Applications of Linear and Integer Programming Models

Chapter 3

WITH APPROXIMATELY $20 BILLION in revenues, FedEx Corporation (http://www.fedex.com) has become a world leader in providing integrated transportation, information, and logistics solutions. As the company has expanded, it has created a high-level management science group to provide senior management with recommendations on a wide variety of issues. This group uses state-of-the-art computer-based mathematical models to analyze a broad range of complex corporate problems with an overall goal of maintaining and increasing company profits while continuing to provide a consistently high level of service to its customers.

One of the models developed by this group is its Global Supply Chain Model, built to redesign its supply chain for revenue packaging. The model has helped management answer the following questions:

- Should FedEx pursue offshore production of packaging? If so, which items and where?
- Should FedEx consolidate nearby warehouses and pick centers into a new form of distribution center?
- Should FedEx pursue expansion of distribution center locations?
- What transportation modes on each link would most reliably get packaging from suppliers to stations while reducing costs?
- What should the service area boundaries be for each distribution center?

The Global Supply Chain Model uses, among other techniques, a large-scale **mixed integer programming** approach. This model has already resulted in a cost savings to FedEx of over $10 million.

3.1 The Evolution of Linear Programming Models in Business and Government

Following World War II, the U.S. Air Force sponsored research for solving military planning and distribution models. In 1947, the simplex algorithm was developed for solving these types of linear models. Not long after, the first commercial uses of linear programming were reported in "large" businesses that had access to digital computers. Seemingly unrelated industries, such as agriculture, petroleum, steel, transportation, and communications, saved millions of dollars by successfully developing and solving linear models for complex problems.

As computing power has become more accessible, the realm of businesses and government entities using linear models has expanded exponentially. In this chapter we present numerous "small" examples selected from a wide variety of applications areas, designed to accomplish four goals:

1. To examine potential applications areas where linear models may be useful
2. To develop good modeling skills
3. To demonstrate how to develop use of the power of spreadsheets to effectively represent the model in unambiguous terms and generate results
4. To gain confidence in interpreting and analyzing results from spreadsheet reports

Although the examples illustrated in this chapter represent "scaled-down" versions of potential real-life situations, today linear and integer programming models proliferate in a wide variety of actual business and government applications. Banking models, large economic/financial models, marketing strategy models, production scheduling and labor force planning models, computer design and networking models, and health care and medical models are but a few notable examples of successful linear programming applications. Below is just a sampling of the thousands of actual documented uses of linear programming models.

- Aircraft fleet assignments
- Telecommunications network expansion
- Air pollution control
- Health care
- Bank portfolio selection
- Agriculture
- Fire protection
- Defense/aerospace contracting
- Land use planning
- Dairy production
- Military deployment

An example of each of the above is detailed in Appendix 3.1 on the accompanying CD-ROM. The reader is encouraged to reference this subfolder for details.

These are but a few of the numerous applications areas of linear optimization models. Additional applications include traffic analysis, fast-food operations, transportation, assignment of medical personnel, coal, steel, gas, chemical, and paper production, recycling, educational assignments, worker evaluations, awarding of contracts, manufacturing, railroads, forestry, school desegregation, government planning, tourism, and sports scheduling. Each year hundreds of new applications appear in the professional literature. Add to that the numerous unreported models that are regularly utilized in business and government, and you can see that linear programming continues to play a significant role in today's world.

3.2 Building Good Linear and Integer Programming Models

Given the widespread use of linear models today, it has become increasingly important for practitioners to be able to develop good, efficient models to aid the manager in the decision-making process. Three factors—familiarity, simplification, and clarity—are important considerations when developing such models.

The greater the modeler's *familiarity* with the relationships between competing activities, the limitations of the resources, and the overall objective, the greater the likelihood of generating a usable model. Viewing the problem from as many perspectives as possible (e.g., those of various management levels, front-line workers, and accounting) helps in this regard.

Linear models are always *simplifications* of real-life situations. Usually, some or all of the required linear programming assumptions discussed in Chapter 2 are violated by an actual situation. Because of the efficiency with which they are solved and the associated sensitivity analysis reports generated, however, linear models are generally preferable to more complicated forms of mathematical models.

When developing a model, it is important to address the following question: "Is a very sophisticated model needed, or will a less sophisticated model that gives fairly good results suffice?" The answer, of course, will guide the level of detail required in the model.

Although a model should reflect the real-life situation, one should not try to model every aspect or contingency of the situation. This could get us bogged down in minutiae, adding little, if any, real value to the model while unnecessarily complicating the solution procedure, delaying solution time, and compromising the usefulness of the model. As George Dantzig, the developer of the simplex algorithm for solving linear programming models, points out, however, "What constitutes the proper simplification, is subject to individual judgment and experience. People often disagree on the adequacy of a certain model to describe the situation."[1] In other words, although experience is the best teacher, you should be aware that even experienced management scientists may disagree as to what level of simplification is realistic or warranted in a model.

Finally, a linear programming model should be *clear*; that is, it should be easy to follow and as transparent as possible to the layperson. From a practitioner's point of view, the model should also be easy to input and yield accurate results in a timely manner.

SUMMATION VARIABLES AND CONSTRAINTS

In an effort to make the model easier to understand and debug, we can introduce **summation variables** and corresponding **summation constraints** into the formulation. A summation variable is the sum of two or more of the decision variables. It is particularly useful when there are constraints involving maximum or minimum percentages for the value of one or more of the decision variables.

To illustrate the use of a summation variable, consider the situation in which X_1, X_2, and X_3 represent the production quantities of three television models to be produced during a production run in which 7000 pounds of plastic are available.

[1] George B. Dantzig, *Linear Programming and Extensions* (Princeton, NJ: Princeton University Press, 1963).

The unit profits are \$23, \$34, and \$45, and the amount of plastic required to produce each is 2 pounds, 3 pounds, and 4 pounds, respectively. In addition, management does not want any model to exceed 40% of total production ($X_1 + X_2 + X_3$).

First, we note that although the proportion of model 1 televisions produced during the production run is $X_1/(X_1 + X_2 + X_3)$. However a constraint of the form: $X_1/(X_1 + X_2 + X_3) \leq .4$ is not a linear constraint. But, because we know that the total production, $X_1 + X_2 + X_3$ is positive, we could multiply both sides of this constraint by the denominator to obtain the equivalent linear constraint: $X_1 \leq .4(X_1 + X_2 + X_3)$. The constraints $X_2 \leq .4(X_1 + X_2 + X_3)$ and $X_3 \leq .4(X_1 + X_2 + X_3)$ require that models 2 and 3 represent no more than 40% of the total production. Thus, the model could be written as:

$$
\begin{array}{ll}
\text{MAX} & 23X_1 + 34X_2 + 45X_3 \\
\text{ST} & 2X_1 + 3X_2 + 4X_3 \leq 7000 \\
& X_1 \leq .4(X_1 + X_2 + X_3) \\
& X_2 \leq .4(X_1 + X_2 + X_3) \\
& X_3 \leq .4(X_1 + X_2 + X_3) \\
& X_1, X_2, X_3 \geq 0
\end{array}
$$

or,

$$
\begin{array}{ll}
\text{MAX} & 23X_1 + 34X_2 + 45X_3 \\
\text{ST} & 2X_1 + 3X_2 + 4X_3 \leq 7000 \\
& .6X_1 - .4X_2 - .4X_3 \leq 0 \\
& -.4X_1 + .6X_2 - .4X_3 \leq 0 \\
& -.4X_1 - .4X_2 + .6X_3 \leq 0 \\
& X_1, X_2, X_3 \geq 0
\end{array}
$$

Written in this form, not only are the coefficients cumbersome to input into a spreadsheet, but the last three constraints do not immediately convey the fact that each television model is not to exceed 40% of the total production.

To clarify the above formulation, a *summation variable* representing the total production and a *summation constraint* expressing this relationship may be introduced into the model as follows:

- Define the summation variable:
  X_4 = the total production of televisions during a production run.
- Add the following summation constraint:
  $X_1 + X_2 + X_3 = X_4$ or equivalently $X_1 + X_2 + X_3 - X_4 = 0$ to the model formulation.
- The 40% production limit constraints can now be written as:
  $X_1 \leq .4X_4$, $X_2 \leq .4X_4$, and $X_3 \leq .4X_4$ respectively.

By subtracting $.4X_4$ from both sides of each of the above production limit constraints, the complete set of constraints can now be written as:

$$
\begin{array}{ll}
\text{MAX} & 23X_1 + 34X_2 + 45X_3 \\
\text{ST} & 2X_1 + 3X_2 + 4X_3 \leq 7000 \\
& X_1 + X_2 + X_3 - X_4 = 0 \text{ (Summation Constraint)} \\
& X_1 - .4X_4 \leq 0 \\
& X_2 - .4X_4 \leq 0 \\
& X_3 - .4X_4 \leq 0 \\
& \text{All X's} \geq 0
\end{array}
$$

Although by adding the summation variable and the summation constraint we have increased the number of constraints and number of variables each by one, the new set of constraints is easier to input and easier to read and interpret when checking the model.[2]

When using spreadsheets, a convenient way of modeling this situation without listing all the percentage constraints explicitly is shown in Figure 3.1. In this figure, cells B2, C2, and D2 are used for the decision variables, while another cell (H2) is set aside to represent total production. The formula in cell H2 is =SUM(B2:D2). Note that summation variable cell H2 is not considered a "Changing Cell." As shown in the accompanying Solver dialogue box, the percentage constraints can be included by $B$2:$D$2 <= .4*$H$2.

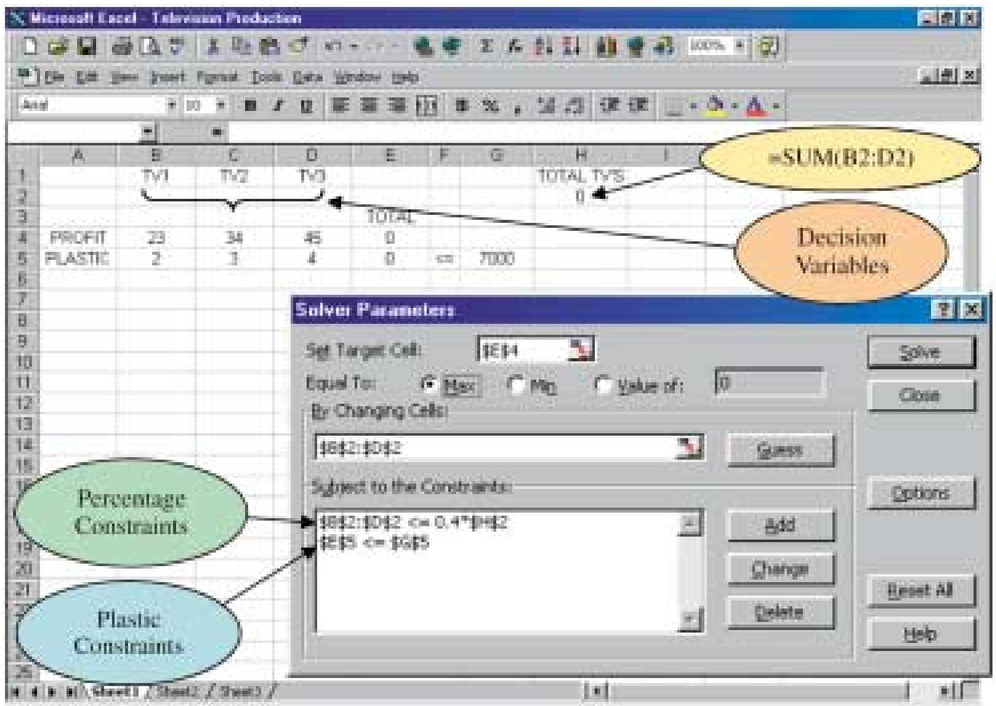


FIGURE 3.1 Solver Spreadsheet for Television Production Model

BINARY VARIABLES

In Chapter 2 we saw that sometimes one or more of the decision variables in a linear model are required to be integer-valued. Some models may also contain **binary variables**, variables that can only assume values of 0 or 1. Any situation that can be modeled by "yes/no," "good/bad," "right/wrong," and so on, can be considered a binary variable. Such situations include whether or not a plant is built, whether or not a particular highway is used when traveling between two cities, and whether or not a worker is assigned to perform a job. (The latter two examples are discussed in Chapter 4.)

A MODELING CHECKLIST

Mathematical modeling is an art that improves with experience. To this point we have suggested several modeling tips that can aid your development of mathematical models. For your convenience we offer a summary of many of these tips in the form of a checklist.

[2] The time needed to solve a linear programming problem typically depends on: (1) the number of constraints; and (2) the percentage of nonzero coefficients in the constraints. Although adding a definitional variable adds one more constraint to the formulation, the efficiency gained in this case by having fewer nonzero coefficients could offset this factor.

A Checklist for Building Linear Models

1. Begin by listing the details of the problem in short expressions. (We have done this in Chapter 2 using "bullets.")
2. Determine the objective in general terms and then determine what is within the control of the decision maker to accomplish this goal. These controllable inputs are *decision variables*.
3. If, during the course of the formulation, you find that another decision variable is needed, add it to the list at that time and include it in the formulation.
4. Define the decision variables precisely using an appropriate time frame (i.e., cars per month, tons of steel per production run, etc.).
5. When writing a constraint or a function, first formulate it in words in the form: (some expression) ⟨has some relation to⟩ (another expression or constant); then convert the words to the appropriate mathematical symbols.
6. Keep the units in the expressions on both sides of the relation consistent (e.g., one side should not be in hours, the other in minutes).
7. If the right-hand side is an expression rather than a constant, do the appropriate algebra so that the end result is of the form:
   (mathematical function) ⟨has a relation to⟩ (a constant)
8. Use *summation variables* when appropriate, particularly when many constraints involve percentages.
9. Indicate which variables are restricted to be nonnegative, which are restricted to be integer valued, and which are binary.

