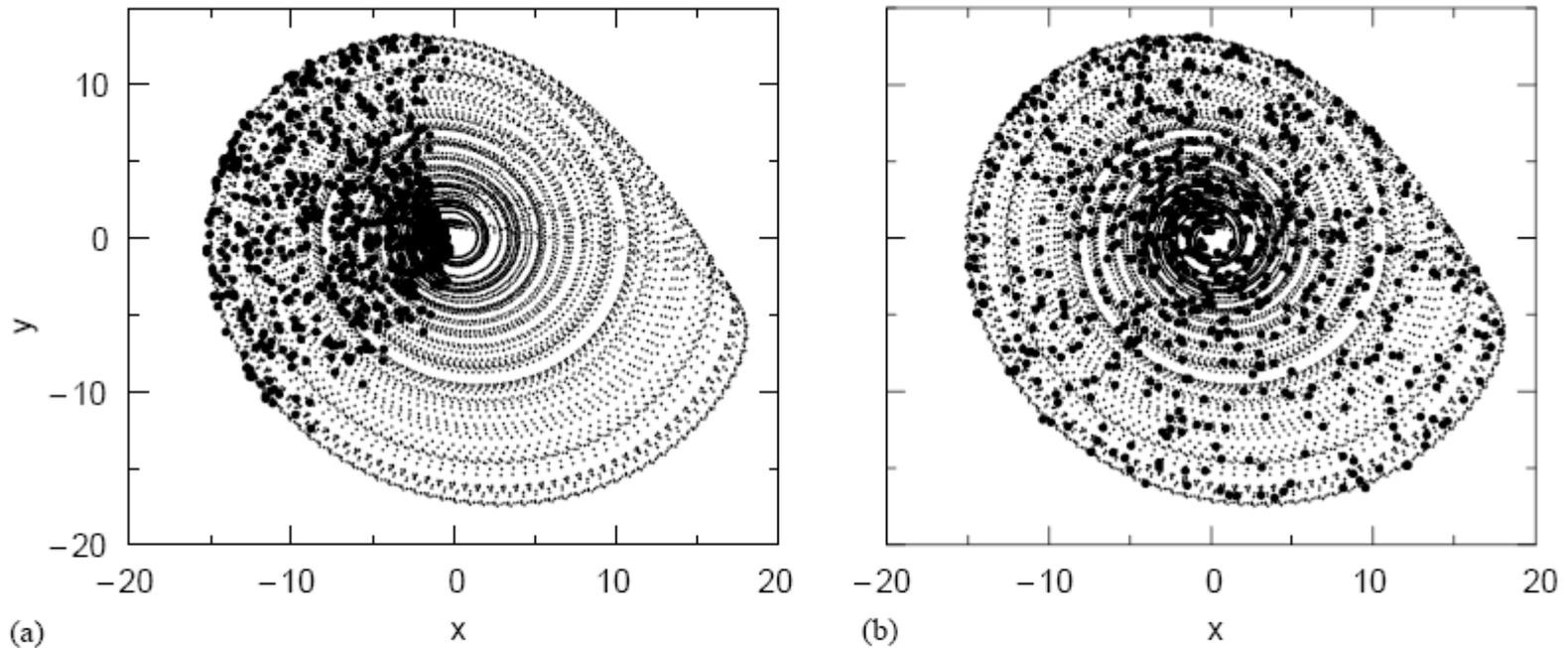
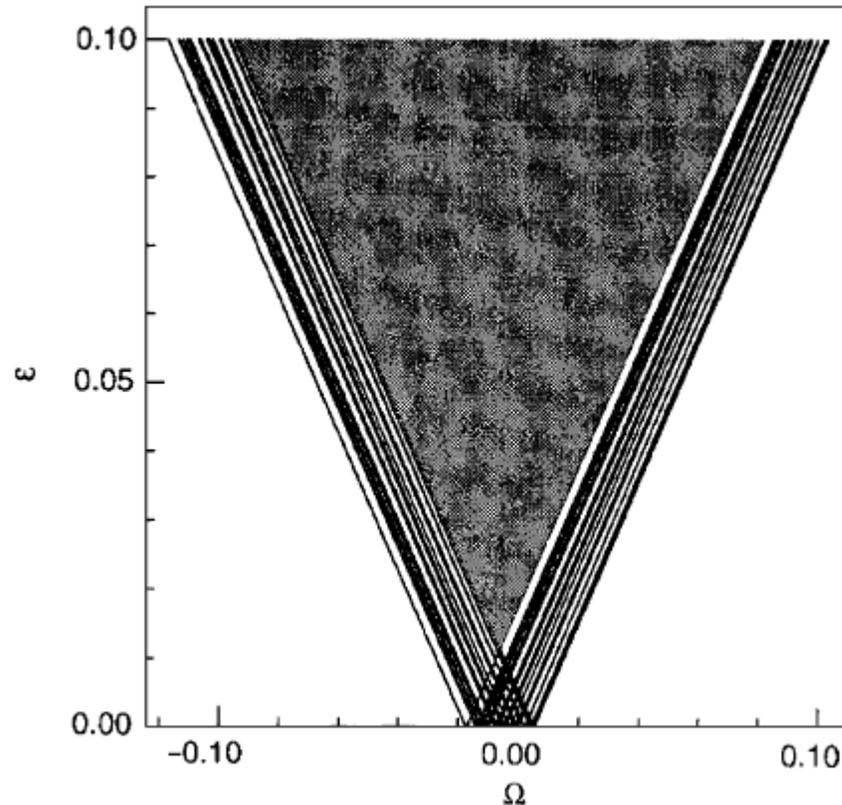


Fig. 3.5. Stroboscopic plot of the Rössler system state (x, y) (filled cycles) at each period of the driving signal (Eq. (3.4)). The dotted background is the unforced chaotic attractor. (a) $E = 0.15$, $\Omega_e = 1.0$, phase is synchronized. (b) $E = 0.15$, $\Omega_e = 1.02$, phase is not synchronized.

Understanding synchronization by means of unstable periodic orbits



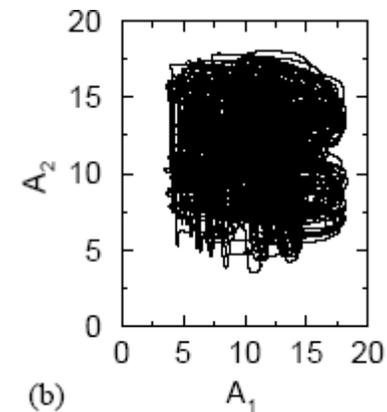
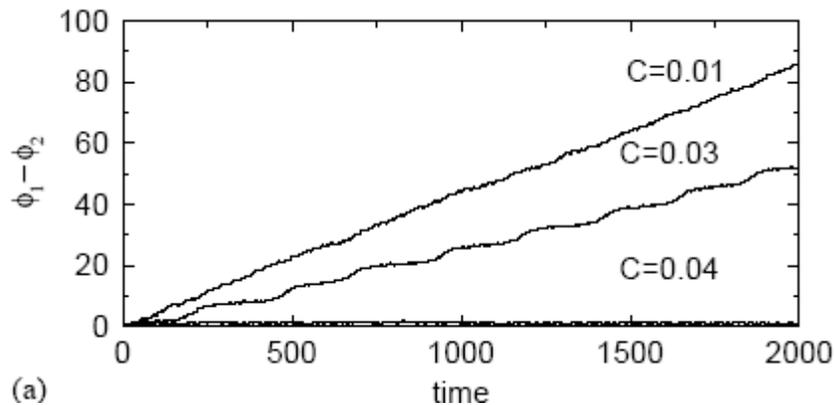
Phase-locking regions for periodic orbits with periods 1-5;
overlapping region – region of full phase synchronization
(dark, = natural frequency of chaotic system – ext force)

Synchronization of two coupled non-identical chaotic oscillators

$$\dot{x}_{1,2} = -\omega_{1,2}y_{1,2} - z_{1,2} + C(x_{2,1} - x_{1,2}) ,$$

$$\dot{y}_{1,2} = \omega_{1,2}x_{1,2} + ay_{1,2} ,$$

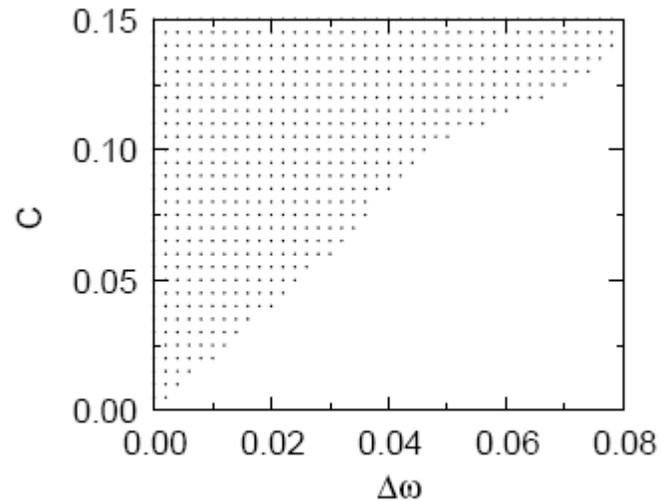
$$\dot{z}_{1,2} = f + z_{1,2}(x_{1,2} - c) ,$$



Phases are synchronized

BUT Amplitudes almost uncorrelated

Synchronization of two coupled chaotic oscillators



Synchronization region

C – coupling strength

$\Delta\omega$ – parameter mismatch

Two coupled non-identical Roessler oscillators

$$\phi = \arctan(y/x), \quad A = (x^2 + y^2)^{1/2},$$

we get

$$\dot{A}_{1,2} = aA_{1,2} \sin^2 \phi_{1,2} - z_{1,2} \cos \phi_{1,2} + C(A_{2,1} \cos \phi_{2,1} \cos \phi_{1,2} - A_{1,2} \cos^2 \phi_{1,2}),$$

$$\begin{aligned} \dot{\phi}_{1,2} = & \omega_{1,2} + a \sin \phi_{1,2} \cos \phi_{1,2} + z_{1,2}/A_{1,2} \sin \phi_{1,2} \\ & - C(A_{2,1}/A_{1,2} \cos \phi_{2,1} \sin \phi_{1,2} - \cos \phi_{1,2} \sin \phi_{1,2}), \end{aligned}$$

$$\dot{z}_{1,2} = f - cz_{1,2} + A_{1,2}z_{1,2} \cos \phi_{1,2}.$$

Equation for the slow phase θ : $\phi_{1,2} = \omega_0 t + \theta_{1,2}$

Averaging yields (Adler-like equation, phase oscillator):

$$\frac{d}{dt}(\theta_1 - \theta_2) = 2\Delta\omega - \frac{C}{2} \left(\frac{A_2}{A_1} + \frac{A_1}{A_2} \right) \sin(\theta_1 - \theta_2)$$

Synchronization threshold

Fixed point solution (by neglecting amplitude fluctuations)

$$\theta_1 - \theta_2 = \arcsin \frac{4\Delta\omega A_1 A_2}{C(A_1^2 + A_2^2)}$$

Fixed point stable (synchronization) if coupling
is larger than

$$C_{PS} = 4\Delta\omega A_1 A_2 / (A_1^2 + A_2^2).$$

Coherence vs. Phase Synchronization

- Coherence – linear technique
Phase analysis - nonlinear
- In several cases leading to similar results, **but**
- Coherence refers only to the **linearized part of the connectivity** (often not sufficient) and
- Coherence may easily lead to **artifacts** (indicates synchro where is none)

Imperfect Phase Synchronization

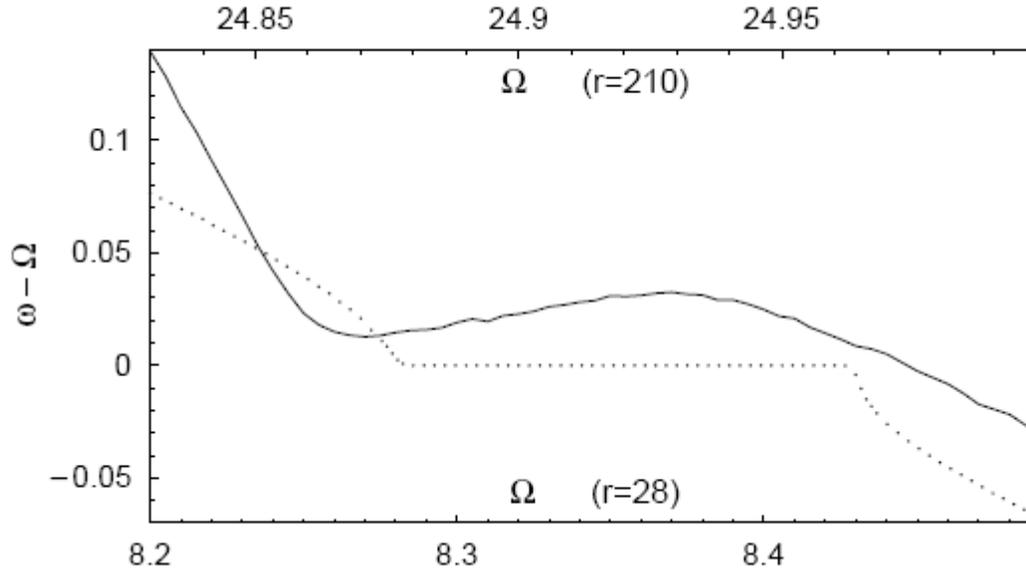
Lorenz system:

