Models in the Physics of Complexity

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1 Introduction

In this class we will discuss and partially analyze three models that arise from physical problems. The treatment will be a simplified version of the one given in the published versions. When the analysis becomes too complicated, we indicate the results in the papers cited. The three parts can be independently read or studied. The models are given in the order of the increasingly complex behavior that we focus in the treatment. We start with very simple coherent behavior used in the theory of flocking, continue with a very simple model of granular flow, and end with a model sometimes used to illustrate the notion of punctuated equilibrium. In each of these case we provide program programs that the students can use to gain insight into these problems.

2 Synchronization of Large Linear Oscillator Arrays

(See [1] and [2].) Synchronization of a large collection of coupled, simple dynamical systems is a problem that has applications from neuroscience to traffic modeling to modeling of consensus formation.

Consider an array of identical linear oscillators, each coupled to its front and rear neighbor. The coupling may be asymmetric. If we kick the front oscillator (the leader), how does this signal propagate through the system? In some isolated cases, for certain values of the parameters, the answer is well-known, but until recently the only general results applicable to large systems were very qualitative. We develop the theory that predicts when the system is well-behaved. In these cases we give the quantitative description of the response.

3 Sand Piles and Staircases

(See [3].) In a granular flows, two angles are particularly important. Rotate half cylinder partially filled sand or rice very slowly. The angle of the slope increases until at some point an avalanche occurs. This avalanche resets the angle of the slope of the surface to a minimum. This minimum angle is the *angle of repose*. If we keep turning the cylinder slowly, just prior to the avalanche, the angle of the surface is maximal. This maximal angle is called the *angle of maximum stability*. These angles (especially the angle of repose) tend to be constant.

Traditional wisdom (physics) says that the existence of these two (constant, but material dependent) angles is a collective phenomenon. We study a *single particle model* that exhibits similar angles. This challenges the above conventional wisdom. We also show that these phenomena are in some sense robust (ie independent of certain parameters), and that, for slopes between the two angles, there is a tendency for the velocity to be close to constant.

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4 Rank Driven Dynamics

(See [4].) We investigate a class of models related to the Bak-Sneppen (BS) model, initially proposed to study evolution. The BS model is extremely simple and yet captures some forms of "complex behavior" such as *punctuated equilibrium* that is often observed in physical and biological systems.

In the BS model, random numbers in $[0, 1]$ (interpreted as fitnesses of agents) distributed according to some cumulative distribution function $R : [0, 1] \to [0, 1]$ are placed at the vertices of a graph G. At every time-step the lowest number and its immediate neighbors are replaced by new random numbers. We approximate this dynamics by making the assumption that the numbers to be replaced are independently distributed. We present the analysis of two main cases: The *exogenous case* where the new fitnesses are taken from an a priori fixed distribution, and the *endogenous case* where the new fitnesses are taken from the current distribution as it evolves.

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References

[1] C. E. Cantos, J. J. P. Veerman, D. K. Hammond, *Signal Velocities in Oscillator Arrays*, Submitted.

[2] C. E. Cantos, J. J. P. Veerman, D. K. Hammond, *Transients in the Synchronization of Oscillator Arrays*, Submitted.

[3] J. J. P. Veerman, D. Daescu, M.-J. Romero-Vallés, P. J. Torres, *A Single Particle Impact Model for Motion in Avalanches*, **Physica D** 238, No 18, 1897-1908, 2009.

[4] J. J. P. Veerman, F. J. Prieto, *On Rank Driven Dynamical Systems*, **Journal of Statistical Physics**, DOI 10.1007/s10955-014-1012-0, 2014.