3.3 Building Good Spreadsheet Models

When we employed spreadsheets to solve linear programming models in Chapter 2, the models were written in a rectangular fashion with columns used for the variables and rows used for constraints. This approach, which is used in many non-spreadsheet software packages, presents a structured approach for inputting the coefficients of the model. Any linear or integer programming model can be represented and solved in this manner.

But with spreadsheets, you are not burdened with such a restrictive format. You can present the data and results in such a way that they do not even look like a linear programming model. This can be very valuable, particularly when information is shared with nonquantitative end-users. With spreadsheets you can:

- Embed formulas that represent required values or subsets of values into individual cells
- Express coefficients as mathematical expressions rather than specific numbers
- Designate various cells scattered throughout the spreadsheet to contain left-hand side and right-hand side values of the constraints without confining them to be in a single column or row

Solver even allows the right-hand side of constraints to be mathematical expressions instead of constants or cell references. Add to this the use of color, various border designs, and other formatting techniques, and this flexibility allows the user to present a spreadsheet in a way that conveys the requisite information in a visually pleasing manner. For instance, the modeler can:

- Group certain types of constraints together
- Designate certain cells for the left sides and right sides of these constraints at positions on the spreadsheet disjoint from the left-hand coefficients

- Highlight the information by using a thick or colored border
- Use a different color background for cells containing the left-hand side values as opposed to the cells that contain the right-hand side restrictions
- Create entries on the spreadsheet that give other relevant information such as subtotals or proportions
- Use various formulas in cells representing the total left-hand side values. This can simply be a cell representing the value of a decision variable, a formula, or the result of a function such as SUMPRODUCT, SUM, and SUMIF

Another option that one might take advantage of is Excel's ability to name cells or sets of cells. Cells can be named by following these steps:

- Highlight the cell(s) to be named.
- Click on the Name box (the far left box immediately above column A) and type in what you wish to call the cells. (The name must start with a letter, and no spaces are allowed.)
- Press Enter.

Then when these cells are referenced in a Solver dialogue box, for instance, the name appears rather than the cell references. This can make it easier to follow the logic of the model. For instance, in the television production model in Section 3.2 which was illustrated in Figure 3.1, we could have assigned the following names:

Cells	***NAME***
B2:D2	SetsProduced
H2	TotalSets
E4	TotalProfit
E5	PlasticUsed
G5	AvailablePlastic

Then by highlighting the same cells as before, the dialogue box would appear as in Figure 3.2. Note that when we input a formula (such as .4*$H$2) for the right side of a constraint, the name does not appear.

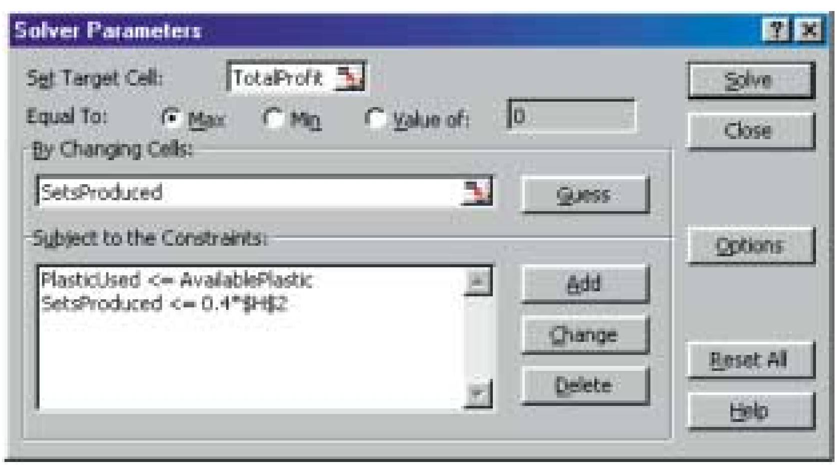


FIGURE 3.2
Dialogue Box Using Named Cells

Although we could do this throughout our illustrations of linear models, we leave cell references as they are so that they can be easily referenced on the spreadsheet itself. But for large models, naming cells certainly has its advantages. In short, a spreadsheet offers a variety of ways other than a matrix format to convey the input coefficients. Solver is still used, but the results will be placed on spreadsheets that make them ripe for discussion or inclusion into business reports or PowerPoint presentations.

In Sections 3.4 and 3.5, the mathematical and spreadsheet modeling tips discussed in the last two sections are illustrated in various formulations of linear and integer models from the private and public sectors. In these models we shall:

- Show how linear models can be applied to different situations arising from the functional areas of business and government
- Illustrate the modeling approach, including some of the problems that might arise in the modeling process
- Employ effective spreadsheet modeling techniques
- Interpret, analyze, and extend the output generated from Excel Solver

In the process we shall illustrate a number of concepts, including how to:

- Choose an appropriate objective function
- Define an inclusive set of decision variables
- Write accurate expressions to model the constraints
- Interpret sensitivity outputs for both maximization and minimization models
- Recognize and address unboundedness
- Recognize and address infeasibility
- Recognize and address alternate optimal solutions

To simplify matters, we identify the applications area and the concepts illustrated for each model introduced in this chapter.

3.4 Applications of Linear Programming Models

In Chapter 2, we introduced the basic concepts of linear programming through the use of two-variable models. These concepts included modeling, using Excel's Solver to generate an optimal solution (or determine that the model is unbounded or infeasible), and interpretation of the output on the Answer and Sensitivity Reports. In this section we illustrate how to model more realistic problems requiring more than two decision variables. However, the concepts developed for two-variable models apply equally as well to these more complex ones.

The models in this section represent small versions of problems one might find in such diverse areas as production, purchasing, finance, and cash flow accounting. Besides spanning a range of applications areas, each model was constructed to illustrate at least one new linear programming concept. Thus, for each model take note of its application area, the model development, the spreadsheet design, and the analysis and interpretation of the output.

3.4.1 PRODUCTION SCHEDULING MODELS

Assisting manufacturing managers in making production decisions that efficiently utilize scarce resources is an area in which a variety of linear programming models have been applied. Determining production levels, scheduling shift workers and overtime, and determining the cost effectiveness of purchasing additional resources for the manufacturing process are just some of the key decisions that these managers must make. The Galaxy Industries model introduced in Chapter 2 is a simplified version of a production scheduling situation. Here, in a slightly larger version of that model, we illustrate how linear programming can help make some of these management decisions.

Galaxy Expansion.xls

GALAXY INDUSTRIES—AN EXPANSION PLAN

Concepts: Maximization
Sensitivity Analysis (All constraint types)
Both Signs in Objective Function
Unit Conversion
Summation Variables, Percentage Constraints

Galaxy Industries has been very successful during its first six months of operation and is already looking toward product expansion and possible relocation within the year to a facility in Juarez, Mexico, where both labor and material costs are considerably lower. The availability of the cheaper labor and a contract with a local distributor to supply up to 3000 pounds of plastic at a substantially reduced cost will effectively double the profit for Space Rays to $16 per dozen and triple the profit for Zappers to $15 per dozen.

The new facility will be equipped with machinery and staffed with workers to facilitate a 40-hour regular time work schedule. In addition, up to 32 hours of overtime can be scheduled. Accounting for wages, benefits, and additional plant operating expenses, each scheduled overtime hour will cost the company $180 more than regular time hours.

Galaxy has been test marketing two additional products, tentatively named the Big Squirt and the Soaker, which appear to be as popular as the Space Ray and Zapper. Table 3.1 summarizes the profit and requirements for each product line.

Galaxy has a signed contract with Jaycee Toys, Inc. to supply it with 200 dozen Zappers weekly once the relocation has taken place. The marketing department has revised its strategy for the post-relocation period. It has concluded that, to keep total demand at its peak, Galaxy's most popular model, the Space Ray, should account for exactly 50% of total production, while no other product line should account for more than 40%. But now, instead of limiting production to at most 700 dozen weekly, the department wishes to ensure that production will total at least 1000 dozen units weekly.

Management would like to determine the weekly production schedule (including any overtime hours, if necessary) that will maximize its net weekly profit.

TABLE 3.1 Profit and Requirements Per Dozen

Product	Profit	Plastic (lb.)	Production Time (min.)
Space Rays	$16	2	3
Zappers	$15	1	4
Big Squirts	$20	3	5
Soakers	$22	4	6
	Available	3000	40 hrs. (Reg.) 32 hrs. (O/T)

SOLUTION

The following is a brief synopsis of the problem.

- Galaxy wants to maximize its Net Weekly Profit = (Weekly Profit from Sales) − (Extra Cost of Overtime).
- A weekly production schedule, including the amount of overtime to schedule, must be determined.
- The following restrictions exist:
  1. Plastic availability (3000 pounds)
  2. Regular time labor (2400 minutes)
  3. Overtime availability (32 hours)
  4. Minimum production of Zappers (200 dozen)
  5. Appropriate product mix

     (Space Rays = 50% of total production)

     (Zappers, Big Squirts, Soakers ≤ 40% of total production)
  6. Minimum total production (1000 dozen)

DECISION VARIABLES

Galaxy must not only decide on the weekly production rates but also determine the number of overtime hours to utilize each week. Thus, we define five decision variables:

X_1 = number of *dozen* Space Rays to be produced each week
X_2 = number of *dozen* Zappers to be produced each week
X_3 = number of *dozen* Big Squirts to be produced each week
X_4 = number of *dozen* Soakers to be produced each week
X_5 = number of *hours* of overtime to be scheduled each week

OBJECTIVE FUNCTION

The total net weekly profit will be the profit from the sale of each product less the cost of overtime. Since each overtime hour costs the company an extra $180, the objective function is:

$$\text{MAXIMIZE } 16X_1 + 15X_2 + 20X_3 + 22X_4 - 180X_5$$

CONSTRAINTS

The following constraints exist in the Galaxy problem:

- *Plastic*: (Amount of plastic used weekly) ≤ 3000 lbs.

$$2X_1 + X_2 + 3X_3 + 4X_4 \qquad \le 3000$$

- *Production Time*: (Number of production *minutes* used weekly) ≤ (Number of regular *minutes* available) + (Overtime *minutes* used)

  Here,

  Number of regular time *minutes* available = 60(40) = 2400
  Number of overtime *minutes* used is 60(O/T hours used) = $60X_5$

  Thus, the production time constraint is:

$$3X_1 + 4X_2 + 5X_3 + 6X_4 \le 2400 + 60X_5, \text{ or}$$
$$3X_1 + 4X_2 + 5X_3 + 6X_4 - 60X_5 \le 2400$$

- *Overtime Hours*: (Number of overtime *hours* used) ≤ 32

$$X_5 \le 32$$

- *Zapper Contract*: (Number of zappers produced weekly) ≥ 200 doz.

$$X_2 \ge 200$$

- *Product Mix*: Since each of the product mix restrictions is expressed as a percentage of the total production, to clarify the model we introduce the following summation *variable*:

$$X_6 = \text{Total weekly production (in dozens of units)}$$

- *Summation Constraint*: Before expressing the product mix constraints, we introduce the *summation constraint* showing that the total weekly production, X_6, is the sum of the weekly production of Space Rays, Zappers, Big Squirts, and Soakers: $X_6 = X_1 + X_2 + X_3 + X_4$, or

$$X_1 + X_2 + X_3 + X_4 - X_6 = 0$$

Now the product mix constraints can be written as

$$\begin{aligned}
&\text{(Weekly production of Space Rays)} &&= \text{(50\% of total production)}\\
&\qquad X_1 &&= .5X_6\\
&\text{(Weekly production of Zappers)} &&\le \text{(40\% of total production)}\\
&\qquad X_2 &&\le .4X_6\\
&\text{(Weekly production of Big Squirts)} &&\le \text{(40\% of total production)}\\
&\qquad X_3 &&\le .4X_6\\
&\text{(Weekly production of Soakers)} &&\le \text{(40\% of total production)}\\
&\qquad X_4 &&\le .4X_6
\end{aligned}$$

or

$$\begin{aligned}
X_1 \quad\quad\quad\quad &- .5X_6 = 0\\
X_2 \quad\quad\quad &- .4X_6 \le 0\\
X_3 \quad\quad &- .4X_6 \le 0\\
X_4 &- .4X_6 \le 0
\end{aligned}$$

- *Total Production*: (Total weekly production) ≥ 1000 dozen.

$$X_6 \ge 1000$$

- *Nonnegativity*: All decision variables ≥ 0

$$X_1, X_2, X_3, X_4, X_5, X_6 \ge 0$$

THE MATHEMATICAL MODEL

Thus, the complete mathematical model for the Galaxy Industries expansion problem is:

$$\begin{array}{llll}
\text{MAXIMIZE} & 16X_1 + 15X_2 + 20X_3 + 22X_4 - 180X_5 & & \text{(Weekly profit)}\\
\text{ST} & & & \\
 & 2X_1 + X_2 + 3X_3 + 4X_4 & \le 3000 & \text{(Plastic)}\\
 & 3X_1 + 4X_2 + 5X_3 + 6X_4 - 60X_5 & \le 2400 & \text{(Production time)}\\
 & X_5 & \le 32 & \text{(Overtime)}\\
 & X_2 & \ge 200 & \text{(Contract)}\\
 & X_1 + X_2 + X_3 + X_4 - X_6 & = 0 & \text{(Definition)}\\
 & X_1 - .5X_6 & = 0 & \text{(Space Rays)}\\
 & X_2 - .4X_6 & \le 0 & \text{(Zappers)}\\
 & X_3 - .4X_6 & \le 0 & \text{(Big Squirts)}\\
 & X_4 - .4X_6 & \le 0 & \text{(Soakers)}\\
 & X_6 & \ge 1000 & \text{(Total)}\\
 & \text{All X's} \ge 0 & & \text{(Nonnegativity)}
\end{array}$$

EXCEL SOLVER INPUT/OUTPUT

There are many ways we could set up a spreadsheet to represent the model. Although we use more sophisticated spreadsheets that generate a great deal of additional information in later models in this chapter, at this point, the one we show in Figure 3.3*a* is only a slight extension of that in Chapter 2. Note that the numbers in row 4 giving the production quantities and scheduled overtime and the numbers in column G giving the total production, the total profit, and the resources used, are the results of executing Solver. The model is constructed as follows:

- Row 4 is set aside for the values of the decision variables.
- Input data for the profit and the first four functional constraints are entered into rows 6 through 10. The SUMPRODUCT function is used to get the total left-hand side values.
- Cell G4 is programmed to be the sum of the other variables. This represents the summation constraint, and since it will be determined by the other variables, it is NOT a changing cell.
- The percentage constraints of exactly 50% for Space Rays and at most 40% for the other models and the restriction of a minimum production of 1000 dozen units are reflected in the Solver dialogue box. Note that the right-hand side of these constraints is not a cell. For the percentage constraints, it is an expression, and for the minimum production constraint, it is a constant.