$$\dot{x} = 10(y - x) ,$$

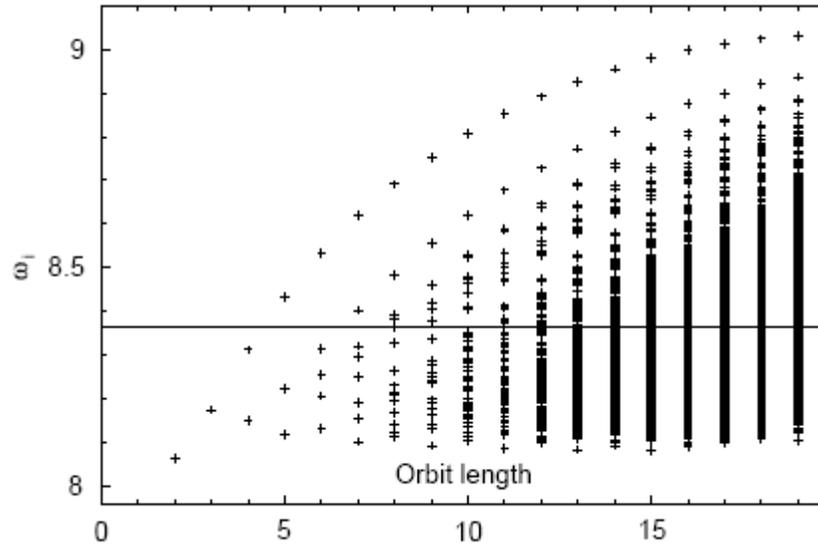
$$\dot{y} = rx - y - xz ,$$

$$\dot{z} = xy - 2.667z + E \cos(\Omega t)$$

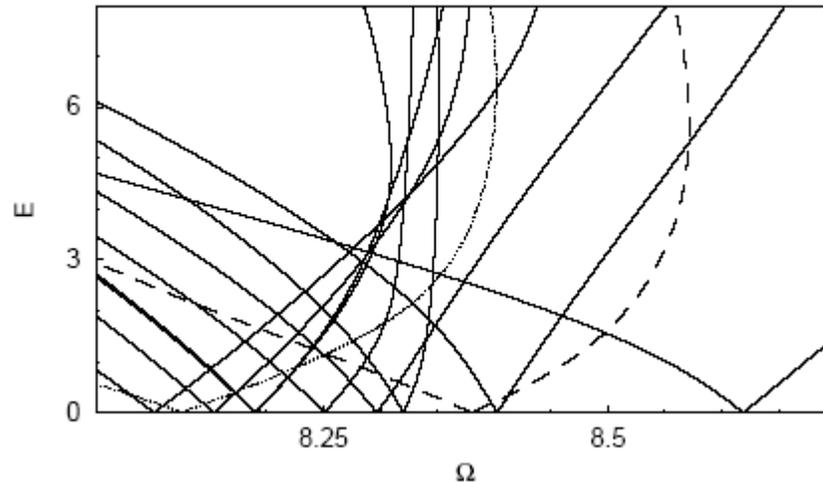
Periodically forced



Unstable Periodic Orbits – usual Lorenz system



Synchronization regions of UPOs in the usual Lorenz system

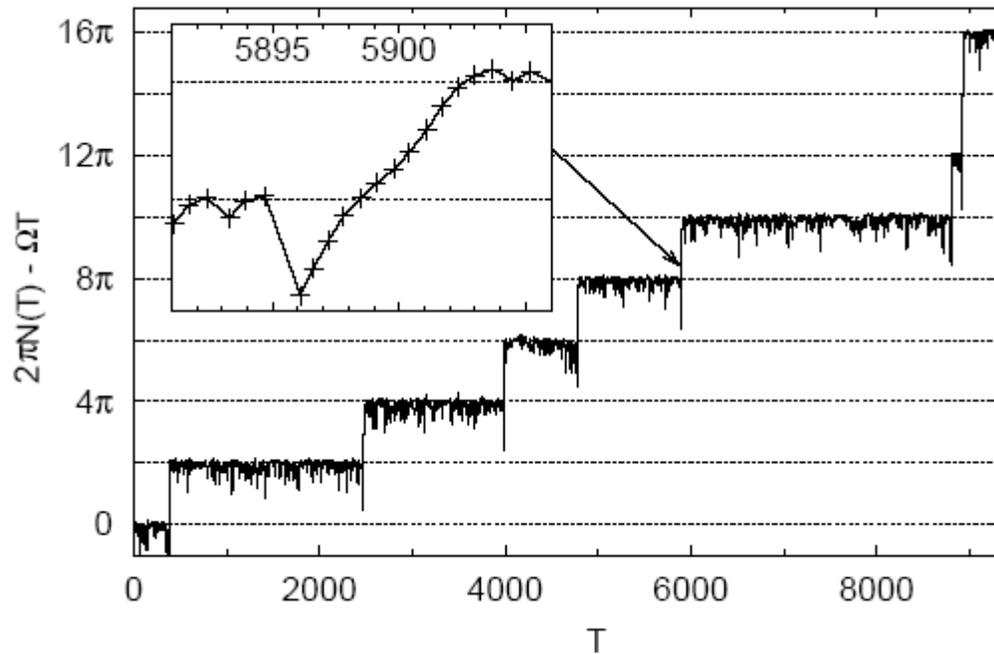


Solid lines – 1:1 synchronization

Dashed line – 14:15 synchronization

Dotted line – 18:20 synchronization

Phase „jumps“ in the forced usual Lorenz model



Problems with phase jumps

- Phase synchronization: difference of phases of two (more) subsystems is bounded
 - Phase jumps of $\pm 2\pi$ occur due to
 - at the borderline of synchro region
 - influence of noise (Stratonovich)
 - broad variety of unstable periodic orbits, as in the Lorenz system (deterministic effect)
- ➔ imperfect phase synchronization
(Zaks, Park, Rosenblum, Kurths: Phys Rev Lett 1999)

Cyclic relative phase

- How to consider this problem?
- Phase synchronization in a statistical sense
- Instead of the strong condition

$$|n\phi_1(t) - m\phi_2(t) - \delta| < \text{const}$$

We consider the cyclic relative phase

$$\Psi_{n,m} = \varphi_{n,m} \bmod 2\pi$$

And analyze the frequency distribution of the cyclic relative phase

Statistical description of phase synchronization

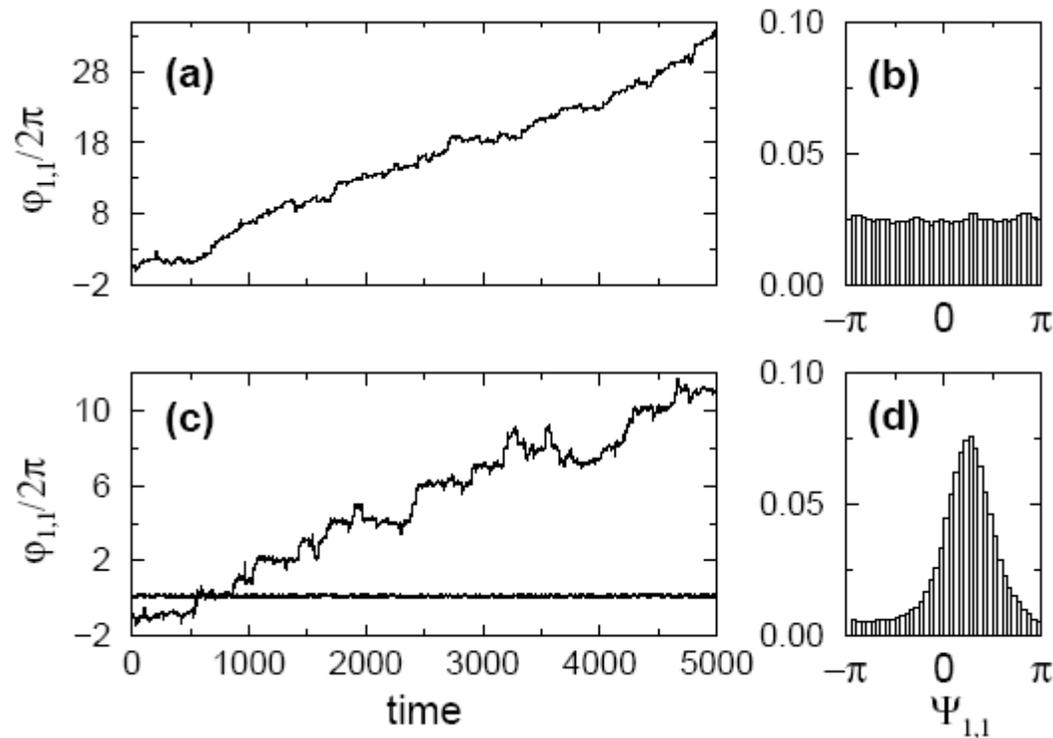


Fig. 3. Relative phase $\varphi_{1,1} = \phi_1 - \phi_2$ and distribution of $\Psi_{1,1} = \varphi_{1,1} \bmod 2\pi$ for the case of uncoupled (a,b) and coupled (c,d) non-identical chaotic systems perturbed by noise. The horizontal line in (c) corresponds to the absence of noise: in this case the phase difference fluctuates around some constant value due to influence of chaotic amplitudes. These fluctuations are rather small (barely seen in this scale), and no phase slips are observed; this fact is explained by the high phase coherence properties of the Rössler attractor. In contrast to the noisy case, here we observe both frequency and phase locking.

Criteria for phase synchronization - 1

- (1) The synchronization index based on the Shannon entropy S of the phase difference distribution [49]. Having an estimate p_k of the distribution of $\Psi_{n,m}$, we define the index ρ as

$$\rho_{n,m} = \frac{S_{max} - S}{S_{max}}, \quad (14)$$

where $S = -\sum_{k=1}^N p_k \ln p_k$, and the maximal entropy is given by $S_{max} = \ln N$; N is the number of bins and p_k is the relative frequency of finding $\Psi_{n,m}$ within the k -th bin⁵. Due to the normalization used,

$$0 \leq \rho_{n,m} \leq 1, \quad (15)$$

whereas $\rho_{n,m} = 0$ corresponds to a uniform distribution (no synchronization) and $\rho_{n,m} = 1$ corresponds to a distribution localized in one point (δ -function). Such distribution can be observed only in the ideal case of phase locking of noise-free quasilinear oscillators.

Criteria for phase synchronization - 2

(2) Intensity of the first Fourier mode of the distribution

$$\gamma_{n,m}^2 = \langle \cos \Psi_{n,m}(t) \rangle^2 + \langle \sin \Psi_{n,m}(t) \rangle^2, \quad (16)$$

where the brackets denote the average over time, can serve as the other measure of the synchronization strength; it also varies from 0 to 1. The advantage of this index is that its computation involves no parameters: we do not need to choose the number of bins as we do not calculate the distribution itself.

Efficiency of these criteria

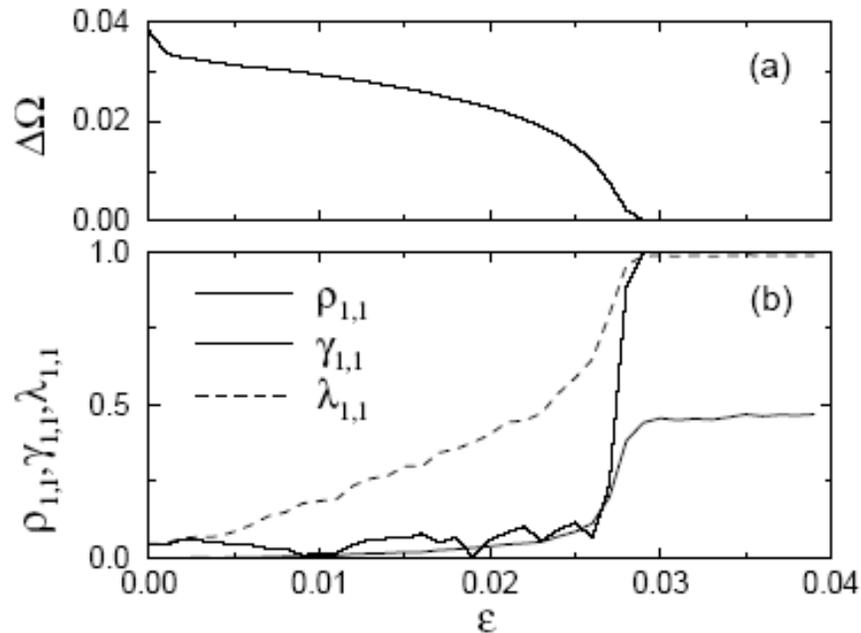


Fig. 9. Comparison of quantitative measures of synchronization using the simulated data from two coupled Rössler oscillators (Eq. (10)). Transition to synchronous state takes place when the difference of frequencies of two oscillators vanishes with increase of coupling coefficient ε (a). Three 1 : 1 synchronization indices are shown as the functions of ε (b).

(Phase) Synchronization –
good or bad???

Context-dependent

Applications in various fields

Lab experiments:

- Electronic circuits (Parlitz, Lakshmanan, Dana...)
- Plasma tubes (Rosa)
- Driven or coupled lasers (Roy, Arecchi...)
- Electrochemistry (Hudson, Gaspar, Parmananda...)
- Controlling (Pisarchik, Belykh)
- Convection (Maza...)

Natural systems:

- Cardio-respiratory system (Nature, 1998...)
- Parkinson (PRL, 1998...)
- Epilepsy (Lehnertz...)
- Kidney (Mosekilde...)
- Population dynamics (Blasius, Stone)
- Cognition (PRE, 2005)
- Climate (GRL, 2005)
- Tennis (Palut)

Posture Control

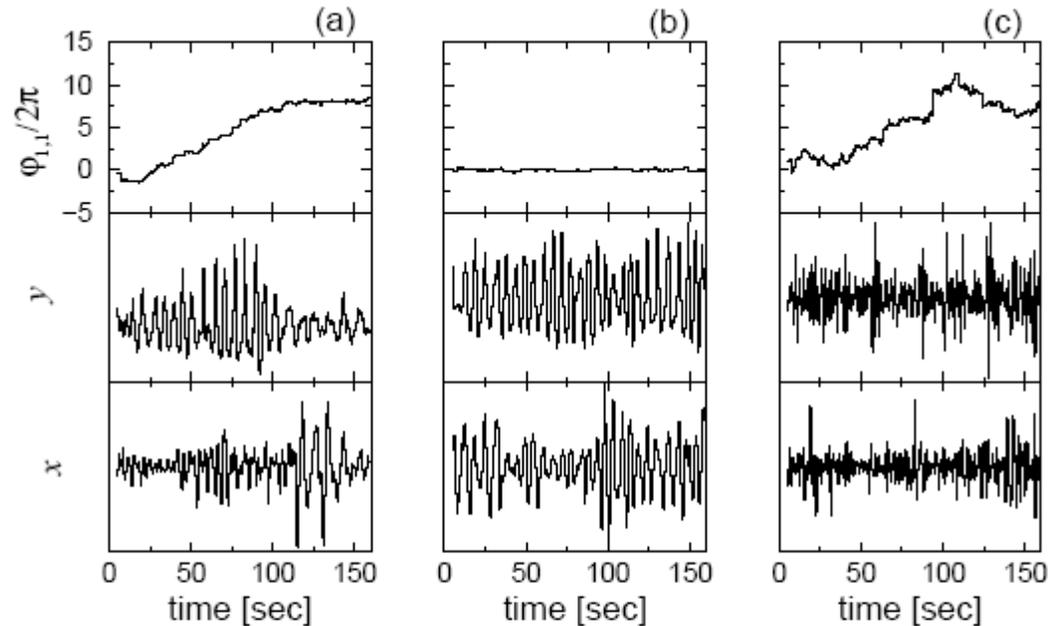


Fig. 7. Stabilograms of a neurological patient for EO (a), EC (b), and AF (c) tests. The upper panels show the relative phase between two signals x and y that are deviation of the center of pressure in anterior–posterior and lateral direction, respectively. During the last 50s of the first test and the whole second test the phases are perfectly locked. No phase entrainment is observed in the AF test.

EO – eyes open, EC – eyes closed, AF – eyes open and visual feedback

Analysis of posturographic measurements of balance

Hypothesis:

Is there a relationship between cognition and motorics?
Ability to control posture and school success



Application:

Cardiovascular System

Cardio-respiratory System

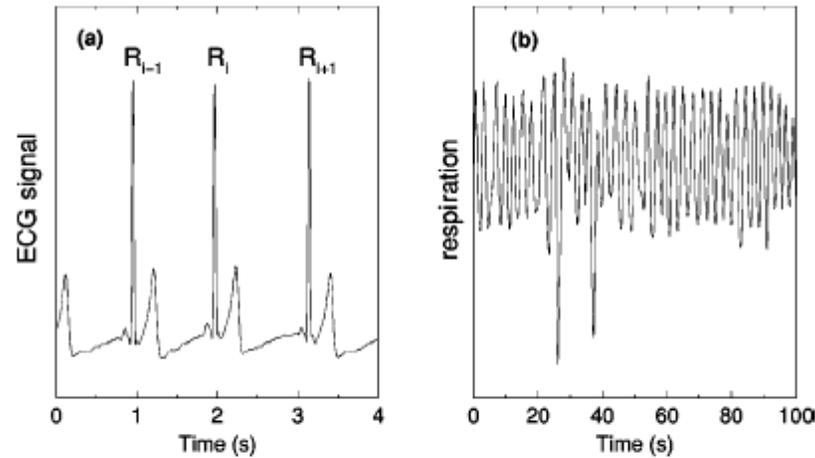


FIG. 1. Short segments of an electrocardiogram with the R peaks marked (a) and of a respiratory signal (b); both signals are in arbitrary units.

Analysis technique: **Synchrogram**

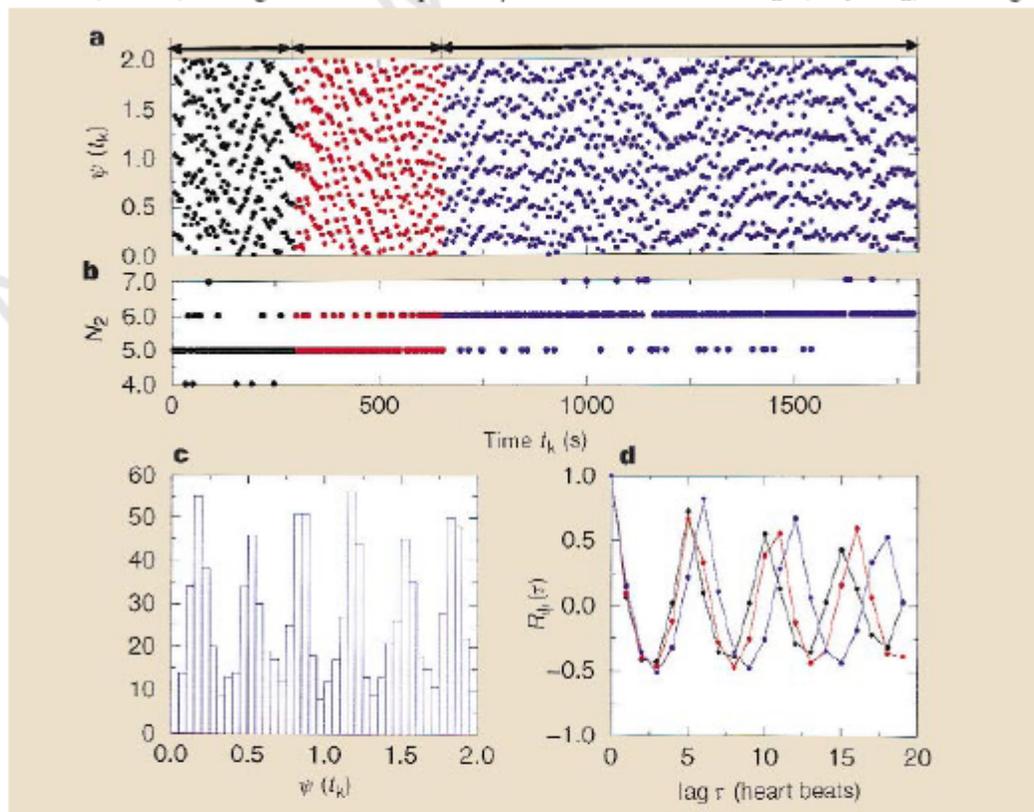


Figure 1 Analysis of cardiorespiratory cycles. a, Cardiorespiratory synchrogram, showing the transition (red) from 5:2 frequency locking (black) to 3:1 phase locking (blue). Each point shows the normalized relative phase of a heartbeat within two adjacent respiratory cycles $\psi(t_k) = (\phi_1(t_k) \text{ mod } 4\pi) / 2\pi$. b, Number of heartbeats within two adjacent respiratory cycles. c, Histogram of phases. The six horizontal stripes in the blue region of the CRS result in six well-pronounced peaks in the distribution of phases. d, Autocorrelation function of phases $R_\psi(\tau) = \sum_i (\psi(t_{i+\tau}) - \langle \psi \rangle) (\psi(t_i) - \langle \psi \rangle) / \sum_i (\psi(t_i) - \langle \psi \rangle)^2$. The coloured curves correspond to respective regions.

Schäfer, Rosenblum, Abel, Kurths: Nature, 1998

Active vs. passive experiments

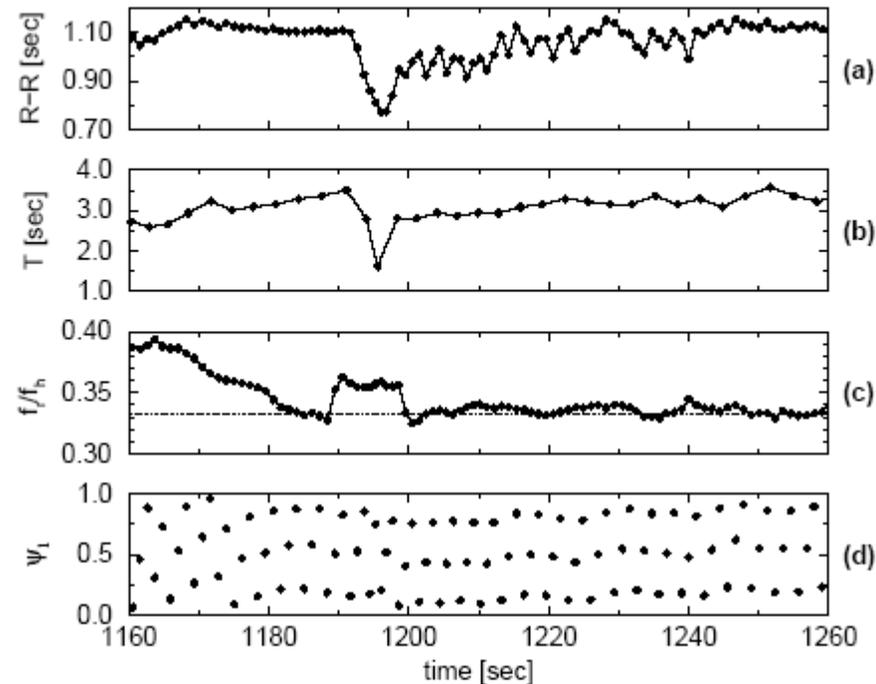
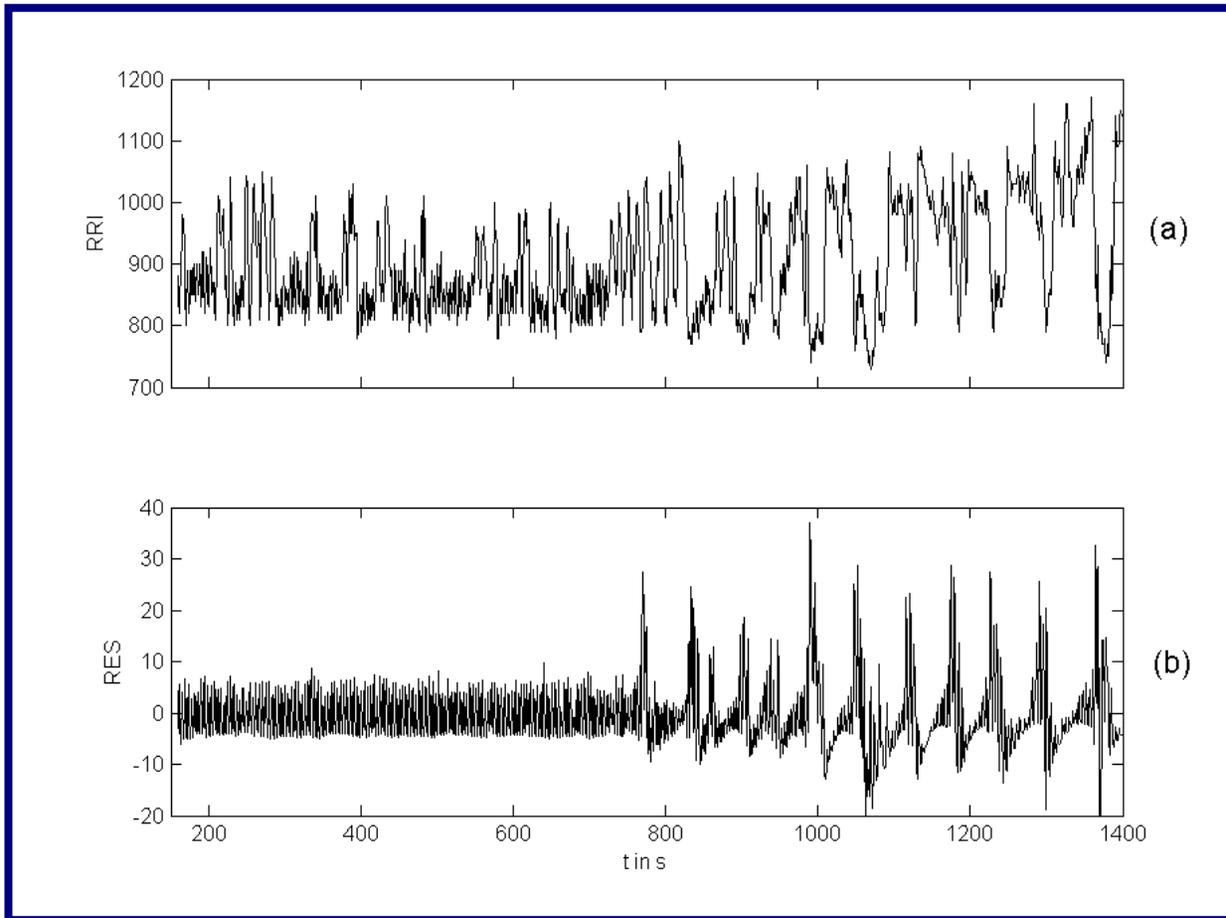


Fig. 23. A transient epoch within the data of subject A confirms the existence of synchronization. The periods of cardiac (R-R) and respiratory cycles (T) are shown in (a) and (b), respectively. After a short epoch of non-synchronous behavior (1150–1200s) the frequencies of heart rate and respiration change, probably due to influence of a certain control mechanism, and become locked, i.e., $f_r/f_h \approx 1/3$. In the next 50s we observe that, although both frequencies decrease, this ratio remains almost constant (c). This means that one of the systems follows the other one, i.e., synchronization takes place. 3 : 1 phase locking is also clearly seen from CRS (d).

Cardiorespiratory Synchronisation during Sleep

NREM

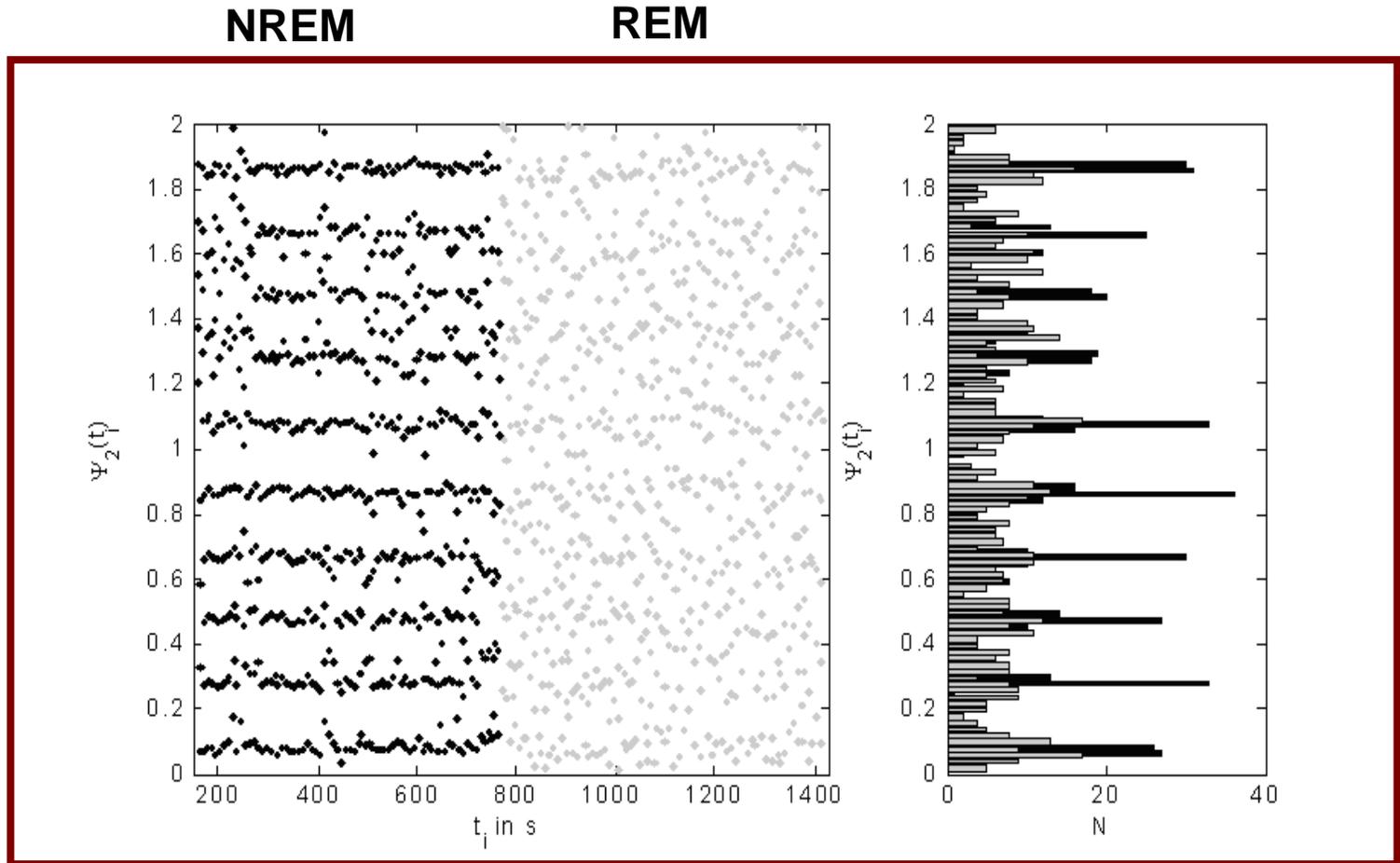
REM



**Beat-to-beat
intervals**

Respiration

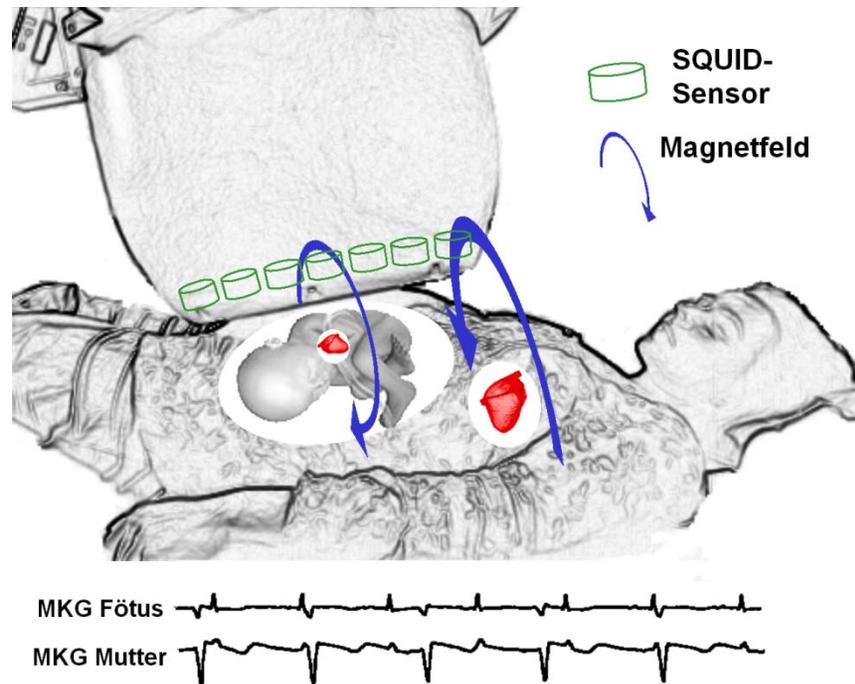
Cardiorespiratory Synchronisation during Sleep



Phil. Trans Roy Soc A, 2009

5:1 synchronization during NREM

Application of synchronization analysis: Mother-Fetus System



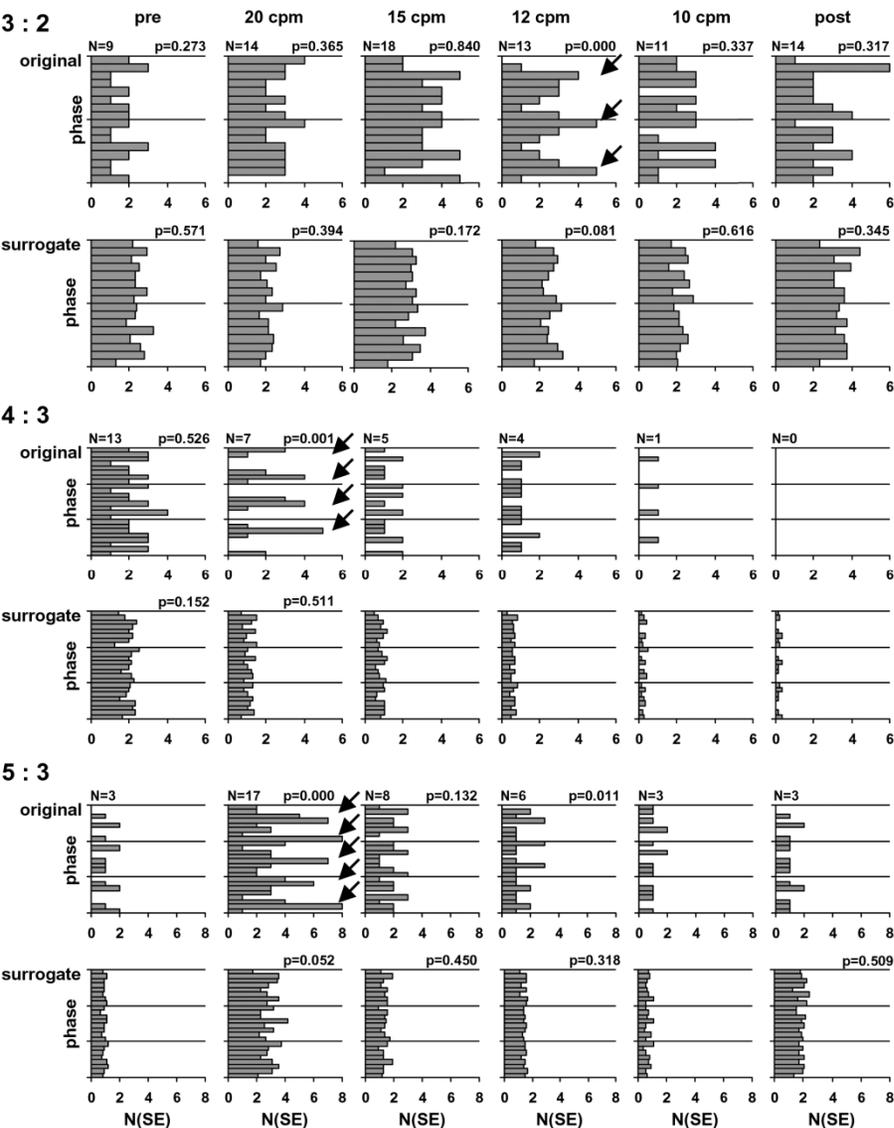
Magnetocardiography

Magnetocardiogram (MCG) Data – Paced Breathing

- 6 pregnant women, aged 33 +/- 4 years
 - 34th – 40th week of gestation
- 6 consecutive 5 min simultaneous fetal and maternal MCGs for the sequence:
spontaneous, 15 cpm, 10 cpm, 20 cpm, 12 cpm, spontaneous breathing of mother;

2-3 min pause between (40 min)

Influence of maternal paced breathing on the connections of the fetal-maternal cardiac systems



Distribution of the synchronization epochs (SE) over the maternal beat phases with respect to the $n:m$ combinations

3:2 (top), 4:3 (middle) and 5:3 (bottom) in the different respiratory conditions.