Clicking Solve gives the result shown in Figure 3.3*a*. Figure 3.3*b* is the corresponding Sensitivity Report.

Galaxy Expansion.xls

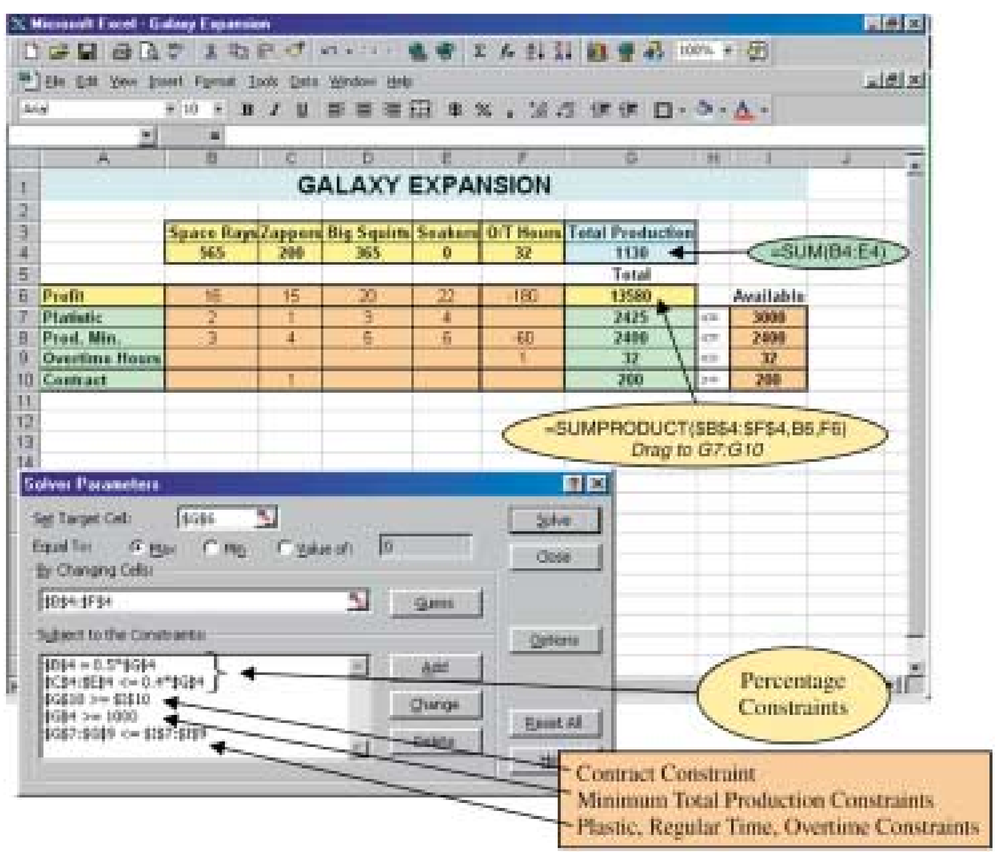
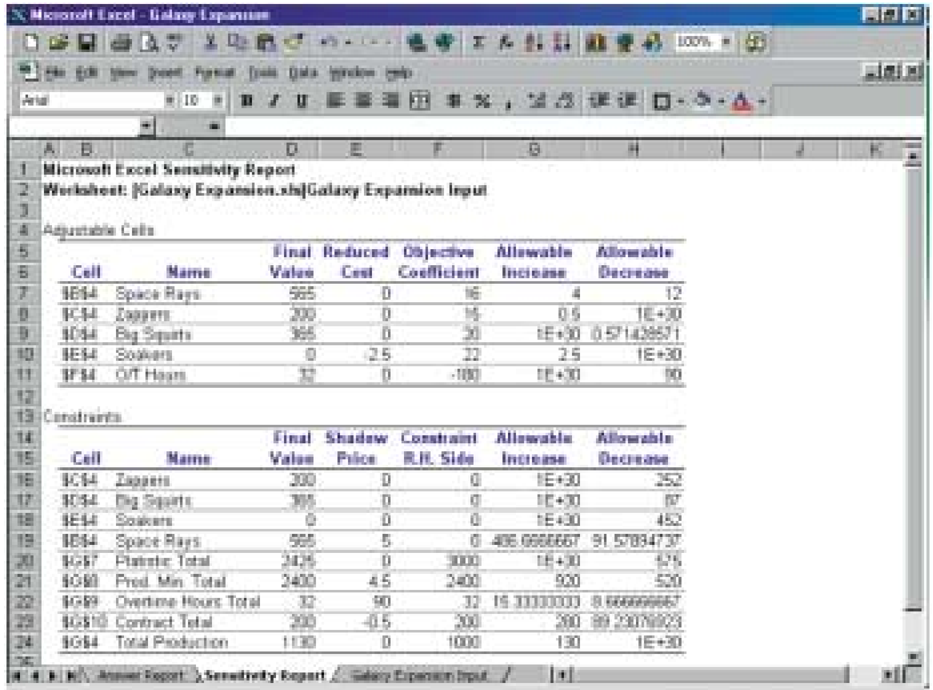


FIGURE 3.3*a* Spreadsheet for the Expansion of Galaxy Industries


Microsoft Excel Sensitivity Report
Worksheet: [Galaxy Expansion.xls]Galaxy Expansion Input

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$4	Space Rays	565	0	16	4	12
$C$4	Zappers	200	0	15	0.5	1E+30
$D$4	Big Squirts	365	0	20	1E+30	0.571428571
$E$4	Soakers	0	-2.5	22	2.5	1E+30
$F$4	O/T Hours	32	0	-180	1E+30	90

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$C$4	Zappers	200	0	0	1E+30	252
$D$4	Big Squirts	365	0	0	1E+30	87
$E$4	Soakers	0	0	0	1E+30	452
$B$4	Space Rays	565	5	0	406.6666667	91.57894737
$G$7	Plastic Total	2425	0	3000	1E+30	575
$G$8	Prod. Min. Total	2400	4.5	2400	920	520
$G$9	Overtime Hours Total	32	90	32	15.33333333	8.666666667
$G$10	Contract Total	200	-0.5	200	200	89.23076923
$G$4	Total Production	1130	0	1000	130	1E+30

FIGURE 3.3*b* Sensitivity Report for the Expansion of Galaxy Industries

Analysis

From Figure 3.3*a* we can determine the gross profit for each model by multiplying the unit profit per dozen times the number of dozens produced. The cost of overtime is 32($180) = $5760. This gives the following results.[3]

Model	Dozens Produced	Total Gross Profit	Percent of Total
Space Rays	565	$ 9,040	50.0%
Zappers	200	$ 3,000	17.7%
Big Squirts	365	$ 7,300	32.3%
Soakers	0	$ 0	0%
Total	1130	$19,340	
	Cost of Overtime	$ 5,760	
	NET PROFIT	$13,580	

Furthermore, we note that:

- The total production of 1130 dozen exceeds the minimum requirement by 130 dozen.
- All 2400 minutes of regular time and all 32 hours of overtime will be used.
- Only 2425 of the 3000 pounds of available plastic will be used.

From the Sensitivity Report information shown in Figure 3.3*b*, we subtract the Allowable Decrease from and add the Allowable Increase to the profit coefficients to determine the following ranges of optimality for which the above solution will remain

[3] In a more elaborate spreadsheet design, such as several of those given in the remainder of this chapter, we could have programmed various cells to generate this information automatically.

optimal. Recall that the range of optimality is the range of values for an objective function coefficient within which the optimal solution remains valid, as long as no other changes are made.

Model	Profit Per Dozen	Minimum Profit Per Dozen	Maximum Profit Per Dozen
Space Rays	$16.00	$ 4.00	$20.00
Zappers	$15.00	No Minimum	$15.50
Big Squirts	$20.00	$19.43	No Maximum
Soakers	$22.00	No Minimum	$24.50

We further note from Figure 3.3*b* that:

- The above solution will remain optimal as long as the cost of overtime hours is less than $270 (Allowable Decrease from −180 is 90 (cell H11)).
- The profit per dozen Soakers must increase by $2.50 (cell E10) to $24.50 before they will be profitable to produce.
- Additional regular time minutes will add $4.50 per minute or $270 per hour to the total profit (cell E21). This holds true for up to an additional 920 minutes or $15\frac{1}{3}$ hours (cell G21).
- Additional (or fewer) overtime hours above or below the 32 scheduled overtime hours will add (or subtract) $90 to the total profit (cell E22). This holds true as long as the total number of overtime hours is between $23\frac{1}{3}$ hours and $47\frac{1}{3}$ hours (obtained from $32 - 8\frac{2}{3}$ (cell H22) and $32 + 15\frac{1}{3}$ (cell G22)).
- Each additional dozen Zappers added to the contract with Jaycee Toys will subtract $0.50 (cell E23) from the total profit (up to an additional 280 dozen (cell G23)). A reduction in the contract amount will save $0.50 per dozen for a reduction not to exceed 89.23 dozen (cell H23). If the contract requirement was outside this range, we would have to re-solve the problem to determine a new shadow price.
- Each dozen Space Rays that are allowed to be produced *above 50% of the total* will add $5.00 (cell E19) to the total profit (up to $486\frac{2}{3}$ dozen more than 50% (cell G19)). Again the problem must be re-solved if the change is outside this range.

Based on this information, the manager might ask for permission to schedule additional overtime hours, increase the percentage of Space Rays produced, reduce the contract with Jaycee Toys, or find ways to increase the profit for Soakers. However, if any changes are made, before proceeding management should first determine if the changes would affect any of the other parameters or basic assumptions underlying the model.

3.4.2 PORTFOLIO MODELS

Numerous mathematical models have been developed for a variety of financial/portfolio models. These models take into account return projections, measures of risk and volatility, liquidity, and long- and short-term investment goals. Some of these models are nonlinear in nature. However, here we present a situation that could be modeled as a linear program.

Jones Investment.xls

JONES INVESTMENT SERVICE

Concepts: Minimization
Sensitivity Analysis (All constraint types)

Charles Jones is a financial advisor who specializes in making recommendations to investors who have recently come into unexpected sums of money from inheritances, lottery winnings, and the like. He discusses investment goals with his clients, taking into account each client's attitude toward risk and liquidity.

After an initial consultation with a client, Charles selects a group of stocks, bonds, mutual funds, savings plans, and other investments that he feels may be appropriate for consideration in the portfolio. He then secures information on each investment and determines his own rating. With this information he develops a chart giving the risk factors (numbers between 0 and 100, based on his evaluation), expected returns based on current and projected company operations, and liquidity information. At the second meeting Charles defines the client's goals more specifically. The responses are entered into a linear programming model, and a recommendation is made to the client based on the results of the model.

Frank Baklarz has just inherited $100,000. Based on their initial meeting, Charles has found Frank to be quite risk-averse. Charles, therefore, suggests the following potential investments that can offer good returns with small risk.

Potential Investment	***Expected Return***	***Jones's Rating***	***Liquidity Analysis***	***Risk Factor***
Savings account	4.0%	A	Immediate	0
Certificate of deposit	5.2%	A	5-year	0
Atlantic Lighting	7.1%	B+	Immediate	25
Arkansas REIT	10.0%	B	Immediate	30
Bedrock Insurance annuity	8.2%	A	1-year	20
Nocal Mining bond	6.5%	B+	1-year	15
Minicomp Systems	20.0%	A	Immediate	65
Antony Hotels	12.5%	C	Immediate	40

Based on their second meeting, Charles has been able to help Frank develop the following portfolio goals.

1. An expected annual return of at least 7.5%
2. At least 50% of the inheritance in A-rated investments
3. At least 40% of the inheritance in immediately liquid investments
4. No more than $30,000 in savings accounts and certificates of deposit

Given that Frank is risk-averse, Charles would like to make a final recommendation that will minimize total risk while meeting these goals. As part of his service, Charles would also like to inform Frank of potential what-if scenarios associated with this recommendation.

SOLUTION

The following is a brief summary of the problem.

- Determine the amount to be placed in each investment.
- Minimize total overall risk.
- Invest all $100,000.
- Meet the goals developed with Frank Baklarz.

THE MATHEMATICAL MODEL

Defining the X's as the amount Frank should allot to each investment, the following linear model represents the situation:

$$
\begin{array}{llr}
\text{MINIMIZE} & 25X_3 + 30X_4 + 20X_5 + 15X_6 + 65X_7 + 40X_8 & \text{(Risk)} \\
\text{ST} & & \\
& X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 = 100{,}000 & \text{(Total)} \\
& .04X_1 + .052X_2 + .071X_3 + .1X_4 + .082X_5 + .065X_6 + .2X_7 + .125X_8 \geq 7500 & \text{(Return)} \\
& X_1 + X_2 + X_5 + X_7 \geq 50{,}000 & \text{(A-Rated)} \\
& X_1 + X_3 + X_4 + X_7 + X_8 \geq 40{,}000 & \text{(Liquid)} \\
& X_1 + X_2 \leq 30{,}000 & \text{(Savings/CD)} \\
& \text{All X's} \geq 0 &
\end{array}
$$

Since Jones Investment Service shares its findings directly with its clients, Charles wants to have a spreadsheet designed with his client in mind. Thus, the spreadsheet should convey all the requisite information without looking like a linear programming model. Accordingly, Charles used the "user-friendly" spreadsheet shown in Figure 3.4*a*. Here the right-hand sides of the constraints are in cells C2, F16, F17, F18, and F19, respectively, and cells C14, B13, D16, D17, D18, and D19 have been programmed as shown to give the quantity designated in the cells to their left. Note that SUMIF formulas in cells D17 and D18 sum only the values that meet the criteria of "A" rating and "Immediate" liquidity, respectively. Cells B5:B12 are reserved for the values of the decision variables.