Special test statistics: twin surrogates

van Leeuwen, Romano, Thiel, Wessel, Kurths, PNAS 106, 13661 (2009) (+ commentary)

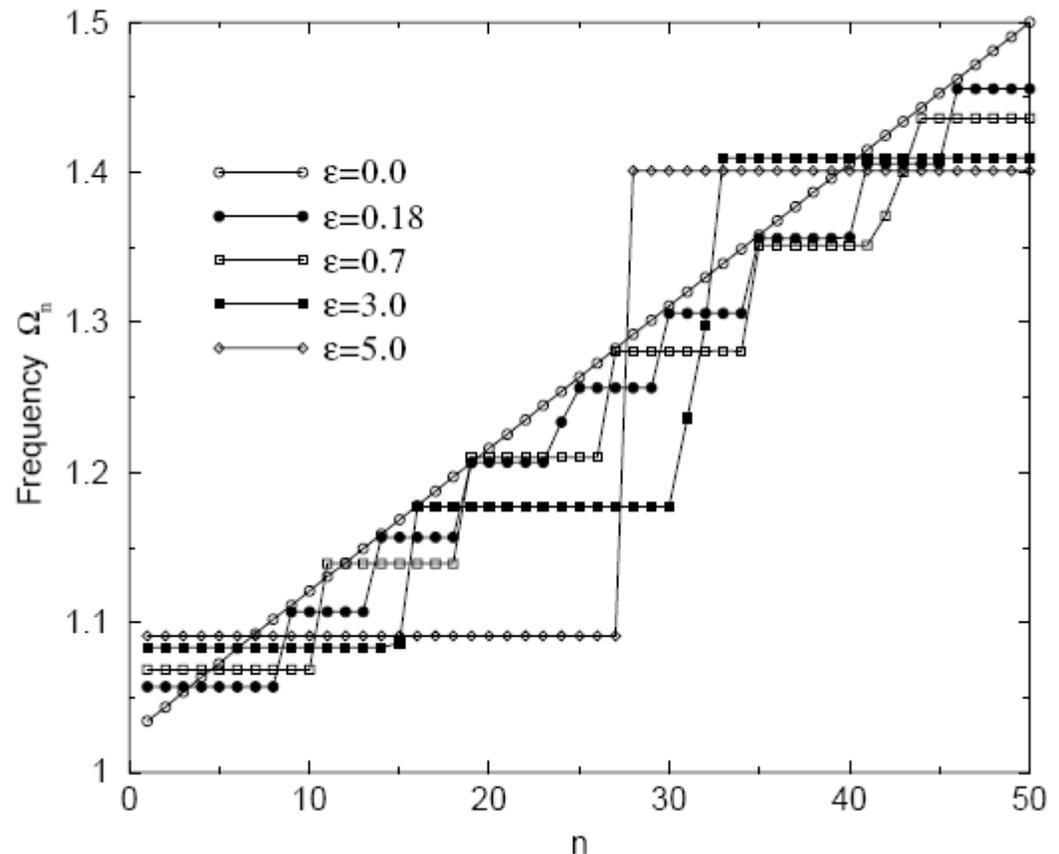
Synchronization in spatially extended systems

$$\dot{x}_n = -\omega_n y_n - z_n ,$$

$$\dot{y}_n = \omega_n x_n + a y_n + \epsilon(y_{n+1} - 2y_n + y_{n-1}) ,$$

$$\dot{z}_n = 0.4 + (x_n - 8.5)z_n .$$

Chain of diffusively
coupled Roessler
oscillators



Soft transition to phase synchronization

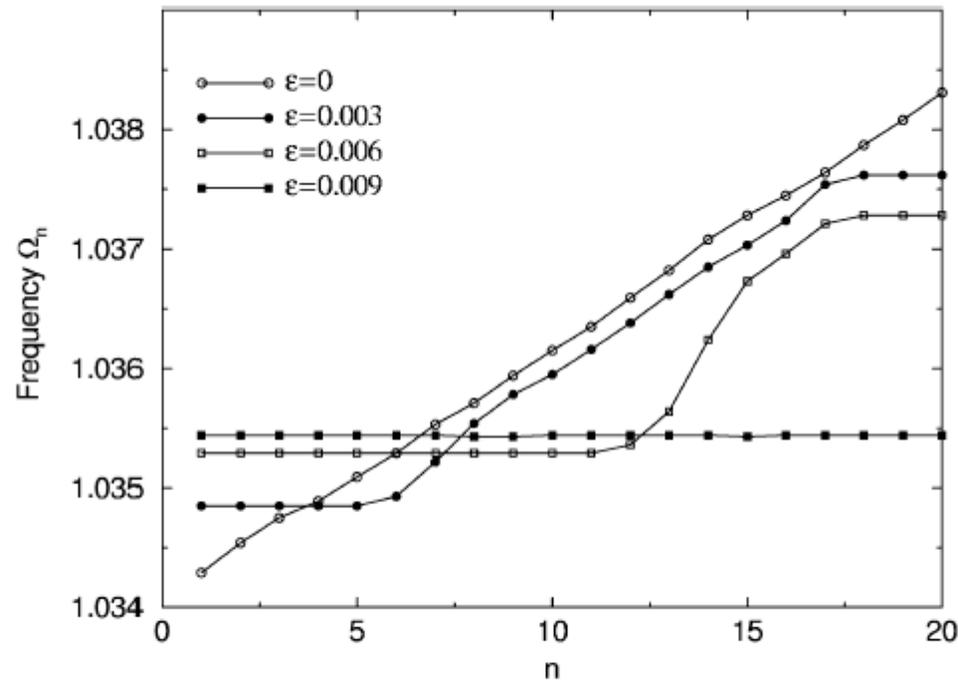


Fig. 6.3. Soft transition to global synchronization in a chain of Rössler oscillators (6.24). Mean frequencies Ω_n for different values of coupling ϵ . The parameters are: $N = 20$, the frequency mismatch $\delta = 2 \times 10^{-4}$ and $\omega_1 = 1$.

Soft transition to PS

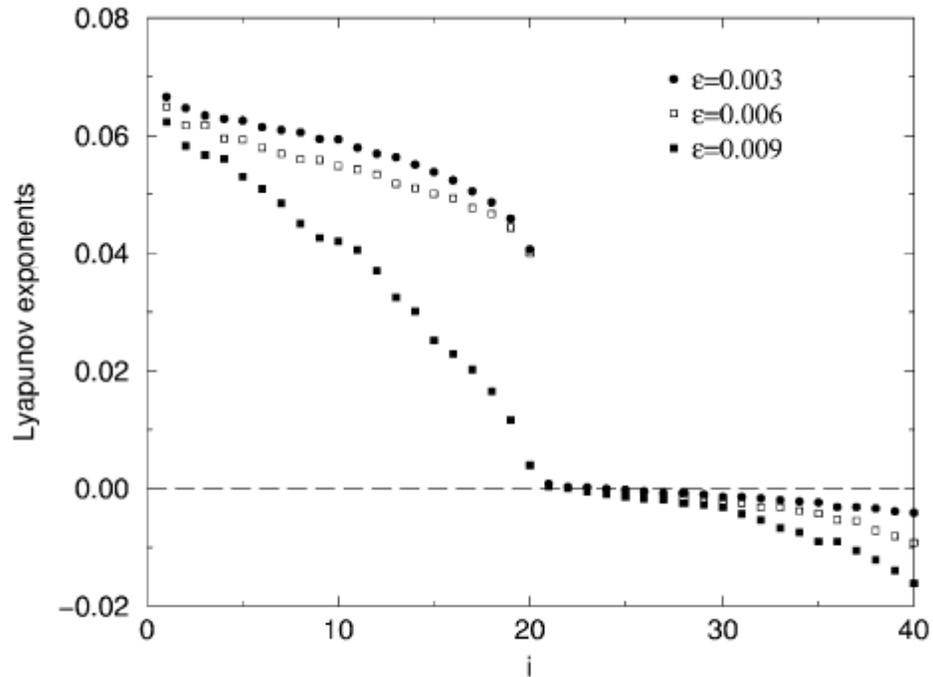


Fig. 6.4. 40 largest Lyapunov exponents λ_i for the regimes reported in Fig. 6.3.

Hard transition to PS

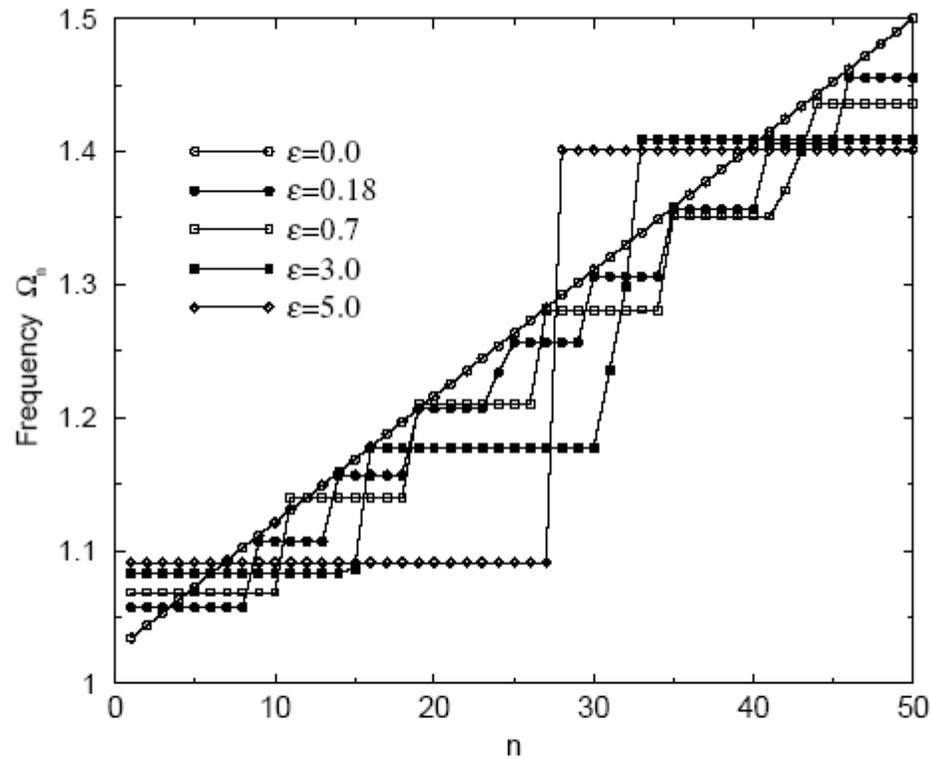


Fig. 6.5. Hard transition to global synchronization in a chain of Rössler oscillators (Eq. (6.24)). Mean frequencies Ω_n for different values of coupling ϵ . The parameters are: $N = 50$, the frequency mismatch $\delta = 9 \times 10^{-3}$ and $\omega_1 = 1$.

Hard transition to PS

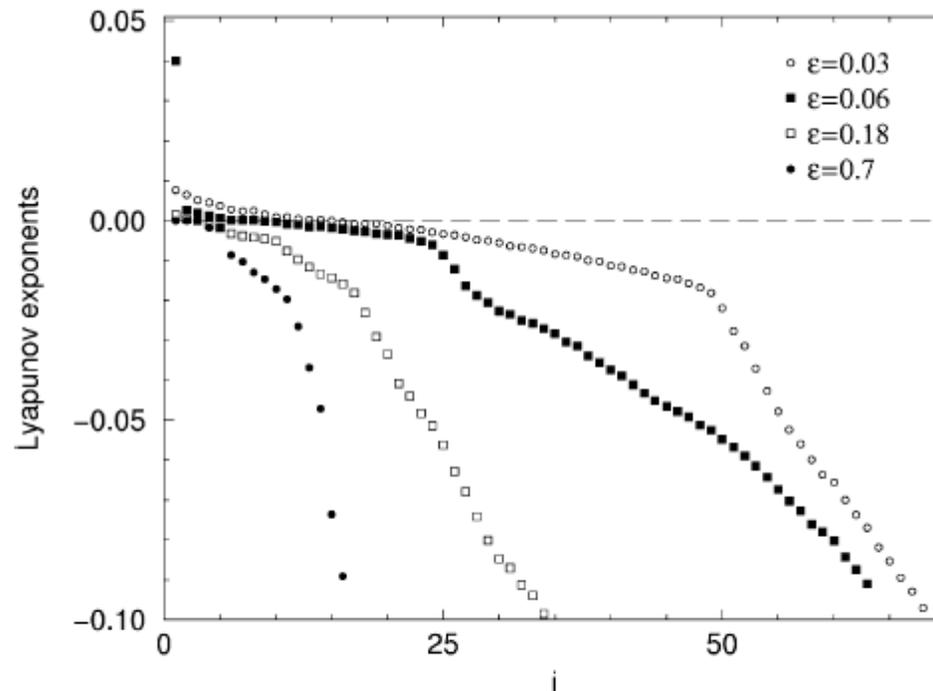


Fig. 6.6. 70 largest Lyapunov exponents λ_i for the regimes reported in Fig. 6.5.

Hard transition to PS

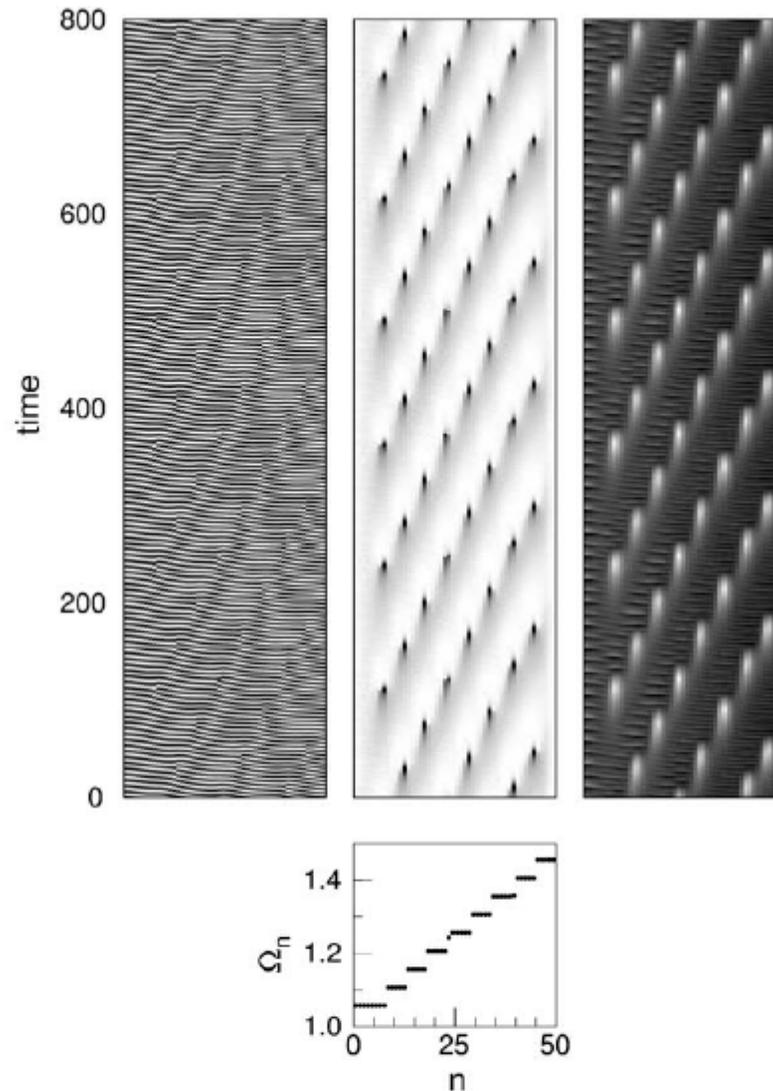


Fig. 6.7. Mean frequencies Ω_n and space-time plots in a chain of 50 coupled Rössler oscillators with a frequency mismatch $\delta = 9 \times 10^{-3}$ and coupling $\epsilon = 0.18$. All plots show a gray-scale representation of corresponding quantities. Minimal values are represented by white and maximal by black.

Randomly distributed initial frequencies

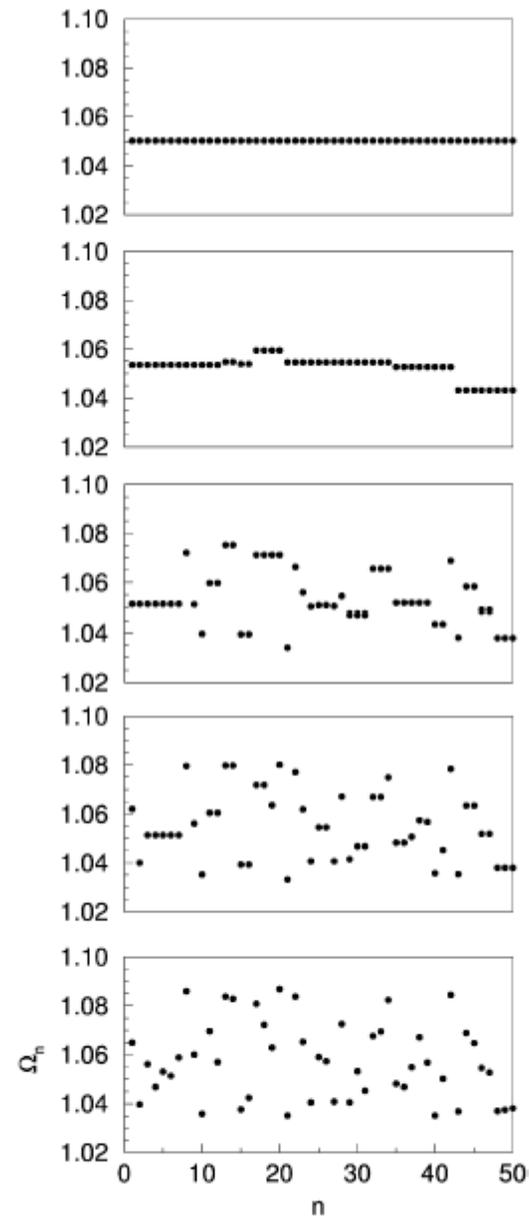


Fig. 6.8. Mean frequencies Ω_n in a chain of Rössler oscillators with randomly distributed natural frequencies ω_n in the interval $[1, 1.05]$. The number of elements $N = 50$, $\omega_1 = 1$. From bottom to top different coupling strengths $\epsilon = 0, 0.01, 0.02, 0.05, 0.2$.

Example: Chain of globally coupled spiking-bursting maps

$$\begin{cases} x(i, n + 1) = \frac{\alpha_i}{1+x(i, n)^2} + y(i, n) + \frac{\varepsilon}{N} \sum_{j=1}^N x(j, n) \\ y(i, n + 1) = y(i, n) - \sigma_i x(i, n) - \beta_i, \end{cases}$$

Rulkov Map

- x fast variable
 - y slow variable
 - α randomly distributed in $[4.1, 4.4]$
- ➔ Spiking and bursting dynamics

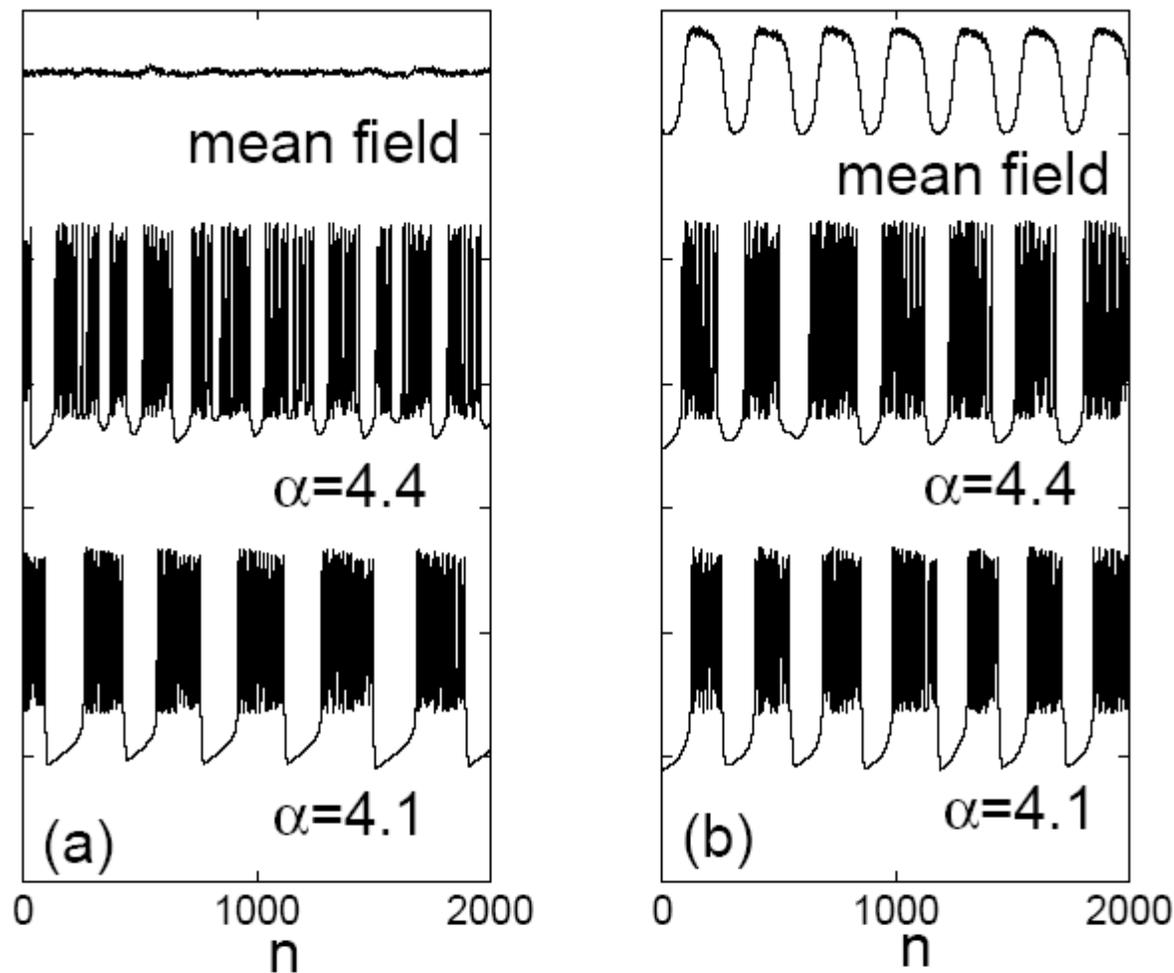


FIG. 1: Realizations of $x(i, n)$ of two neurons and the mean field from (a) an uncoupled ensemble ($\varepsilon = 0$) and (b) a coupled ensemble ($\varepsilon = 0.04$, synchronization of bursts is achieved), in the absence of external signal ($d = 0$). Different values of α_i are implemented, $\sigma = \beta = 0.001$. Here $N = 1000$.

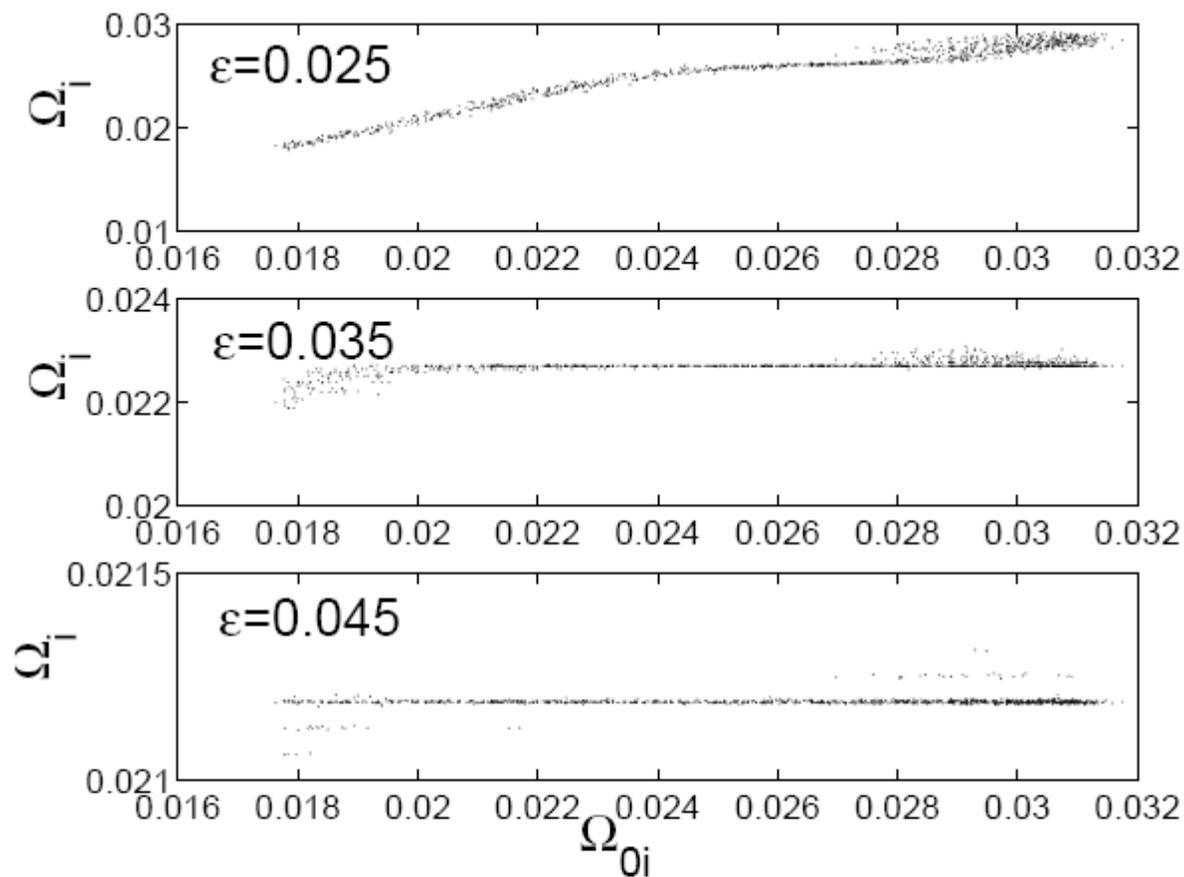
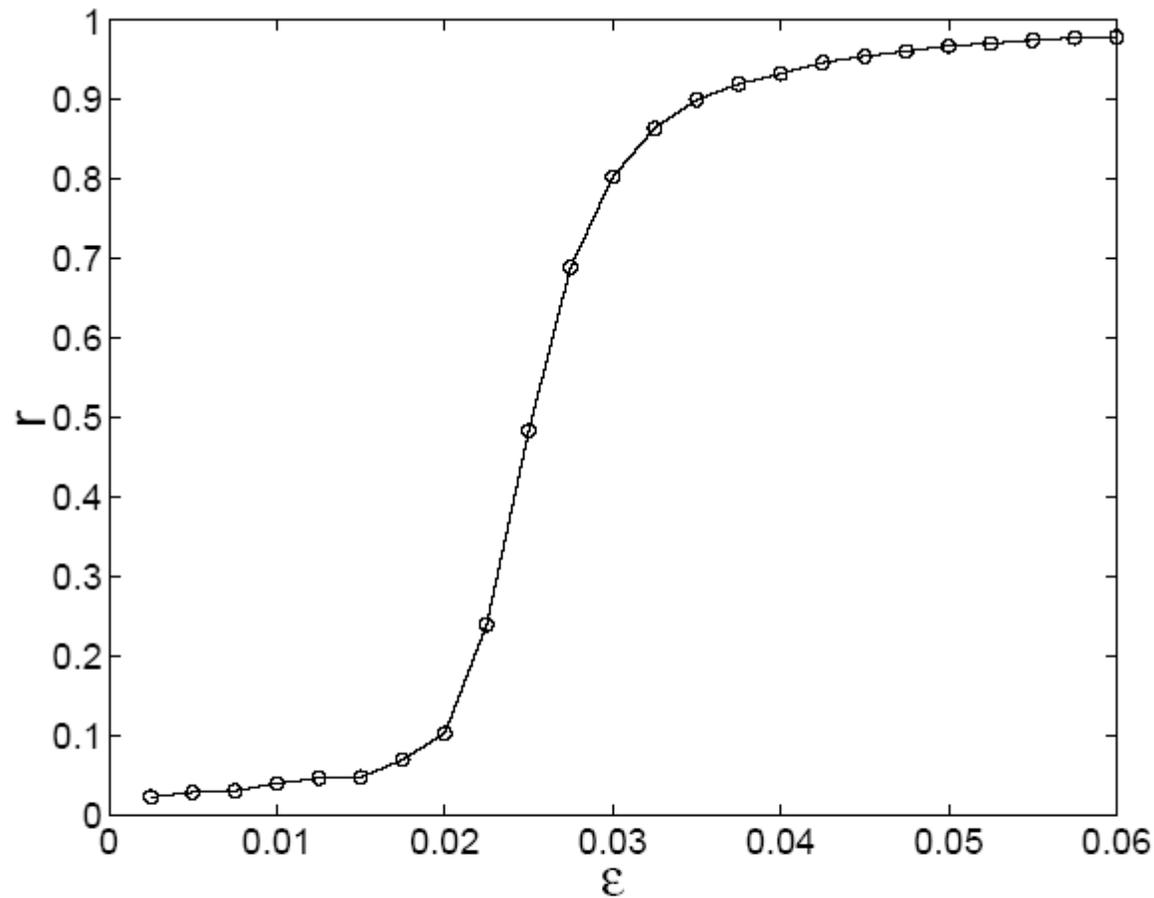


FIG. 2: Frequencies of bursting in the mean field coupled ensemble vs. those at the zero mean field coupling show a growth of a synchronization cluster as the coupling coefficient ε is gradually increased. The external signal is absent ($d = 0$), $N = 1000$.



Synchronization
only of bursts
and not of
spikes!

FIG. 3: The order parameter r vs. mean field coupling coefficient ϵ indicates a second-order phase transition to CPS of bursting ($N = 1000$).