Jones Investment.xls

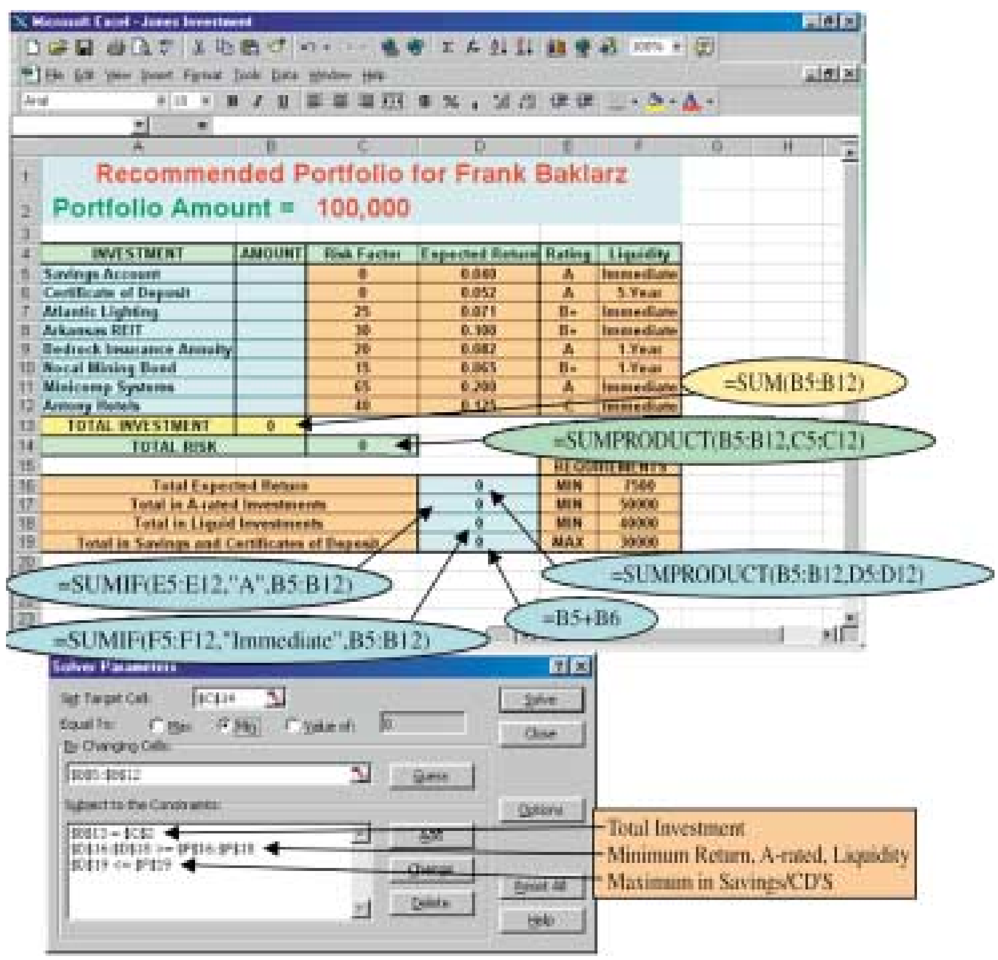
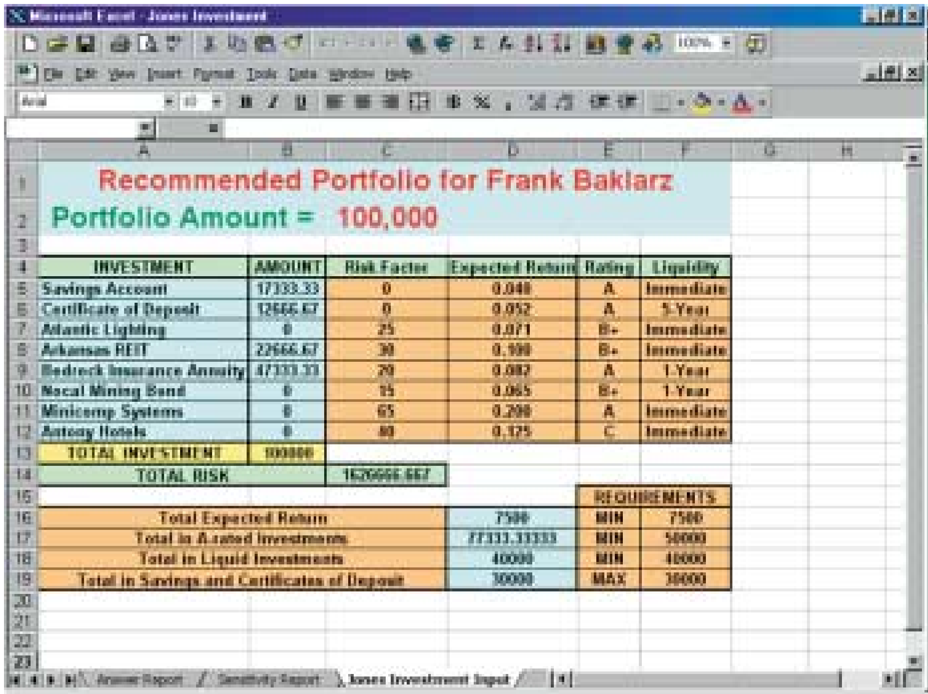


FIGURE 3.4*a*
Excel Spreadsheet and Dialogue Box for Frank Baklarz



Recommended Portfolio for Frank Baklarz

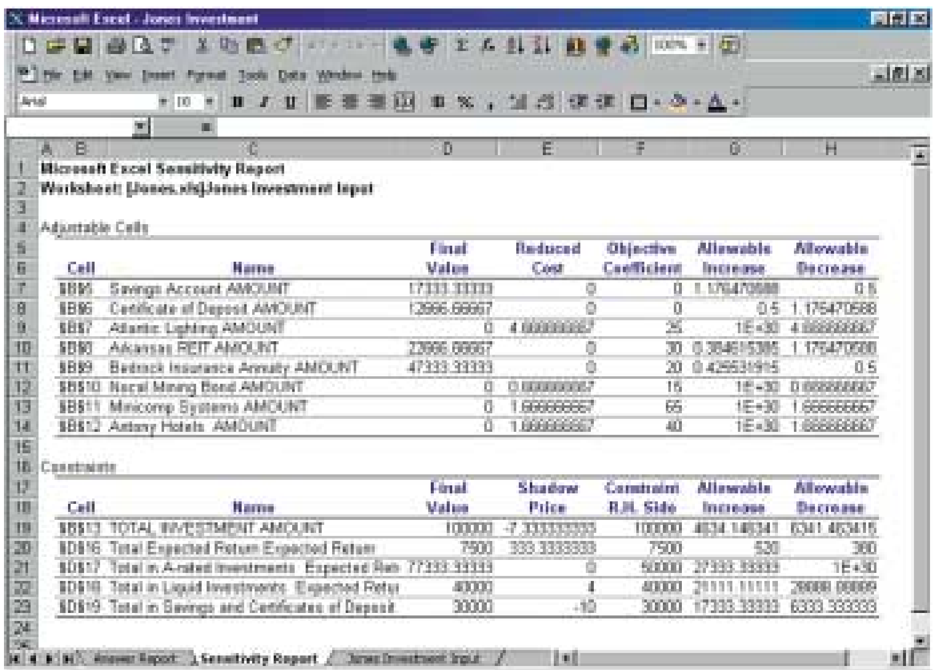
Portfolio Amount = 100,000

INVESTMENT	AMOUNT	Risk Factor	Expected Return	Rating	Liquidity
Savings Account	17333.33	0	0.048	A	Immediate
Certificate of Deposit	12666.67	0	0.052	A	5-Year
Atlantic Lighting	0	25	0.071	B+	Immediate
Arkansas REIT	22666.67	30	0.100	B+	Immediate
Bedrock Insurance Annuity	47333.33	20	0.082	A	1-Year
Nocal Mining Bond	0	15	0.065	B+	1-Year
Minicomp Systems	0	65	0.200	A	Immediate
Antony Hotels	0	40	0.125	C	Immediate
TOTAL INVESTMENT	100000				
TOTAL RISK		1626666.667			

		REQUIREMENTS	
Total Expected Return	7500	MIN	7500
Total in A-rated Investments	77333.33333	MIN	50000
Total in Liquid Investments	40000	MIN	40000
Total in Savings and Certificates of Deposit	30000	MAX	30000

FIGURE 3.4*b*
Optimal Portfolio for Frank Baklarz

After verifying all requirements with Frank Baklarz, Charles clicked on Excel Solver giving the results and Sensitivity Report in Figures 3.4*b* and 3.4*c*, respectively.



Microsoft Excel Sensitivity Report
Worksheet: [Jones.xls]Jones Investment Input

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$5	Savings Account AMOUNT	17333.33333	0	0	1.176470588	0.5
$B$6	Certificate of Deposit AMOUNT	12666.66667	0	0	0.5	1.176470588
$B$7	Atlantic Lighting AMOUNT	0	4.666666667	25	1E+30	4.666666667
$B$8	Arkansas REIT AMOUNT	22666.66667	0	30	0.384615385	1.176470588
$B$9	Bedrock Insurance Annuity AMOUNT	47333.33333	0	20	0.425531915	0.5
$B$10	Nocal Mining Bond AMOUNT	0	0.666666667	15	1E+30	0.666666667
$B$11	Minicomp Systems AMOUNT	0	1.666666667	65	1E+30	1.666666667
$B$12	Antony Hotels AMOUNT	0	1.666666667	40	1E+30	1.666666667

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$13	TOTAL INVESTMENT AMOUNT	100000	-7.333333333	100000	4634.146341	6341.463415
$D$16	Total Expected Return Expected Return	7500	333.3333333	7500	520	380
$D$17	Total in A-rated Investments Expected Ret	77333.33333	0	50000	27333.33333	1E+30
$D$18	Total in Liquid Investments Expected Retu	40000	4	40000	21111.11111	28888.88889
$D$19	Total in Savings and Certificates of Deposit	30000	-10	30000	17333.33333	6333.333333

FIGURE 3.4*c*
Sensitivity Report for Frank Baklarz

Analysis

The spreadsheet is designed for easy reading and interpretation. In addition to the optimal solution, it is easy to see that the binding constraints are those requiring an expected annual return of at least $7500, a minimum amount of $40,000 in immediately liquid investments, and a maximum amount of $30,000 in the savings account and the certificate of deposit. This portfolio exceeds Frank's minimum requirement of at least $50,000 in A-rated investments by $27,333.

RECOMMENDATION

According to the spreadsheet in Figure 3.4*b*, Charles should recommend to Frank that he invest $17,333 in a savings account, $12,667 in a certificate of deposit, $22,667 in Arkansas REIT, and $47,333 in the Bedrock Insurance Annuity. This gives an overall risk value of 1,626,667 (an average risk factor of 16.27 per dollar invested). Any other combination of investments will give a higher risk value.

REDUCED COSTS

According to the Sensitivity Report in Figure 3.4*c*, for Atlantic Lighting to be included in the portfolio, its risk factor would have to be lowered by 4.67 to 20.33. Similarly, to include Nocal Mining, Minicomp Systems, or Antony Hotels requires a reduction in their risk factors of 0.67, 1.67, and 1.67, respectively.

RANGE OF OPTIMALITY

For each investment, the "Allowable Increase" and "Allowable Decrease" columns in the Sensitivity Report give the minimum and maximum amounts that the risk factors can change without altering Charles's recommendation. For example, the range of optimality of the risk factor for the Bedrock Insurance annuity is between 19.5(=20 − 0.5) and 20.43(=20 + 0.43). Recall that the range of optimality applies to changing one investment at a time. Since negative risk factors do not make sense, the minimum risk factors for the savings account and certificate of deposit would be 0.

SHADOW PRICES

The shadow prices in the Sensitivity Report give us the following information:

- If an extra dollar were invested above the $100,000, the risk value would improve (decrease) by 7.33.
- For every extra dollar increase to the minimum expected annual return, the overall risk value would increase by 333.33.
- For every extra dollar that must be made immediately liquid, the overall risk value would increase by 4.
- For every extra dollar that is allowed to be invested in a savings account or a certificate of deposit, the risk value would decrease by 10.
- No change in total risk value would result from requiring that additional dollars be invested in A-rated investments.

RANGE OF FEASIBILITY

The Allowable Increase and Allowable Decrease to the original right-hand side coefficients give the range of feasibility of individual changes to the right-hand side within which the shadow prices remain constant. For example, the range of feasibility corresponding to the $7500 minimum return is ($7500 − $380) to ($7500 + $520) or from $7120 to $8020. An Allowable Decrease of 1E+30 is effectively infinity; thus, $-\infty$ is the minimum right-hand side value in the range of feasibility for the amount invested in A-rated investments.

3.4.3 PUBLIC SECTOR MODELS

National, state, and local governments and agencies are charged with distributing resources for the public good. Frequently, political pressures and conflicts cause these entities to try to do more than the available resources will allow. When that happens, several remedies are possible. One is to try to meet a subset, but not all, of a perceived set of constraints, as is the case of one of the homework exercises in this chapter. A second approach is to treat several of the constraints as "goals" and to prioritize and weight these goals. This is called a "goal programming" approach, which is discussed in Chapter 13 on the accompanying CD-ROM. A third approach, and the one illustrated here, is simply to scale back and try to "live within one's means."