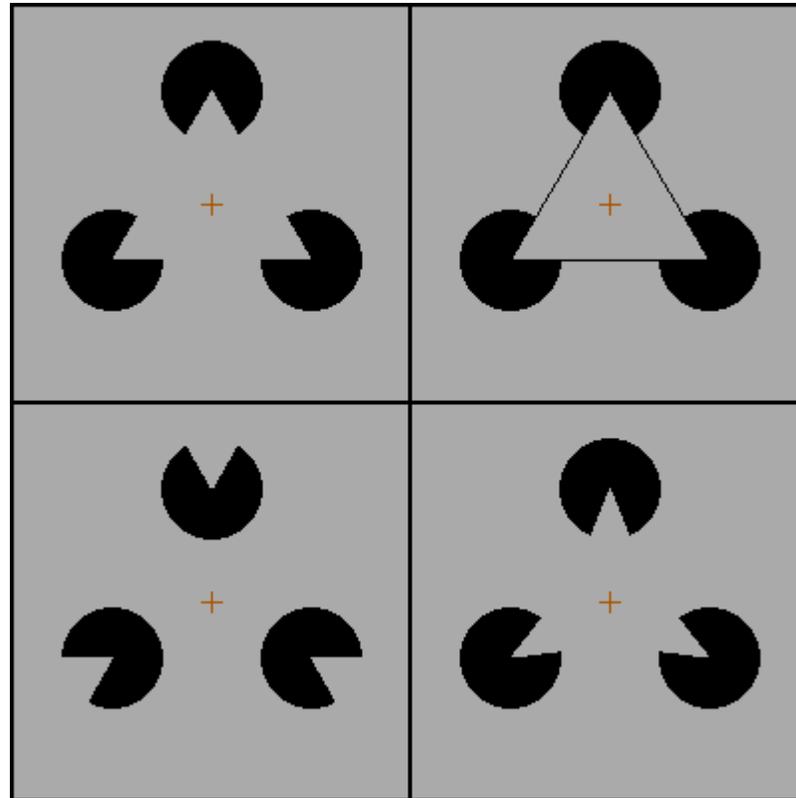
Ivanchenko, Osipov, Shalfeev, Kurths: Phys Rev Lett
92, 234101 (2004)

Cognitive Processes

Processing of visual stimuli

- Kanizsa-figure as stimulus (virtual figure vs. control figure)
- EEG-measurements (500 Hz, 30 channels)
- Multivariate synchronization analysis to identify synchronized clusters (Allefeld, Kurths, 2004)

Kanizsa Figures



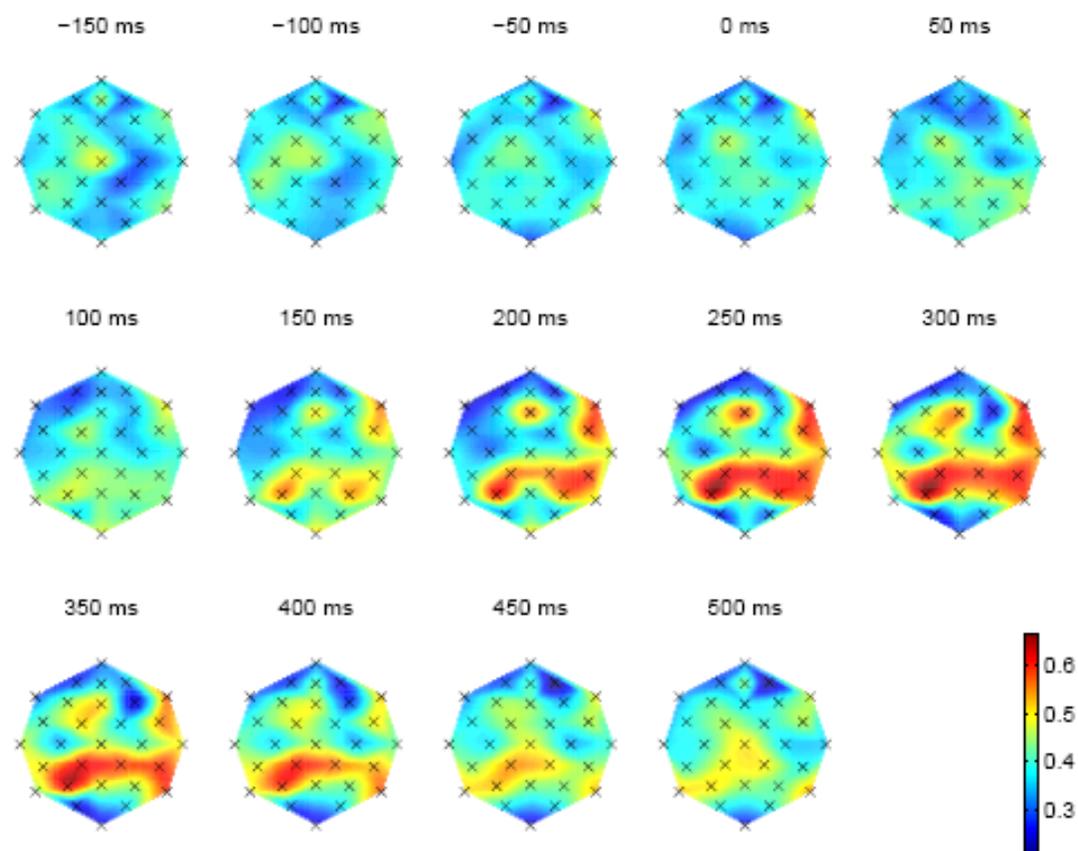


Figure 3: Time evolution of the cluster synchronization topography at $f = 13$ Hz for the Kanizsa condition. The continuous colors correspond to an interpolation of the ρ_{IC} -values attributed to the electrodes, whose positions are marked by \times -symbols. For a chart of the electrode names, see Fig. 5.

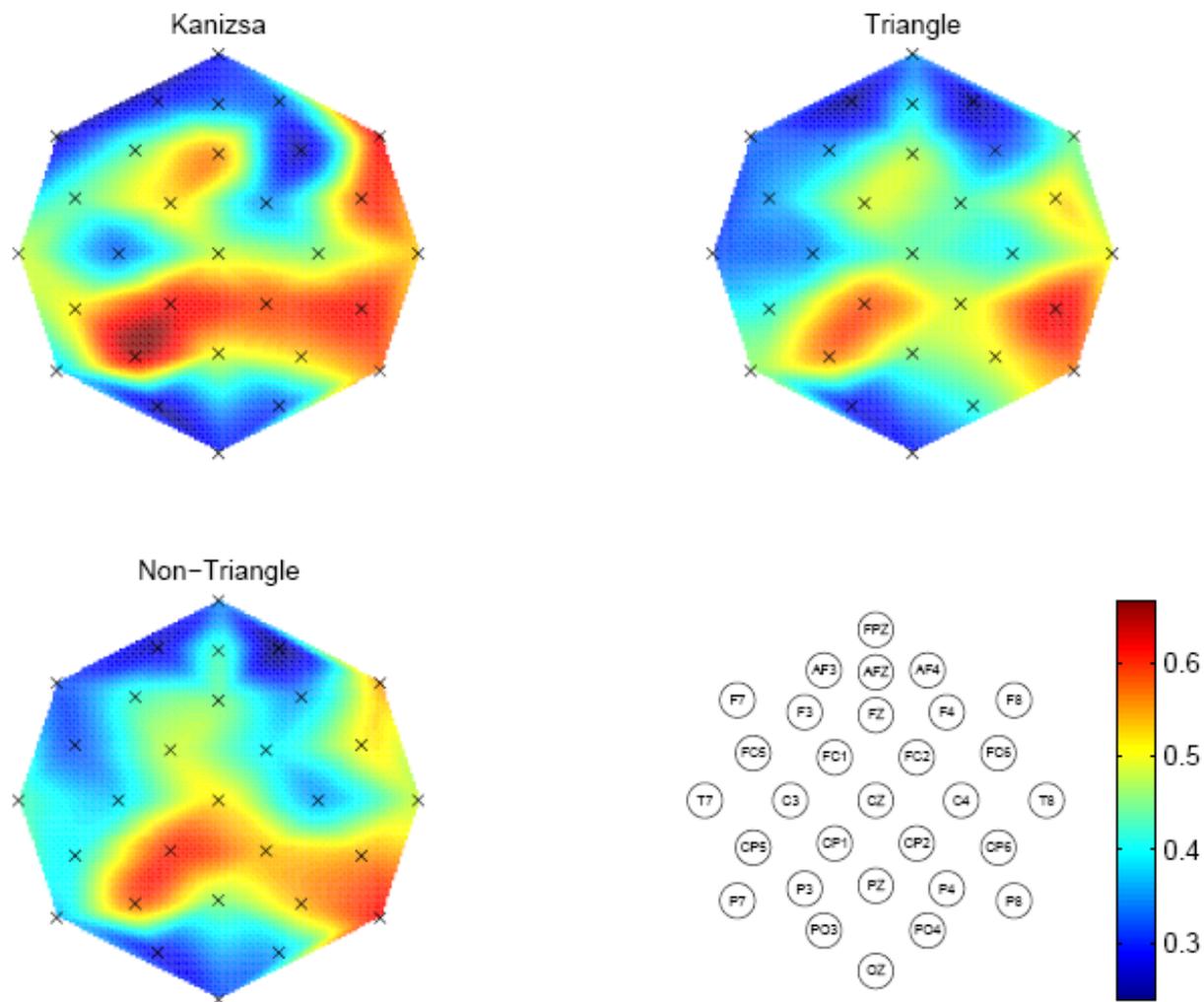


Figure 5: Differences in the synchronization topography at $f = 13$ Hz and $t = 300$ ms for the three experimental conditions.

Possible Artefacts

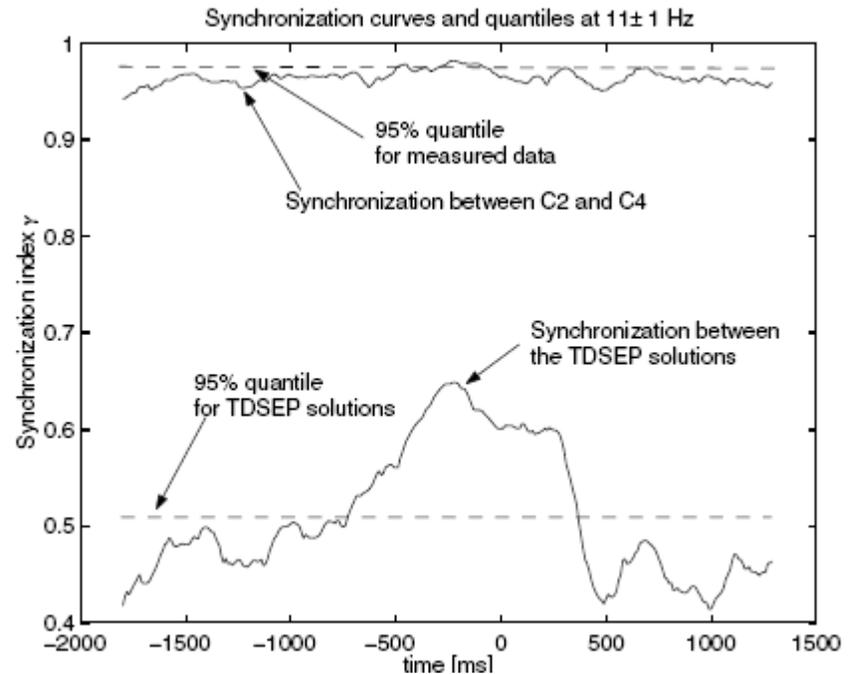
- Superimposed signals (EEG...)
 - Cocktail-party problem (source separation)
 - ICA - independent component analysis
 - Temporal decorrelation separation method
- ➔ very necessary as a 1st step (before synchronization analysis) to avoid pitfalls

Cognitive Experiment

Press a key 2 s

Activity: -500 ... 300 ms

Synchro effect in mu-band
(11 Hz)



Meinecke, Ziehe, Kurths,
Müller: Phys. Rev. Lett.
94, 084102 (2005)

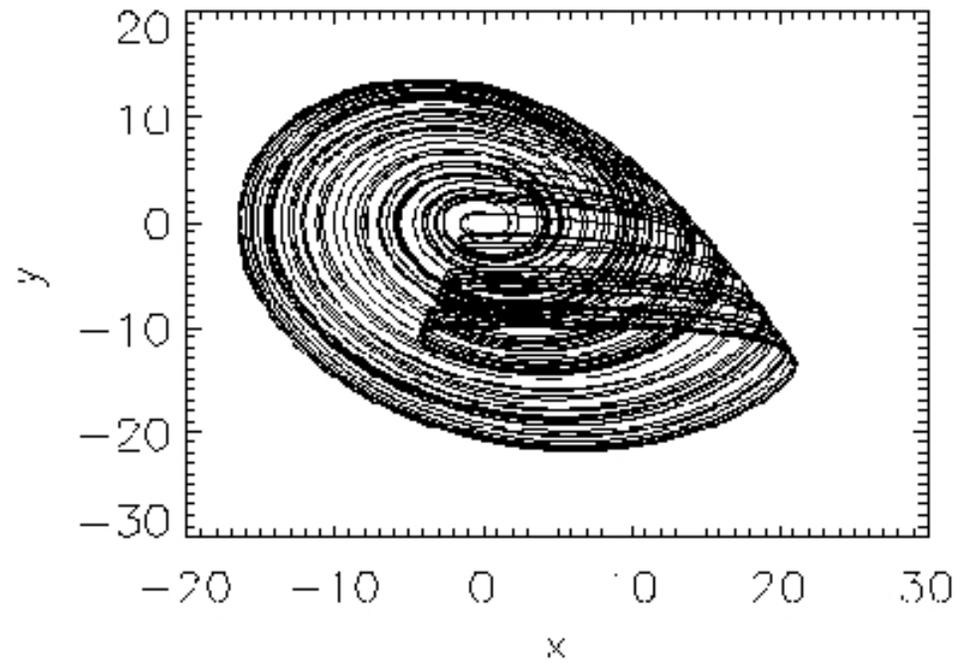
FIG. 3. Synchronization between EEG signals during finger movements measured by electrodes located over the motor cortex. The high phase locking values of the originally measured signals (upper curve) do not reveal any interesting temporal structure and can in large part be explained by superposition effects. The TDSEP-source estimates show a significant synchronization peak directly before the movement.

Synchronization in more complex topology

- Systems are often non-phase-coherent (e.g. funnel attractor – much stronger phase diffusion)
- How to study phase dynamics there?
- 1st Concept: **Curvature**
(Osipov, Hu, Zhou, Ivanchenko, Kurths: Phys. Rev. Lett., 2003)

$$\phi = \arctan \frac{\dot{y}}{\dot{x}}.$$

Roessler Funnel – Non-Phase coherent



Phase basing on curvature

curve $\vec{r}_1 = (u, v)$ the angle velocity at each point is

$$\nu = \frac{ds}{dt} / R,$$

where

$$ds/dt = \sqrt{\dot{u}^2 + \dot{v}^2}$$

is the speed along the curve and

$$R = (\dot{u}^2 + \dot{v}^2)^{3/2} / [\dot{v}\ddot{u} - \dot{u}\ddot{v}]$$

is the radius of the curvature. If $R > 0$ at each point, then

$$\nu = \frac{d\phi}{dt} = \frac{\dot{v}\ddot{u} - \dot{u}\ddot{v}}{\dot{u}^2 + \dot{v}^2},$$

is always positive and therefore the variable ϕ defined as

$$\phi = \int \nu dt = \arctan \frac{\dot{v}}{\dot{u}},$$

Dynamics in non-phase-coherent oscillators

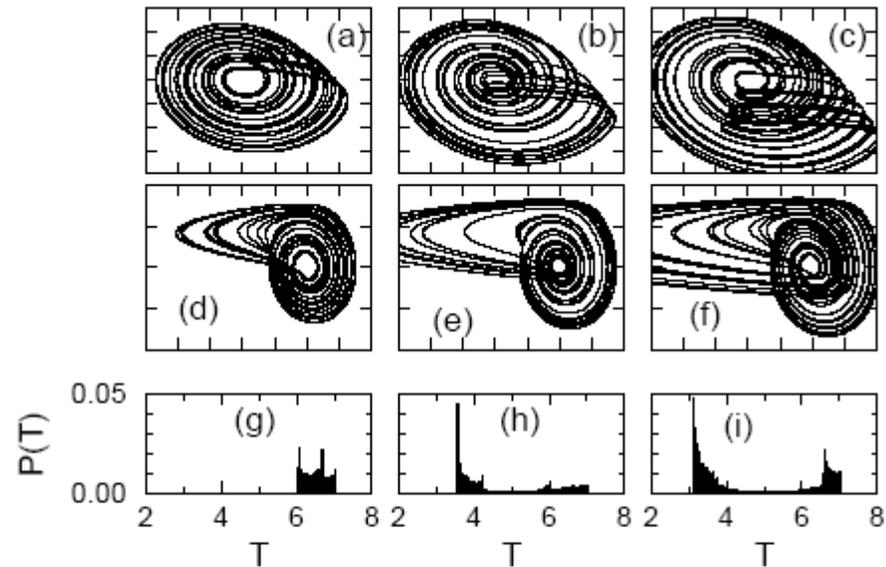


FIG. 1: Upper panel (a,b,c): projections of the attractors of the Rössler systems (1) onto the plane (x, y) ; middle panel: (d,e,f): projections onto (\dot{x}, \dot{y}) ; lower panel (g,h,i): distribution of the return times T . The parameters are $\omega = 0.98$ and $a = 0.16$ (a,d,g), $a = 0.22$ (b,e,h) and $a = 0.28$ (c,f,i).