Saint Joseph.xls
Saint Joseph (Revised).xls

ST. JOSEPH PUBLIC UTILITY COMMISSION

Concepts: Ignoring Integer Restrictions
Summation Variable
Infeasibility
Multiple Optimal Solutions

The St. Joseph Public Utilities Commission has been charged with inspecting and reporting utility problems that have resulted from recent floods in the area. Concerns have been raised about the damage done to electrical wiring, gas lines, and insulation. The Commission has one week to carry out its inspections. It has been assigned three electrical inspectors and two gas inspectors, each available for 40 hours, to analyze structures in their respective areas of expertise. In addition, the Commission has allocated $10,000 for up to 100 hours (at $100 per hour) of consulting time from Weathertight Insulation, a local expert in home and industrial insulation.

These experts are assigned to inspect private homes, businesses (office complexes), and industrial plants in the area. The goal is to thoroughly inspect as many structures as possible during the allotted time in order to gather the requisite information. However, the minimum requirements are to inspect at least eight office buildings and eight industrial plants, and to make sure that at least 60% of the inspections are of private homes.

Once the total number of each type of structure to be inspected has been determined, the actual inspections will be done by choosing a random sample from those that are served by the St. Joseph Public Utility Commission. The Commission has mandated the following approximate inspection hours for each type of inspection:

	Electrical	***Gas***	***Insulation***
Homes	2	1	3
Offices	4	3	2
Plants	6	3	1

A team of management science consultants has been hired to suggest how many homes, office buildings, and plants should be inspected.

SOLUTION

The following is a brief summary of the problem faced by the St. Joseph Public Utilities Commission.

- St. Joseph must determine the number of homes, office complexes, and plants to be inspected.
- It wishes to maximize the total number of structures inspected.
- At least eight offices and eight plants are to be inspected.
- At least 60% of the inspections should involve private homes.
- At most, 120 hours (3×40) can be allocated for electrical inspections, 80 hours (2×40) for gas inspections, and 100 consulting hours for insulation inspection

The management science team has decided to formulate the problem as a linear program, although the results should be integer-valued. If the linear program does not generate an integer solution, another method such as integer programming or dynamic programming (which is discussed in Chapter 13 on the accompanying CD-ROM) must be used, or St. Joseph could accept a feasible rounded solution.

DECISION VARIABLES

The team defined the following variables:

$$X_1 = \text{number of homes to be inspected}$$
$$X_2 = \text{number of office complexes to be inspected}$$
$$X_3 = \text{number of industrial plants to be inspected}$$

and they used the following summation variable

$$X_4 = \text{total number of structures to be inspected}$$

OBJECTIVE FUNCTION

The problem is to determine the maximum number of structures that can be inspected, subject to the constraints. Thus, the objective function of the model is simply:

$$\text{MAXIMIZE } X_4$$

CONSTRAINTS

The summation constraint for X_4 is:

$$X_1 + X_2 + X_3 - X_4 = 0$$

The minimum number of office complexes and plants to be inspected are simply modeled as

$$X_2 \geq 8$$
$$X_3 \geq 8$$

The fact that at least 60% of the inspections must be of homes is modeled as

$$X_1 \geq 0.6X_4$$

or

$$X_1 - 0.6X_4 \geq 0$$

Finally, the constraints on the time limits for electrical, gas, and insulation inspections are:

$$
\begin{aligned}
2X_1 + 4X_2 + 6X_3 &\le 120 \text{ (Electrical)}\\
1X_1 + 3X_2 + 3X_3 &\le 80 \text{ (Gas)}\\
3X_1 + 2X_2 + 1X_3 &\le 100 \text{ (Insulation)}
\end{aligned}
$$

THE LINEAR PROGRAMMING MODEL

The complete linear programming model for the St. Joseph Public Utility Commission is:

$$
\begin{array}{llll}
\text{MAXIMIZE} & X_4 & & \text{(Total structures)}\\
\text{ST} & & & \\
 & X_1 + X_2 + X_3 - X_4 & = & 0 \text{ (Summation)}\\
 & X_2 & \ge & 8 \text{ (Minimum offices)}\\
 & X_3 & \ge & 8 \text{ (Minimum plants)}\\
 & X_1 - 0.6X_4 & \ge & 0 \text{ } (\ge 60\% \text{ Homes})\\
 & 2X_1 + 4X_2 + 6X_3 & \le & 120 \text{ (Electrical)}\\
 & X_1 + 3X_2 + 3X_3 & \le & 80 \text{ (Gas)}\\
 & 3X_1 + 2X_2 + 1X_3 & \le & 100 \text{ (Insulation)}\\
 & \text{All X's} \ge 0 & &
\end{array}
$$

EXCEL SOLVER INPUT/OUTPUT AND ANALYSIS

Figure 3.5*a* shows the Excel spreadsheet and Solver dialogue box used by management science consultants. Note that the summation constraint is included in cell B9. The constraint requiring a minimum limit of 60% of the inspections to be houses can be expressed as $X_1 \ge .6X_4$. By entering the formula =.6*B9 into cell C5, this constraint requires (cell B5) ≥ (cell C5), which is part of the first set of constraints in the dialogue box.

Saint Joseph.xls

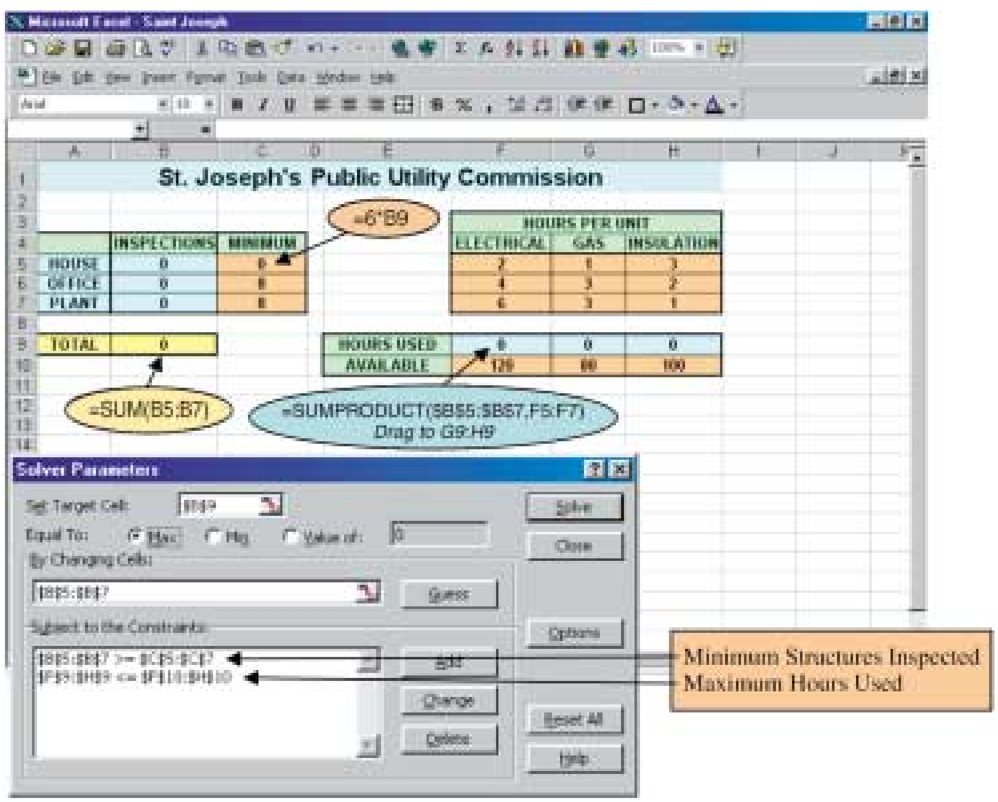


FIGURE 3.5*a* Input for the St. Joseph Public Utility Commission

But when Solve was clicked, instead of an optimal solution, Solver returned the dialogue box shown in Figure 3.5*b*.

FIGURE 3.5*b*
Solver Found the St. Joseph Utility Model to Be Infeasible

Needless to say, the Commission was not too pleased with this analysis. In fact, it was beginning to conclude that the management science consultants (and perhaps management science itself) could not be trusted to give the desired results.

When the consultants were asked to explain this result, they pointed out the reason for infeasibility. Even if only the minimum eight offices and eight plants were inspected, 80 of the 120 electrical hours [4(8) + 6(8)] would be used, leaving only 40 hours to inspect homes. At two hours per home, a maximum of 20 homes could be inspected. Thus, a total of 36 structures would be inspected, only 20 of which would be homes. This represents only 55.56% of the total homes (=20/36), not the minimum 60% the Commission desired.

In other words, the problem had been formulated correctly by the management science team, but the Commission had simply given them a set of constraints that were impossible to meet. Given this situation, after much debate the Commission decided that it could get by with inspecting a minimum of six office buildings and six plants. This would use up 60 electrical hours, leaving 60 electrical hours to inspect 30 homes, which far exceeds the 60% minimum limit on homes.

The Commission was about to initiate this action when it was pointed out that inspecting 30 homes, six office buildings, and six plants would use up 108 hours for insulation inspection [3(30) + 2(6) + 1(6)], exceeding the 100 available inspection hours. What to do?

The Commission asked the management science consultants to reconsider their problem in light of these relaxed constraints and offer a recommendation. The consultants changed the values in cells C6 and C7 of their spreadsheet from 8 to 6 and again called Solver to determine an optimal solution. The resulting spreadsheet and the Sensitivity Report are shown in Figures 3.6*a* and 3.6*b*, respectively.

The optimal solution turned out to have integer values, and thus the consultants could now report that a maximum of 40 structures (27 houses, 6 office buildings, and 7 plants) could be inspected; 67.5% (=27/40) of the inspected structures would be houses. All 120 electrical inspection hours and all 100 insulation inspection hours would be used. A total of 12 hours of gas inspection time would remain unused.

An Alternate Optimal Solution

But the consultants noted from the Sensitivity Report in Figure 3.6*b* that the Allowable Increase of Office Inspections and the Allowable Decrease for both House Inspections and Plant Inspections were all 0. From our discussion in Chapter 2, recall that this is an indication that there may be alternate optimal solutions. Fol-

Saint Joseph (Revised).xls

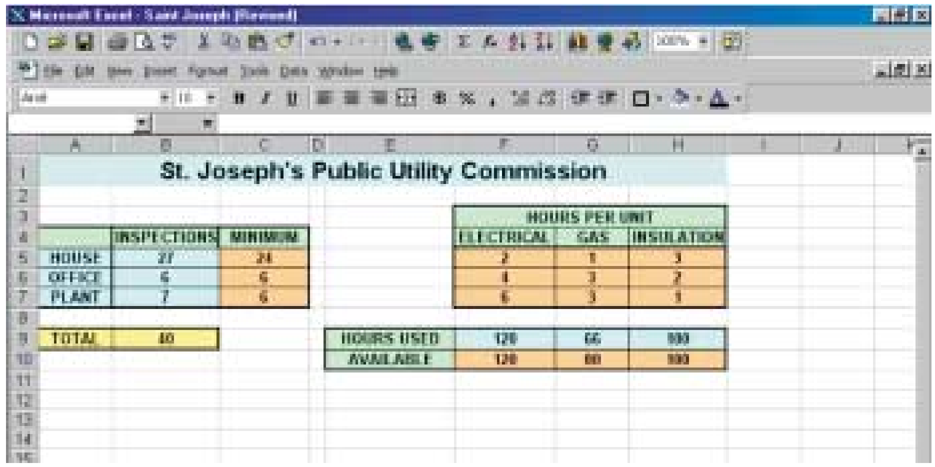


FIGURE 3.6*a* Optimal Solution for the St. Joseph Public Utility Commission

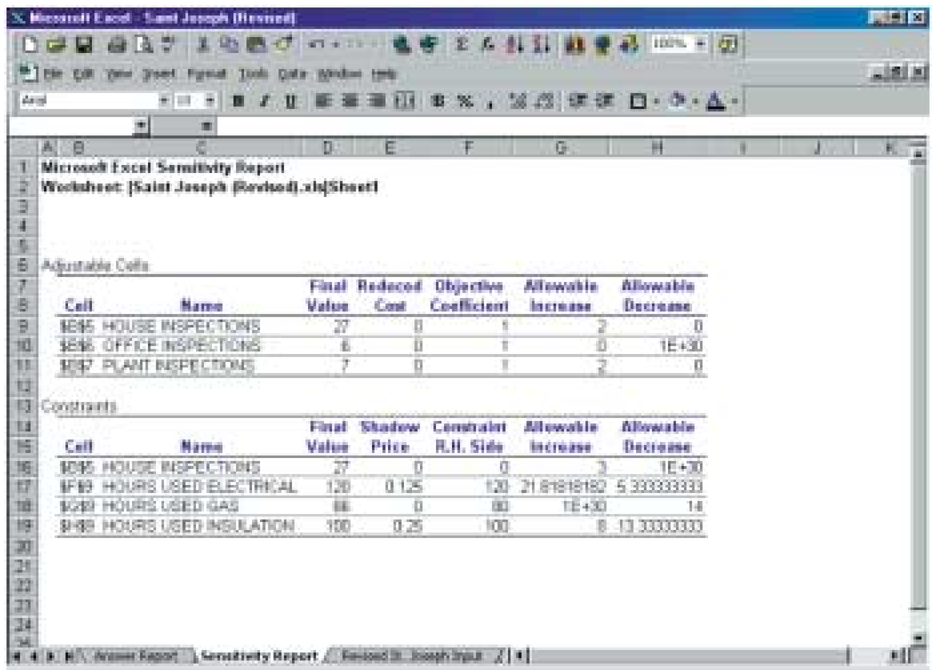


FIGURE 3.6*b* Sensitivity Report for the St. Joseph Public Utility Commission

lowing the procedure for generating alternate optimal solutions outlined in Chapter 2, the consultants:

- Added a constraint requiring the total number of inspections to be 40
- Changed the objective function to MAX X_2 (cell B6), since the Allowable *Increase* for office building inspections is 0. (Alternatively, they could have chosen to *minimize* either cell B5 or B7 since the Allowable *Decrease* for house inspections or plant inspections is also 0.)

Figure 3.7 shows the dialogue box and the resulting spreadsheet.

This spreadsheet shows that inspecting 26 houses, 8 office buildings, and 6 plants would be an alternative way of inspecting 40 structures while staying within the constraints of the model. In this solution, 65% (=26/40) of the structures inspected would be houses.

Saint Joseph (Revised).xls (Alternative Solution Worksheet)

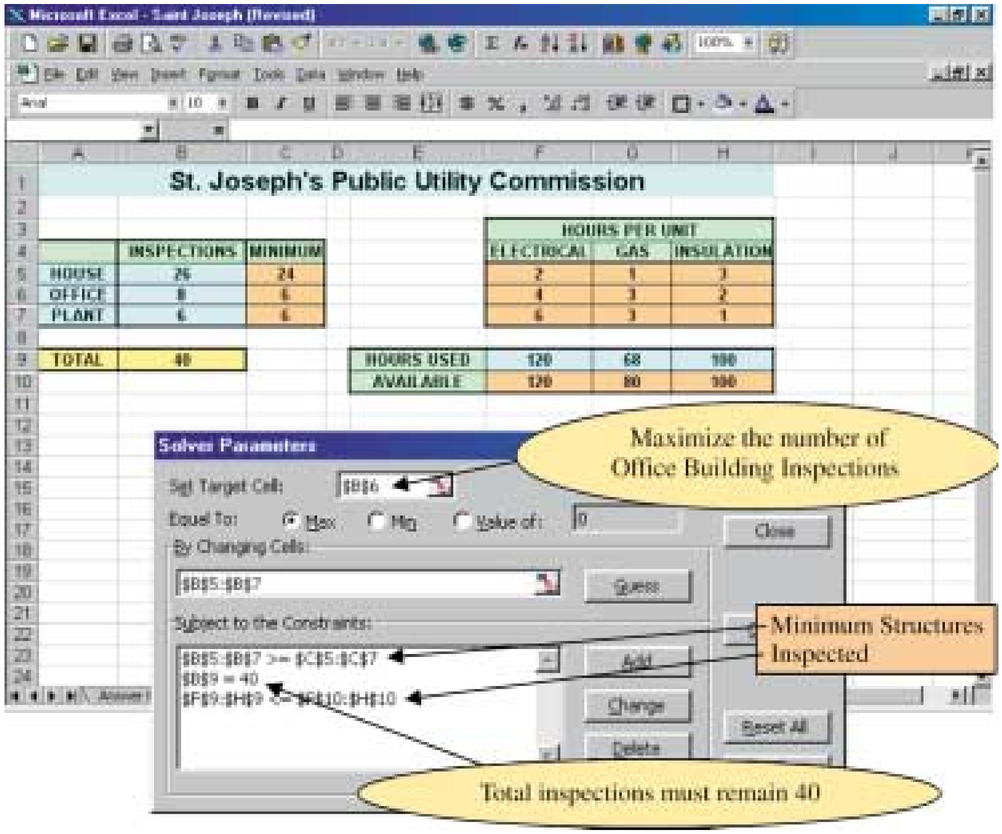


FIGURE 3.7 Alternate Optimal Solution for St. Joseph's Public Utility Commission

Although any weighted average of these two solutions would also be optimal, since the first solution calls for inspecting 26 homes and the second 27 homes, any weighted average of the two solutions would yield a *fractional* solution of between 26 and 27 homes to be inspected. Thus, the consultants reported that these two solutions are the only optimal solutions that yield integers for the number of homes, office complexes, and plants to be inspected.

Faced with two feasible alternatives, the Commission had the opportunity to inject some political preferences into the decision process while still inspecting 40 structures.

3.4.4 PURCHASING MODELS

Purchasing models can take into account customer demand, budgets, cash flow, advertising, and inventory restrictions. In today's global economy, purchasing models play a key role in balancing customer satisfaction levels within the limited resources of the business enterprise. In the following application we present a very simplified model that takes only a few of these factors into account. We have purposely presented this problem in such a way as to illustrate another situation that can arise when building mathematical models—that of failing to consider all (or at least not enough) of the limiting factors in the original formulation.

Euromerica Liquors.xls
Euromerica Liquors (Revised).xls

EUROMERICA LIQUORS

Concepts: Choosing an Objective
Lower Bound Constraints
Unboundedness
"Slightly" Violated Constraints
Interpretation of Reduced Costs for Bounded Variables

Euromerica Liquors of Jersey City, New Jersey purchases and distributes a number of wines to retailers. See Table 3.2. Purchasing manager Maria Arias has been asked to order at least 800 bottles of each wine during the next purchase cycle. The only other direction Maria has been given is that, in accordance with a long-standing company policy, she is to order at least twice as many domestic (U.S.) bottles as imported bottles in any cycle. Management believes that this policy promotes a steady sales flow that keeps inventory costs at a minimum. Maria must decide exactly how many bottles of each type of wine the company is to purchase during this ordering cycle.

TABLE 3.2 Euromerica Liquors' Wine Purchases and Distribution

Wine	Country	Cost	Selling Price
Napa Gold	U.S.	\$2.50	\$4.25
Cayuga Lake	U.S.	\$3.00	\$4.50
Seine Soir	France	\$5.00	\$8.00
Bella Bella	Italy	\$4.00	\$6.00

SOLUTION

To summarize, Maria must:

- determine the number of bottles of each type of wine to purchase
- order at least 800 of each type
- order at least twice as many domestic bottles as imported bottles
- select an appropriate objective function

DECISION VARIABLES

The four decision variables can be defined as:

X_1 = bottles of Napa Gold purchased in this purchase cycle
X_2 = bottles of Cayuga Lake purchased in this purchase cycle
X_3 = bottles of Seine Soir purchased in this purchase cycle
X_4 = bottles of Bella Bella purchased in this purchase cycle

OBJECTIVE FUNCTION

At first, Maria reasoned that since Euromerica Liquors' goal is to make good profits, her objective should be to maximize the profit from the purchases made during this purchase cycle. Because inventory costs are assumed to be small due to the company's ordering policy, she defined the profit coefficients in terms of the selling price minus the purchase cost per bottle. Thus, the unit profits for the respective decision variables are \$1.75, \$1.50, \$3, and \$2, and the objective function is:

$$\text{MAX } 1.75X_1 + 1.50X_2 + 3X_3 + 2X_4$$

CONSTRAINTS

The following constraints must be considered

Minimum Production: At least 800 bottles of each of the wines are to be purchased:

$$X_1 \geq 800$$
$$X_2 \geq 800$$
$$X_3 \geq 800$$
$$X_4 \geq 800$$

These constraints could be entered in linear programming software either as functional constraints or as lower bound constraints that would replace the nonnegativity constraints for the variables.

Mix Constraint: (The number of bottles of domestic wine purchased) should be at least (twice the number of bottles of imported wine purchased):

$$X_1 + X_2 \geq 2(X_3 + X_4)$$

or

$$X_1 + X_2 - 2X_3 - 2X_4 \geq 0$$

THE MATHEMATICAL MODEL

The complete model can now be formulated as

$$\begin{array}{llllll} \text{MAXIMIZE} & 1.75X_1 & + 1.50X_2 & + 3X_3 & + 2X_4 & \\ \text{ST} & X_1 & & & & \geq 800 \\ & & X_2 & & & \geq 800 \\ & & & X_3 & & \geq 800 \\ & & & & X_4 & \geq 800 \\ & X_1 & + X_2 & - 2X_3 & - 2X_4 & \geq 0 \end{array}$$

EXCEL INPUT/OUTPUT AND ANALYSIS

Maria created the Excel spreadsheet and Solver dialogue box shown in Figure 3.8*a*, with cells C4:C7 set aside for the number of bottles to order.

When she clicked Solve, however, she got the result shown in Figure 3.8*b*. Recall that the message "The Set Cell values do not converge" is Excel's way of stating that the problem is unbounded.

But Euromerica cannot make an infinite profit! When Maria examined the model, she realized that she had ignored the following considerations when building the model:

- Euromerica has a finite budget for the procurement of bottles of wine during the purchase cycle.
- The suppliers have a finite amount of product available.
- There is a limit on demand from the wine-buying public.

Undaunted, Maria discussed the situation with management and discovered that they wished to commit no more than $28,000 to purchase wine during this cycle. She then contacted the wine producers and found that there were ample supplies of Cayuga Lake and Bella Bella, but that only 300 cases of Napa Gold and

Euromerica Liquors.xls

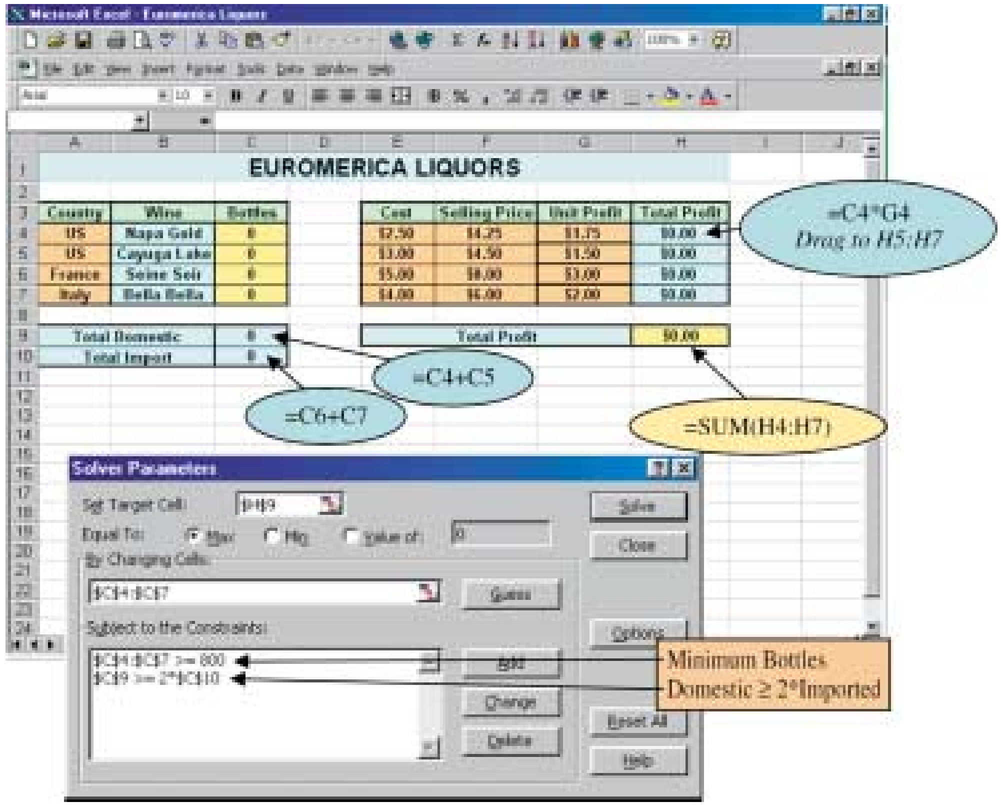


FIGURE 3.8*a* Spreadsheet and Dialogue Box for Euromerica Liquors

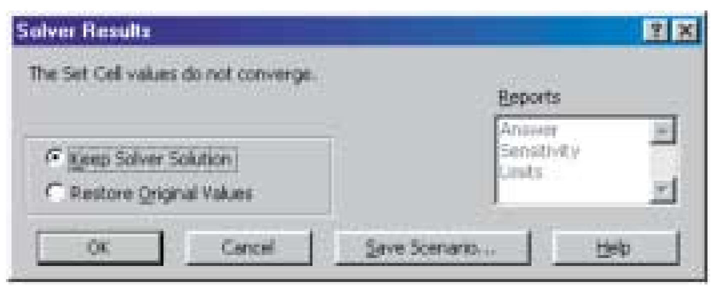


FIGURE 3.8*b* Solver Result for Euromerica Liquors—Unbounded Solution

200 cases of Seine Soir were available (each case contains 12 bottles). Finally, she performed a market survey and, based on the results, concluded that no more than 10,000 total bottles should be purchased. Thus, the revised model is:

$$
\begin{array}{llcl}
\text{MAXIMIZE} & 1.75X_1 + 1.50X_2 + 3X_3 + 2X_4 & & \\
\text{ST} & X_1 & \geq & 800 \\
 & X_2 & \geq & 800 \\
 & X_3 & \geq & 800 \\
 & X_4 & \geq & 800 \\
 & X_1 + X_2 - 2X_3 - 2X_4 & \geq & 0 \\
 & 2.50X_1 + 3.00X_2 + 5X_3 + 4X_4 & \leq & 28{,}000 \text{ (Budget)} \\
 & X_1 & \leq & 3600 \text{ (Napa)} \\
 & X_3 & \leq & 2400 \text{ (Seine)} \\
 & X_1 + X_2 + X_3 + X_4 & \leq & 10{,}000 \text{ (Total)}
\end{array}
$$

Maria revised her spreadsheet to reflect these changes by adding cells C11 and C13 to reflect the total number of bottles purchased and the amount of budget spent and cells G11 and G13 to reflect the limits on the maximum number of bottles purchased and the cycle budget. When the constraints for the maximum number of bottles purchased and the maximum budget expenditure along with limits on the availability of Napa Gold and Seine Soire were added to the Solver dialogue box, clicking Solve generated the optimal spreadsheet and Sensitivity Report shown in Figures 3.9*a* and 3.9*b*.