Mutually coupled Rössler oscillators

$$\dot{x}_{1,2} = -\omega_{1,2}y_{1,2} - z_{1,2},$$

$$\dot{y}_{1,2} = \omega_{1,2}x_{1,2} + ay_{1,2} + d(y_{2,1} - y_{1,2}),$$

$$\dot{z}_{1,2} = 0.1 + z_{1,2}(x_{1,2} - 8.5),$$

d – coupling strength

a – system parameter

Synchronization Transitions

- Types of synchronization:
 - phase synchronization
 - phase difference bounded
 - generalized synchronization
 - a positive Lyapunov exponent becomes negative
 - complete synchronization

Different types of synchronization transitions

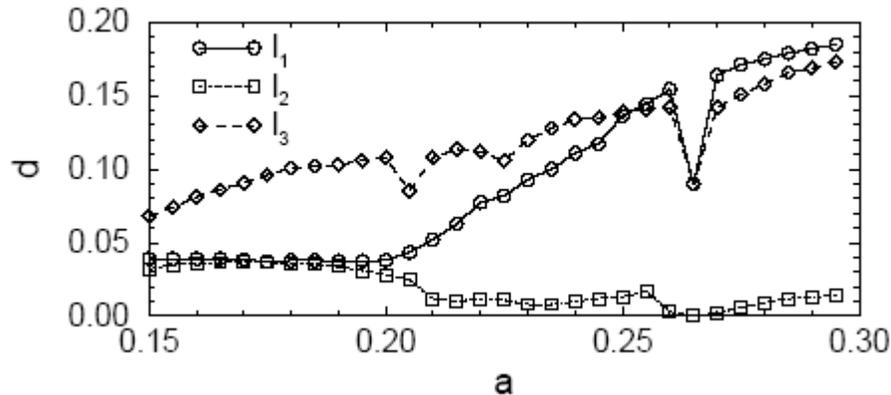


FIG. 3: Critical coupling curves. l_1 corresponds to the onset of CPS, i.e., below this line the oscillations are not synchronized, and above the phase and frequency locking conditions are fulfilled; l_2 to the transition of one of zero LEs to negative value and l_3 to zero-crossing of one of the positive LEs. Note: In this figure we do not separate the cases, where the synchronization occurs between regular and chaotic oscillations.

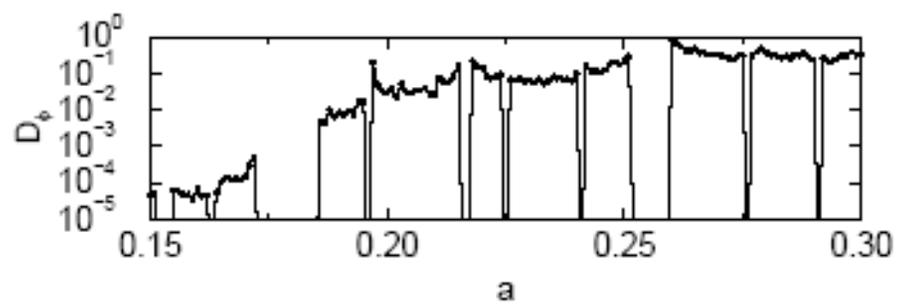


FIG. 2: Phase diffusion coefficient D_ϕ (3) vs a . $\omega = 0.98$.

Weakly Non-Phase-Coherent

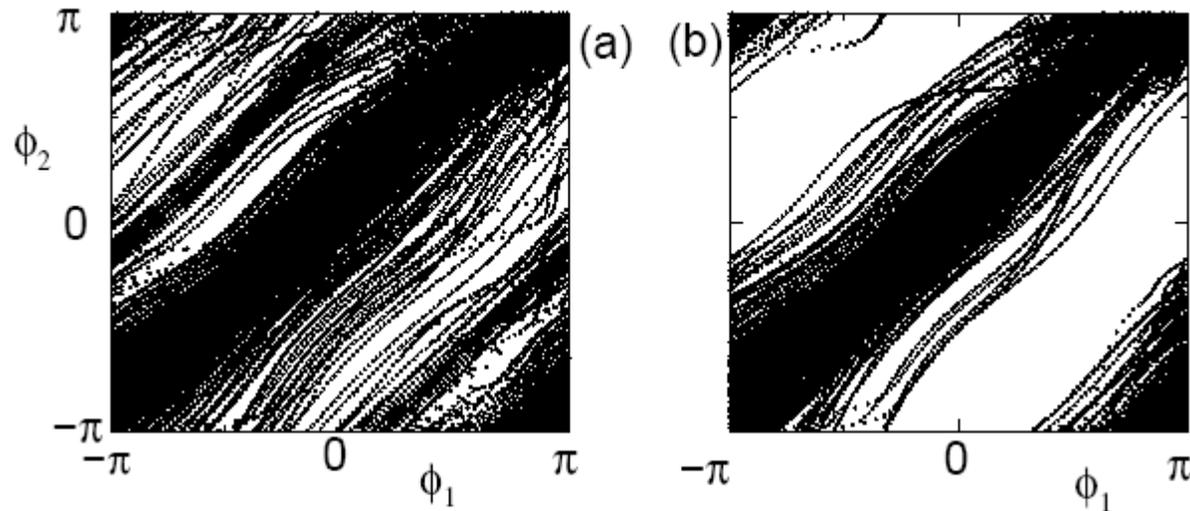


FIG. 4: Projections of trajectories of the Rössler systems for $a = 0.22$ on the plane (ϕ_1, ϕ_2) for the coupling strength d outside (a) ($d = 0.055$) and within (b) ($d = 0.075$) the synchronization region.

Three types of transition to phase synchronization

- **Phase-coherent:** one zero Lyapunov exponent becomes negative (small phase diffusion); phase synchronization to get for rather weak coupling, whereas generalized synchronization needs stronger one
- **Weakly non-phase-coherent:** inverse interior crises-like
- **Strongly non-phase-coherent:** one positive Lyapunov exponent becomes negative (strong phase diffusion) – also amplitudes are interrelated

The Earth as a Complex System

- **Components**

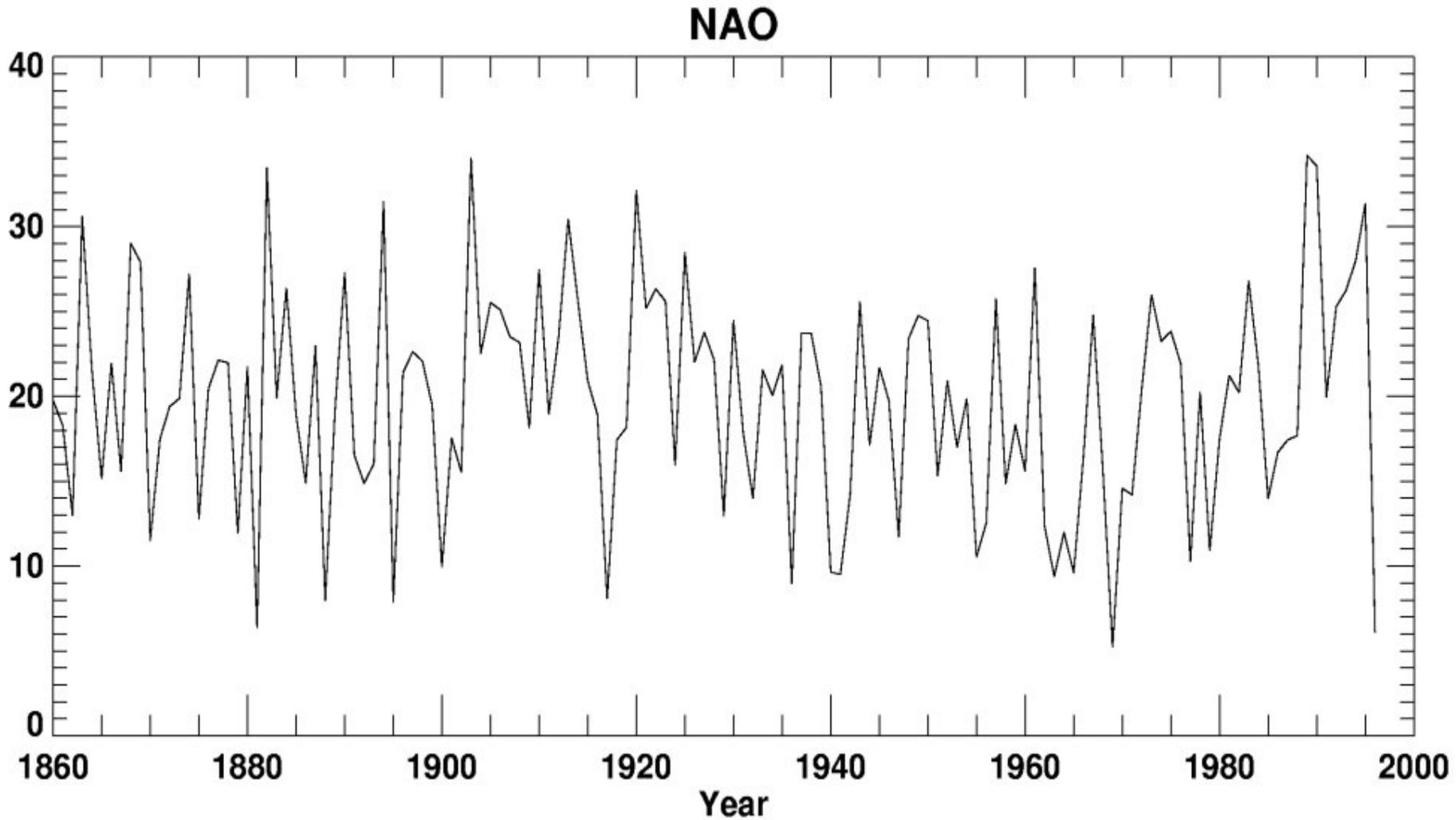
- Solid Earth (crust, mantle)
- Fluid Envelopes (atmosphere, ocean, snow, ice)
- Living Parts (fauna, flora, people)

- **Complex Feedbacks**

- Positive and Negative
- Nonlinearities

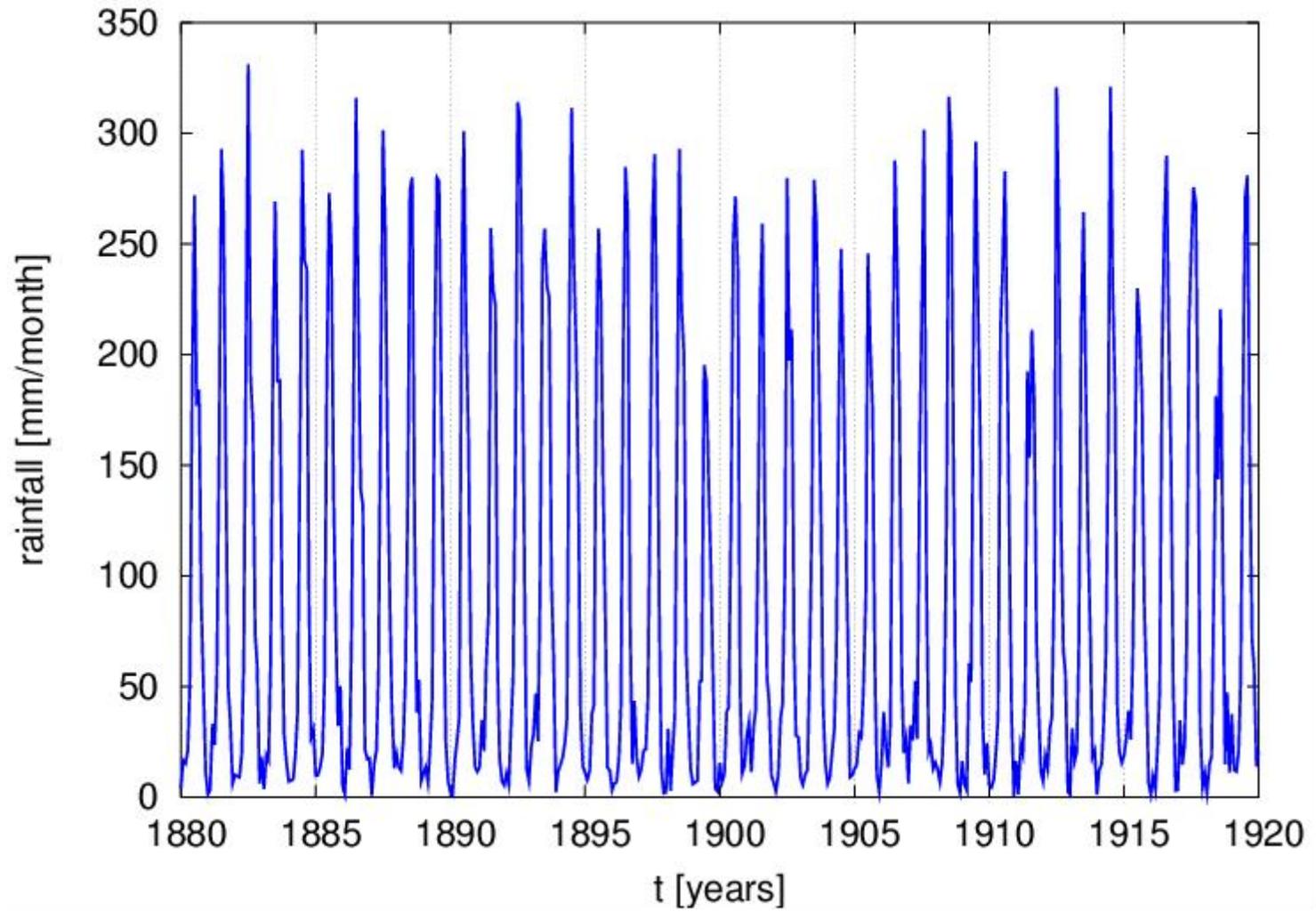
- **How to approach?**

North Atlantic Oscillations



Which type of oscillations (periodic, noisy, chaotic)?

Monsoon-Data



Teleconnections

- (Weak) Connections of meteorological conditions/ regimes between largely distant regions
- Examples: NAO – El Nino
El Nino – Indian Monsoon

How to study such interactions?