Euromerica Revised.xls

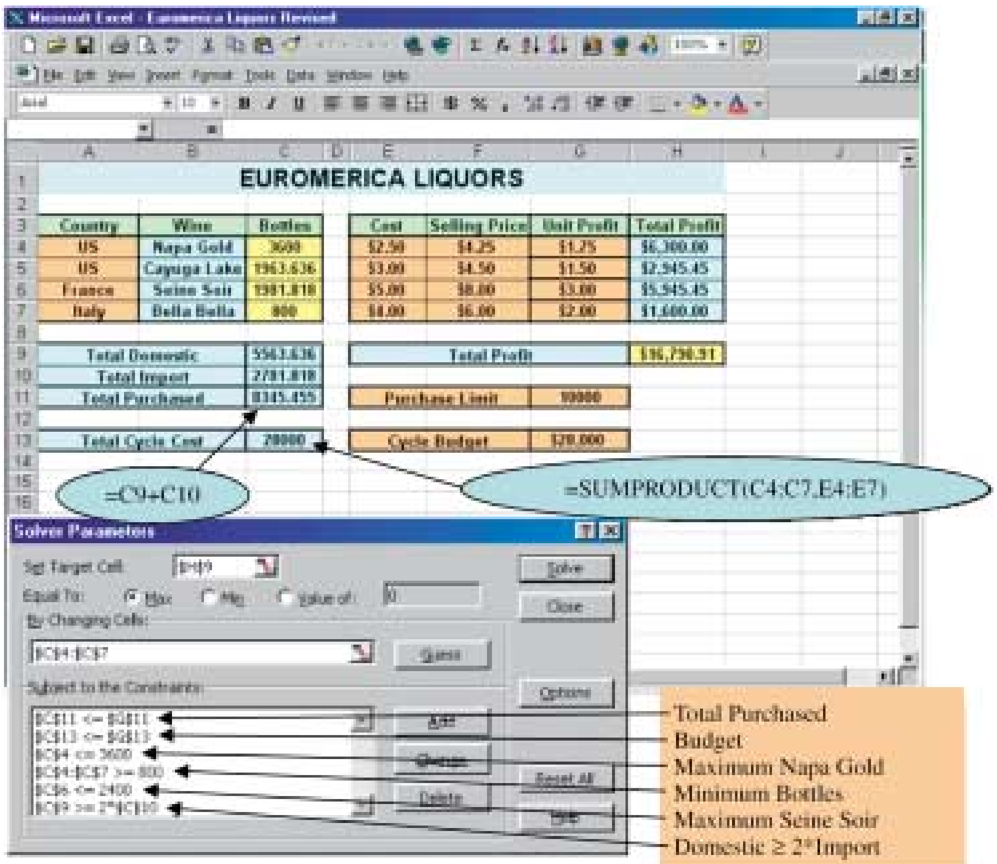
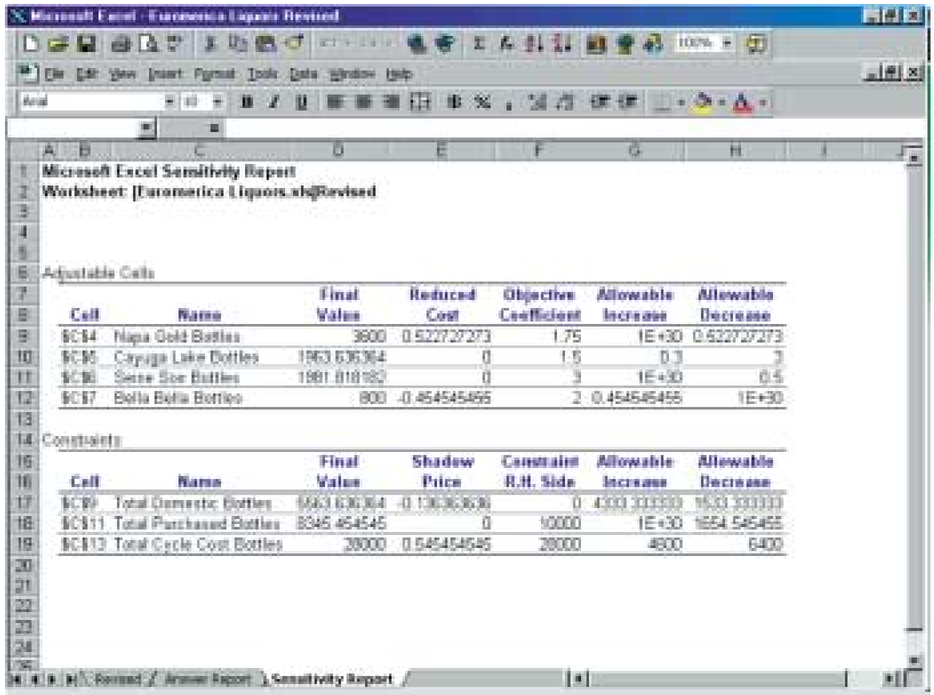


FIGURE 3.9a Revised Spreadsheet for Euromerica Liquors

Maria rounded off the solution to *full cases* and placed them in the order shown in Table 3.3. Note that this proposal is $20 over the budget limit of $28,000. Although the $28,000 limit was a restriction, it was probably a strong guideline rather than a hard and fast value. Hence, Maria had no qualms about recommending this solution.

TABLE 3.3 Euromerica Liquors' Solution

Wine	Bottles	Cases	Cost	Profit
Napa Gold	3600	300	$ 9,000	$ 6,300
Cayuga Lake	1968	164	$ 5,904	$ 2,952
Seine Soir	1980	165	$ 9,900	$ 5,940
Bella Bella	804	67	$ 3,216	$ 1,608
Total	8352	696	$28,020	$16,800



Microsoft Excel Sensitivity Report
Worksheet: [Euromerica Liquors.xls]Revised

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$4	Napa Gold Bottles	3600	0.522727273	1.75	1E+30	0.522727273
$C$5	Cayuga Lake Bottles	1963.636364	0	1.5	0.3	3
$C$6	Seine Soir Bottles	1981.818182	0	3	1E+30	0.5
$C$7	Bella Bella Bottles	800	-0.454545455	2	0.454545455	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$C$9	Total Domestic Bottles	5563.636364	-0.136363636	0	4000.333333	1533.333333
$C$11	Total Purchased Bottles	8345.454545	0	10000	1E+30	1654.545455
$C$13	Total Cycle Cost Bottles	28000	0.545454545	28000	4800	6400

FIGURE 3.9*b* Sensitivity Report for Revised Euromerica Liquors

Reduced Cost for Bounded Variables

In Chapter 2 we defined the reduced cost for a variable as the amount the objective function would change if the value of that variable were increased from 0 to 1. There we implicitly assumed that the lower bound for the variable was 0 and that there was no upper bound. When a variable is defined to be a **bounded variable** by restricting its cell value in the spreadsheet to be at least or at most some nonzero constant, the reduced cost indicates the change in the objective function value if that bound were changed by 1.

In Figure 3.9*b* Maria noticed that the number of Napa Gold bottles purchased would be its upper bound of 3600. She also noticed that the number of Bella Bella bottles purchased would be at its lower bound of 800. Thus, she reported to management that if she were allowed to *increase* the number of bottles of Napa Gold (above 3600), overall profit would increase by slightly more than $0.52 per bottle, whereas if she were allowed to *decrease* the number of bottles of Bella Bella ordered (below 800), profit would increase by slightly more than $0.45 per bottle.

3.4.5 BLENDING MODELS

One of the early successful applications of linear programming models was that of aiding executives in the oil industry in determining how much raw crude oil to purchase from various sources and how to blend these oils into useful gasoline and other byproducts. Each of these products has certain specifications that must be met such as a minimum octane rating or a maximum vapor pressure level. The United Oil Company model presented here is a simplified version of such a model. Other industries where similar blending models are useful include the garment and food industries, which blend several raw materials from various sources into finished products.

United Oil.xls

UNITED OIL COMPANY

Concepts: Variable Definitions for Blending Models
Calculation of Objective Coefficients
Ratio Constraints
Summation Variables
Alternate Optimal Solutions
Hidden Cells on Spreadsheet

United Oil blends two input streams of crude oil products—alkylate and catalytic cracked (c.c.)—to meet demand for weekly contracts for regular (12,000 barrels), mid-grade (7500 barrels), and premium (4500 barrels) gasolines. Each week United can purchase up to 15,000 barrels of alkylate and up to 15,000 barrels of catalytic cracked. Because of demand, it can sell all blended gasolines, including any production that exceeds its contracts.

To be classified as regular, mid-grade, or premium, gasolines must meet minimum octane and maximum vapor pressure requirements. The octane rating and vapor pressure of a blended gasoline is assumed to be the weighted average of the crude oil products in the blend. Relevant cost/pricing, octane, and vapor pressure data are given in Tables 3.4 and 3.5.

United must decide how to blend the crude oil products into commercial gasolines in order to maximize its weekly profit.

TABLE 3.4 Cost/pricing, Octane, and Vapor Pressure Data—United Oil

	Crude Oil Product Data		
Product	Octane Rating	Vapor Pressure (lb./sq. in.)	Cost per Barrel
Alkylate	98	5	$19
Catalytic cracked	86	9	$16

TABLE 3.5 Gasoline Octane Rating, Vapor Pressure, and Barrel Profit

	Blended Gasoline Requirements		
Gasoline	Minimum Octane Rating	Maximum Vapor Pressure	Selling Price per Barrel
Regular	87	9	$18
Mid-grade	89	7	$20
Premium	92	6	$23

SOLUTION

The problem for United Oil is to:

- determine how many barrels of alkylate to blend into regular, mid-grade, and premium and how many barrels of catalytic cracked to blend into regular, mid-grade, and premium each week
- maximize total weekly profit

- remain within raw gas availabilities
- meet contract requirements
- produce gasoline blends that meet the octane and vapor pressure requirements

DECISION VARIABLES (FIRST PASS)

The pending decision is to determine how much of each crude oil (X, Y) to blend into each of the three grades (1, 2, 3) each week:

X_1 = number of barrels of alkylate blended into regular weekly
X_2 = number of barrels of alkylate blended into mid-grade weekly
X_3 = number of barrels of alkylate blended into premium weekly
Y_1 = number of barrels of catalytic cracked blended into regular weekly
Y_2 = number of barrels of catalytic cracked blended into mid-grade weekly
Y_3 = number of barrels of catalytic cracked blended into premium weekly

OBJECTIVE FUNCTION

The profit made on a barrel of crude product blended into a commercial gasoline is the difference between the selling price of the blended gasoline and the cost of the crude product. Table 3.6 gives the profit coefficients. The objective function is:

$$\text{MAX} - 1X_1 + 1X_2 + 4X_3 + 2Y_1 + 4Y_2 + 7Y_3$$

TABLE 3.6 Profit Coefficients for Oil

Variable	Crude Product Cost	Gasoline Selling Price	Barrel Profit
X_1	$19	$18	−$1
X_2	$19	$20	$1
X_3	$19	$23	$4
Y_1	$16	$18	$2
Y_2	$16	$20	$4
Y_3	$16	$23	$7

CONSTRAINTS

United must consider the following constraints in its analysis:

Crude Availability: United cannot blend more than the product available from either input source. The total amount blended from a source is simply the sum of the amounts blended into regular, mid-grade, and premium gasoline:

$$X_1 + X_2 + X_3 \leq 15{,}000$$
$$Y_1 + Y_2 + Y_3 \leq 15{,}000$$

Contract Requirements: Although the contract requirements must be met, they may be exceeded; thus, although at least 12,000 barrels of regular must be produced, the actual amount produced will be the sum of the amounts of alkylate and catalytic cracked blended into regular: $X_1 + Y_1$. Similarly, the amount of mid-grade gas produced will be $X_2 + Y_2$, and the amount of premium gas produced will be $X_3 + Y_3$. Since these quantities are of interest to United Oil (and will figure into the remaining constraints), to simplify the formulation, summation variables can be used.

DECISION VARIABLES (SECOND PASS)

Define the following summation variables

R = barrels of regular gasoline produced weekly
M = barrels of mid-grade gasoline produced weekly
P = barrels of premium gasoline produced weekly

Doing so requires adding the following summation constraints:

$$\begin{aligned} X_1 + Y_1 - R &= 0 \\ X_2 + Y_2 - M &= 0 \\ X_3 + Y_3 - P &= 0 \end{aligned}$$

Now the contract constraints can then be written as

$$\begin{aligned} R &\geq 12{,}000 \\ M &\geq 7500 \\ P &\geq 4500 \end{aligned}$$

Octane and Vapor Constraints: The octane rating for regular gasoline is the weighted average of the octane ratings for alkylate and catalytic cracked blended into regular. The appropriate weights are the ratios of the amount of alkylate to the amount of regular and the amount of catalytic cracked to the amount of regular, respectively:

98 (Amount of alkylate in regular/Total amount of regular) +
86 (Amount of catalytic cracked in regular/Total amount of regular) =

$$98(X_1/R) + 86(Y_1/R)$$

Since this must be at least 87, the constraint is:

$$98(X_1/R) + 86(Y_1/R) \geq 87$$

The terms (X_1/R) and (Y_1/R) make this a *nonlinear* constraint. Since R will be positive in the optimal solution, however, multiplying both sides by R gives the following linear constraint:

$$98X_1 + 86Y_1 \geq 87R$$

or

$$98X_1 + 86Y_1 - 87R \geq 0$$

The remaining octane and vapor pressure constraints are constructed similarly; thus, the complete set of octane and vapor pressure restrictions is:

$$\begin{aligned} 98X_1 + 86Y_1 - 87R &\geq 0 \\ 98X_2 + 86Y_2 - 89M &\geq 0 \\ 98X_3 + 86Y_3 - 92P &\geq 0 \\ 5X_1 + 9Y_1 - 9R &\leq 0 \\ 5X_2 + 9Y_2 - 7M &\leq 0 \\ 5X_3 + 9Y_3 - 6P &\leq 0 \end{aligned}$$

THE MATHEMATICAL MODEL

The complete model is as follows:

$$
\begin{array}{lrrrrrrrrrl}
\text{MAXIMIZE} & -1X_1 + & 1X_2 + & 4X_3 + & 2Y_1 + & 4Y_2 + & 7Y_3 & & & & \\
\text{ST} & X_1 + & X_2 + & X_3 & & & & & & & \le 15{,}000 \\
 & & & & Y_1 + & Y_2 + & Y_3 & & & & \le 15{,}000 \\
 & X_1 & & & + Y_1 & & & - R & & & = 0 \\
 & & X_2 & & & + Y_2 & & & - M & & = 0 \\
 & & & X_3 & & & + Y_3 & & & - P & = 0 \\
 & & & & & & & R & & & \ge 12{,}000 \\
 & & & & & & & & M & & \ge 7500 \\
 & & & & & & & & & P & \ge 4500 \\
 & 98X_1 & & & - 86Y_1 & & & - 87R & & & \ge 0 \\
 & & 98X_2 & & & + 86Y_2 & & & - 89M & & \ge 0 \\
 & & & 98X_3 & & & + 86Y_3 & & & - 92P & \ge 0 \\
 & 5X_1 & & & + 9Y_1 & & & - 9R & & & \le 0 \\
 & & 5X_2 & & & + 9Y_2 & & & - 7M & & \le 0 \\
 & & & 5X_3 & & & + 9Y_3 & & & - 6P & \le 0 \\
\end{array}
$$

All variables ≥ 0

EXCEL INPUT/OUTPUT AND ANALYSIS

Figure 3.10*a* shows the completed worksheet for United Oil. On the left side in columns A:E are the parameter inputs. On the right side, columns H:K give output values generated when Solver solves the model. The values of the decision variables are given in cells H7:J8. Cell formulas used in the spreadsheet are described in Table 3.7.