Concept of Synchronization

Application: El Niño vs. Indian monsoon

- El Niño/Southern Oscillation (ENSO) – self-sustained oscillations of the tropical Pacific coupled ocean-atmosphere system
- Monsoon - oscillations driven by the annual cycle of the land vs. Sea surface temperature gradient
- ENSO could influence the amplitude of Monsoon – Is there phase coherence?
- Monsoon failure coincides with El Niño
- (Maraun, Kurths, Geophys Res Lett (2005))

El Niño vs. Indian Monsoon

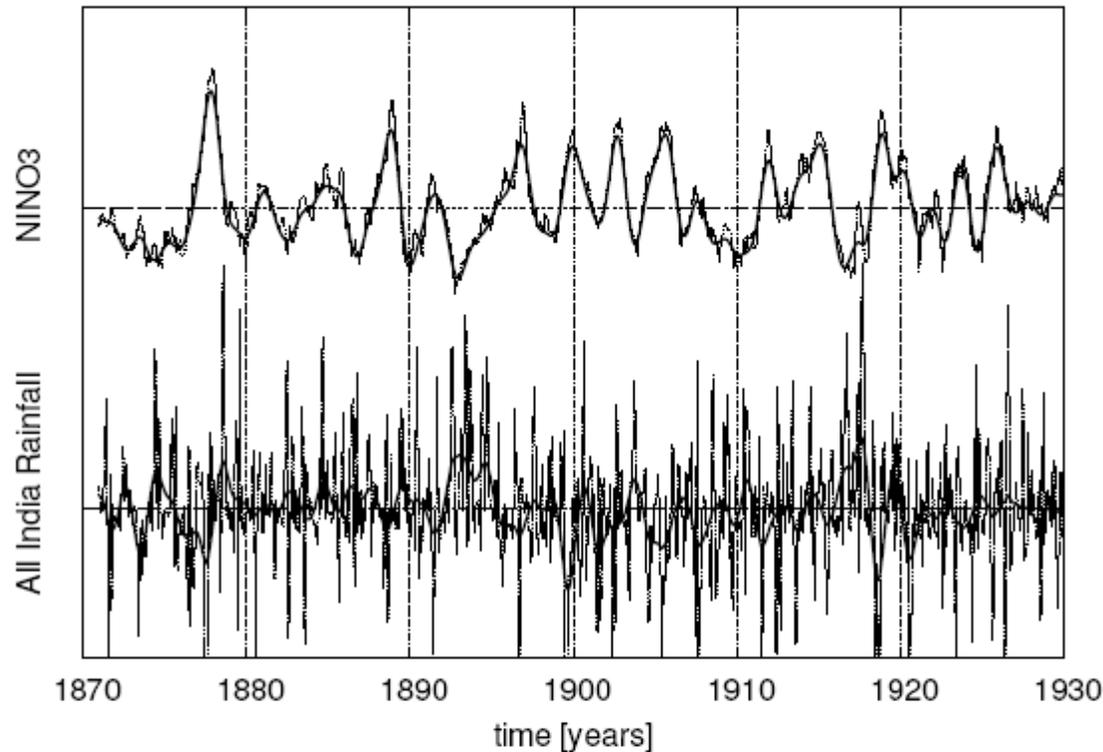
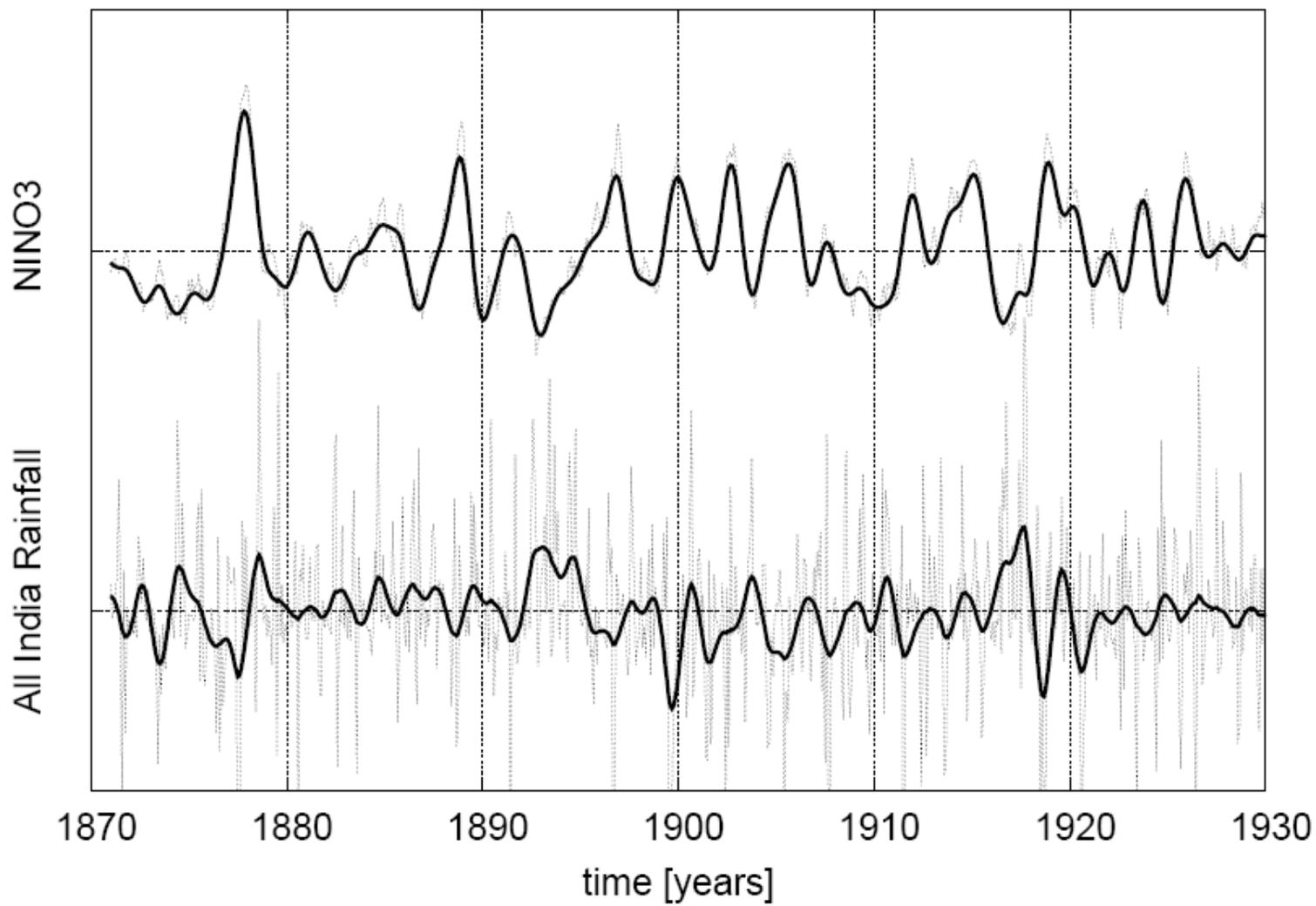


Figure 1. Section of the NINO3 (upper graph) and AIR anomalies (lower graph) time series. The dotted lines depict the raw data, the **solid lines show** the low-pass filtered data used for the further analysis.

Filtered Data



El Niño – non phase-coherent

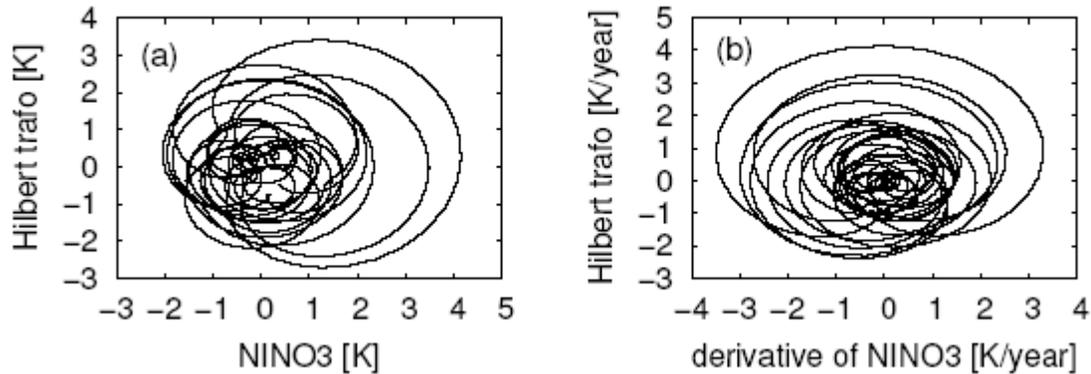


Figure 2. (a) Embedding of low-pass filtered NINO3 time series by Hilbert transformation. Many oscillations are not centered around a common center. (b) The same, but for the time derivative of the NINO3 time series. All pronounced oscillations circle around the origin.

Phase coherence between El Niño and Indian monsoon

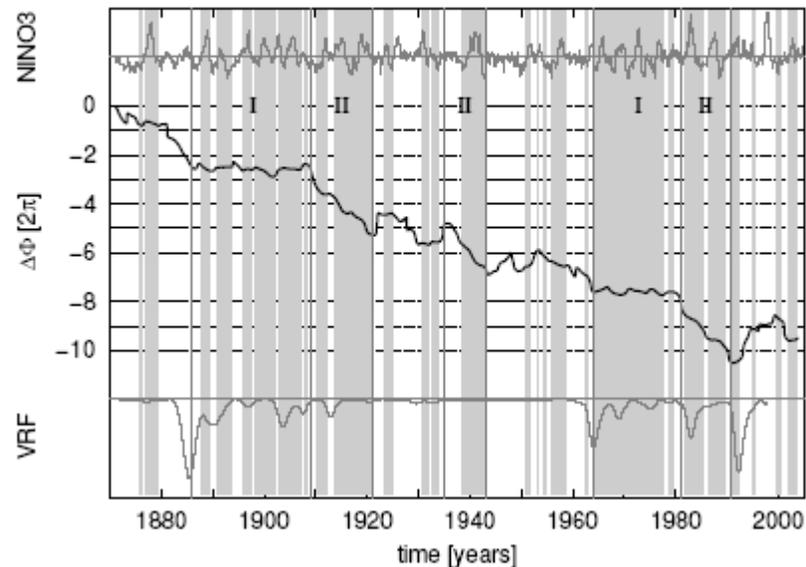


Figure 5. Phase difference of ENSO and Monsoon (black). Grey shading marks intervals of jointly well defined phases. 1886-1908 and 1964-1980 (I): plateaus indicate phase coherence. 1908-1921, 1935-1943 and 1981-1991 (II): Monsoon oscillates with twice the phase velocity of ENSO. During these intervals, both systems exhibit distinct oscillations (NINO3 time series, upper graph). 1921-1935 and 1943-1963: phases are badly defined, both processes exhibit irregular oscillations of low variance (upper graph). Lower graph shows volcanic radiative forcing index (VRF).

Distribution of phase differences

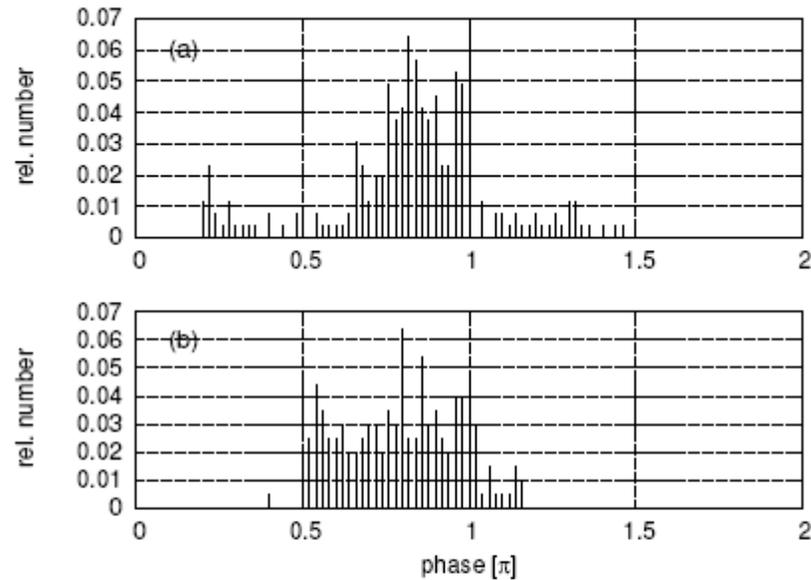
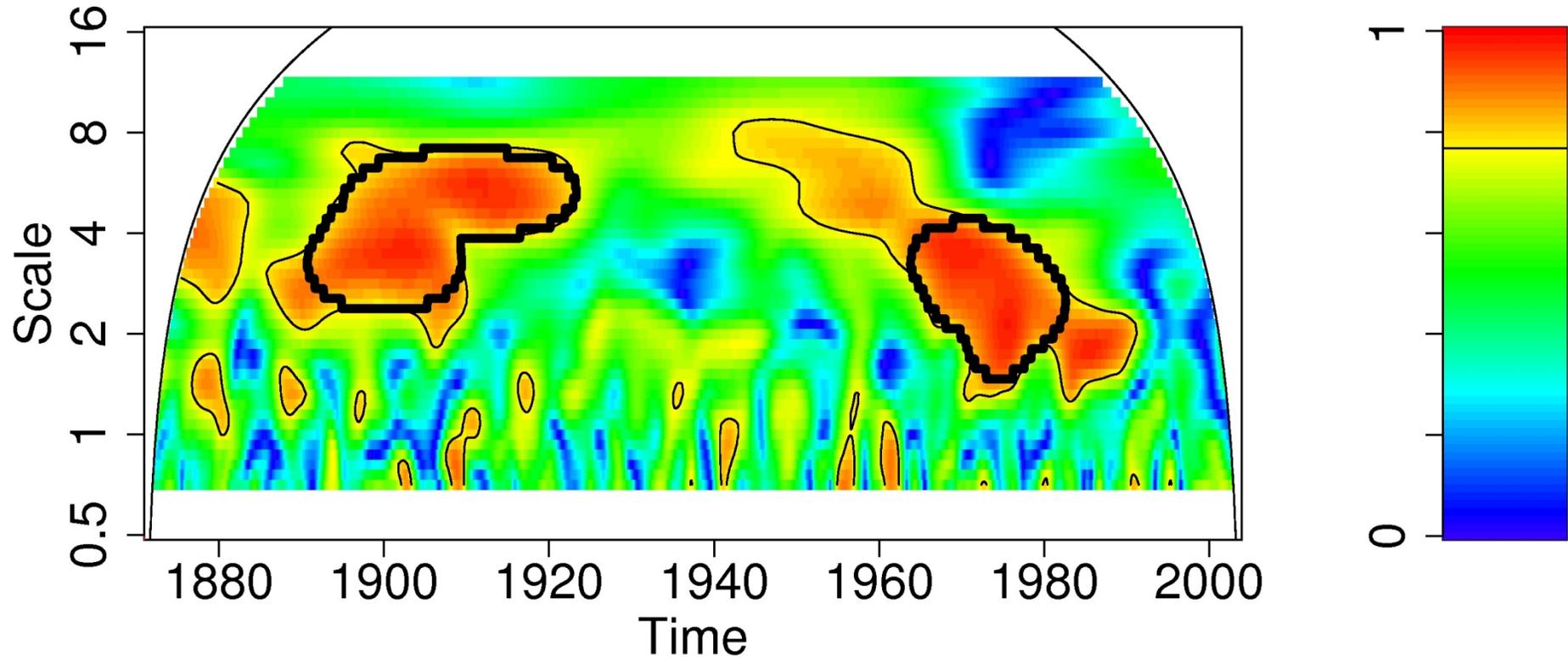


Figure 6. Histogram of phase differences for the two phase coherent intervals (a) 1886-1908, (b) 1964-1980. Both diagrams show peaks between $\pi/2$ and π , reflecting that ENSO and Monsoon are anti-correlated.

Wavelet Analysis



Thick black curve – statistically significant

Thin black curve – not-correct statistical significance region

Directionality Analysis based on Granger Causality

- Linear (AR) and nonlinear (polynomials) dependences
- Result: **bidirectional alternating dependence**
(Geoph. Res. Lett. 2011, 38, L00F04)

ENSO → Monsoon 1890-1920 and 1950-1980

Monsoon → ENSO 1917-1927 and 1980-1990

Granger causality

- ENSO → Monsoon

$$x_1(t) = a_{1,1}x_1(t-1) + b_{1,1}x_2(t-1) \\ + c_{1,1}x_1^2(t-1)x_2(t-1) + c_{1,2}x_2^3(t-1) + \eta_1(t),$$

- Monsoon → ENSO

$$x_2(t) = a_{2,1}x_2(t-1) + a_{2,5}x_2(t-5) + b_{2,1}x_1(t-1) \\ + b_{2,2}x_1(t-2) + b_{2,3}x_1(t-3) + \eta_2(t),$$



ELSEVIER

Physics Reports 366 (2002) 1–101

PHYSICS REPORTS

www.elsevier.com/locate/physrep

The synchronization of chaotic systems

S. Boccaletti^{a,b,*}, J. Kurths^c, G. Osipov^d, D.L. Valladares^{b,e}, C.S. Zhou^c

^a*Istituto Nazionale di Ottica Applicata, Largo E. Fermi, 6, I50135 Florence, Italy*

^b*Department of Physics and Applied Mathematics, Institute of Physics, Universidad de Navarra, Irunlarrea s/n,
31080 Pamplona, Spain*

^c*Institut für Physik, Universität Potsdam, 14415 Potsdam, Germany*

^d*Department of Radiophysics, Nizhny Novgorod University, Nizhny Novgorod 603600, Russia*

^e*Department of Physics, Univ. Nac. de San Luis, Argentina*

Received 2 January 2002

editor: I. Procaccia

Synchronization

A Universal Concept in Nonlinear Sciences

Arkady Pikovsky, Michael Rosenblum and
Jürgen Kurths

Synchronization
A Universal Concept in Nonlinear Sciences

Cambridge Nonlinear Science Series 12

Pikovsky, Rosenblum and Kurths

Cambridge