United Oil.xls

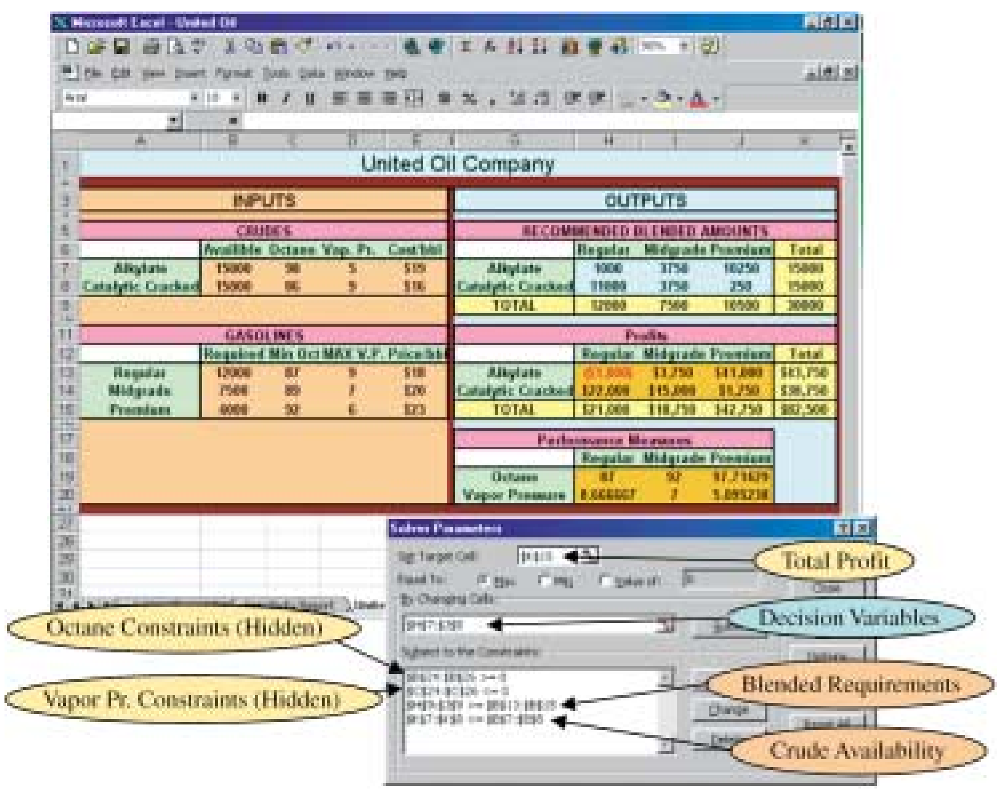


FIGURE 3.10*a* Optimal Spreadsheet for United Oil Company

TABLE 3.7 Cell Formulas and Analysis of Spreadsheet in Figure 3.10*a*

Spreadsheet Formulas/Analysis			
Cell	Quantity	Formula	Observations
K7	Alkylate Used	=SUM(H7:J7)	*All 15,000 barrels of each are used.*
K8	Catalytic Cracked Used	=SUM(H8:J8)	
H9	Regular Produced	=SUM(H7:H8)	*Regular and mid-grade are produced at the minimum required levels. The 10,500 barrels of premium exceed the minimum requirement of 4,000 barrels by 6,500 barrels.*
I9	Midgrade Produced	=SUM(I7:I8)	
J9	Premium Produced	=SUM(J7:J8)	
K9	Total Gasoline Produced	=SUM(H9:J9)	*30,000 barrels are produced.*
H13	Profit from Alk. in Reg.	=(E13-E7)*H7	*There is actually a loss for blending alkylate into regular.*
H14	Profit from Alk. in Mid.	=(E14-E7)*I7	
H15	Profit from Alk. in Prem.	=(E15-E7)*J7	
I13	Profit from C.C. in Reg.	=(E13-E8)*H8	
I14	Profit from C.C. in Mid.	=(E14-E8)*I8	
I15	Profit from C.C. in Prem.	=(E15-E8)*J8	
K13	Total Profit from Alk.	=SUM(H13:J13)	*Profit from alkylate = $43,750*
K14	Total Profit from C.C.	=SUM(H14:J14)	*Profit from c.c. = $38,750*
H15	Total Profit from Reg.	=SUM(H13:H14)	*Profit from Reg. = $21,000*
I15	Total Profit from Mid.	=SUM(I13:I14)	*Profit from Mid. = $18,750*
J15	Total Profit from Prem.	=SUM(J13:J14)	*Profit from Prem. = $42,750*
K15	Total Profit (Maximized)	=SUM(H15:J15)	*Max. Total Profit = $82,500*
H19	Octane Rating Regular	=C7*(H7/H9)+C8*(H8/H9)	*Octane ratings are found by weighting the octane ratings of alkylate and cc by the proportion in each blended gasoline. Regular and mid-grade meet minimum octane ratings; the premium rating of 97.5 exceeds its minimum rating of 92.*
I19	Octane Rating Midgrade	=C7*(I7/I9)+C8*(I8/I9)	
J19	Octane Rating Premium	=C7*(J7/J9)+C8*(J8/J9)	
H20	Vapor Pressure Regular	=D7*(H7/H9)+D8*(H8/H9)	*Vapor pressure of a blended gasoline is found by weighting the vapor pressures of alkylate and cc by the proportion in each blended gasoline. Mid-grade is produced at its highest possible vapor pressure level. Regular and premium vapor pressures are less than their maximum allowable limits.*
I20	Vapor Pressure Midgrade	=D7*(I7/I9)+D8*(I8/I9)	
J20	Vapor Pressure Premium	=D7*(J7/J9)+D8*(J8/J9)	

Construction and Analysis of the Spreadsheet
Hidden Cells

The octane ratings and vapor pressures of the blended gasolines in cells H19:J20 were found by taking the weighted averages of the octane ratings and vapor pressures of alkylate and catalytic cracked blended into each grade. For example, as seen in Table 3.7, the formula in cell H19 is: =C7*(H7/H9)+C8*(H8/H9). But the denominator, cell H9, is the sum of the six decision variables in H7:J8, making the expression in cell H19 nonlinear in terms of the decision variables. There are similar formulas in cells H20, I19, I20, J19, and J20. Thus, if this nonlinear term were included in the left side of a constraint, a linear programming approach could not be used. That is why we wrote the mathematical model the way we did—to obtain a linear programming formulation!

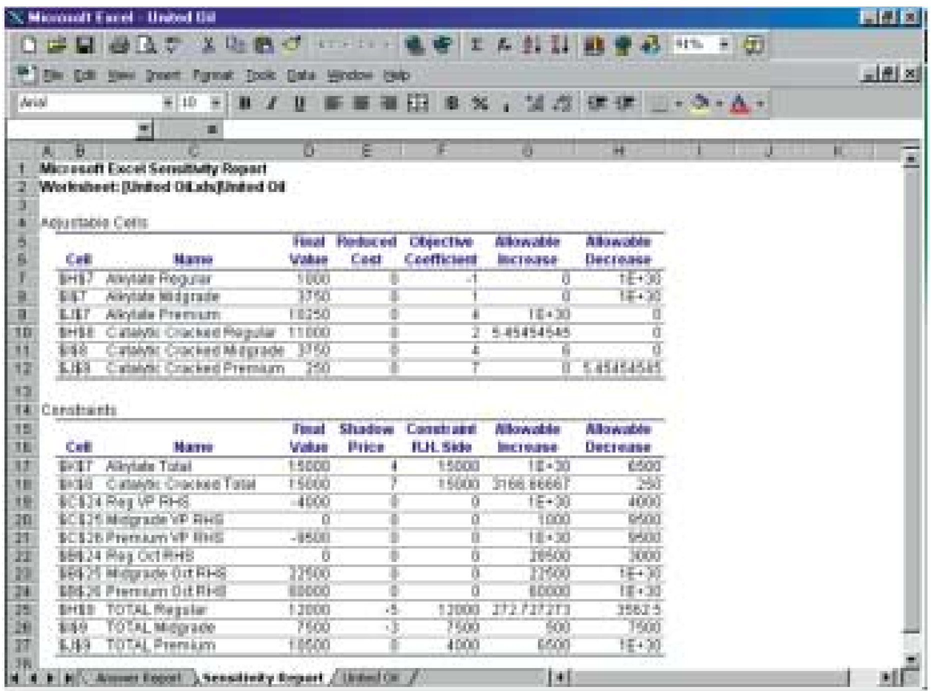
Accordingly, the formulas for the left side of the last six functional constraints of the model for the octane and vapor pressure constraints are entered into cells B24:B26 and C24:C26, respectively, as shown in Table 3.8. Although these values are required to be nonnegative for octane and nonpositive for vapor pressure, their precise values are meaningless and add nothing to the manager's analysis of the results. Thus, the corresponding rows were hidden in Figure 3.10*a* by holding down the left mouse key over the selected row numbers in the left margin and then clicking "Hide" with the right mouse key.

TABLE 3.8 Hidden Cells—Left-Hand Sides of the Last Six Functional Constraints

Left-Hand Side Values of the Last Six Functional Constraints (Hidden in Rows 24–26)			
Cell	**Left Side Quantity**	**Left-Hand Side**	**Spreadsheet Formula**
B24	Reg. Octane Rating	$98X_1+86Y_1-87R$	=SUMPRODUCT(C7:C8,H7:H8)-C13*H9
B25	Mid. Octane Rating	$98X_2+86Y_2-89M$	=SUMPRODUCT(C7:C8,I7:I8)-C14*I9
B26	Prem. Octane Rating	$98X_3+86Y_3-92P$	=SUMPRODUCT(C7:C8,J7:J8)-C15*J9
C24	Reg. Vapor Pressure	$5X_1+9Y_1-9R$	=SUMPRODUCT(D7:D8,H7:H8)-D13*H9
C25	Mid. Vapor Pressure	$5X_2+9Y_2-7M$	=SUMPRODUCT(D7:D8,I7:I8)-D14*I9
C26	Prem. Vapor Pressure	$5X_3+9Y_3-6P$	=SUMPRODUCT(D7:D8,J7:J8)-D15*J9

Analysis of the Sensitivity Report

Figure 3.10*b* shows the corresponding Sensitivity Report for this model.


Microsoft Excel Sensitivity Report
Worksheet: [United Oil.xls]United Oil

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$H$7	Alkylate Regular	1000	0	-1	0	1E+30
$I$7	Alkylate Midgrade	3750	0	1	0	1E+30
$J$7	Alkylate Premium	10250	0	4	1E+30	0
$H$8	Catalytic Cracked Regular	11000	0	2	5.45454545	0
$I$8	Catalytic Cracked Midgrade	3750	0	4	6	0
$J$8	Catalytic Cracked Premium	250	0	7	0	5.45454545

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$K$7	Alkylate Total	15000	4	15000	1E+30	6500
$K$8	Catalytic Cracked Total	15000	7	15000	3166.66667	250
$C$24	Reg VP RHS	-4000	0	0	1E+30	4000
$C$25	Midgrade VP RHS	0	0	0	1000	9500
$C$26	Premium VP RHS	-9500	0	0	1E+30	9500
$B$24	Reg Oct RHS	0	0	0	26500	3000
$B$25	Midgrade Oct RHS	22500	0	0	22500	1E+30
$B$26	Premium Oct RHS	60000	0	0	60000	1E+30
$H$9	TOTAL Regular	12000	-5	12000	272.727273	3562.5
$I$9	TOTAL Midgrade	7500	-3	7500	500	7500
$J$9	TOTAL Premium	10500	0	4000	6500	1E+30

FIGURE 3.10*b*
Sensitivity Report for United Oil

From this Sensitivity Report we observe the following.

Effects of Extra Crude All additional barrels of alkylate will add $4 each to the total profit. Additional barrels of catalytic cracked will add $7 each for each of the next 3166.67 barrels. That is, United should be willing to pay up to $19 + $4 = $23 for an additional barrel of alkylate and up to $16 + $7 = $23 for additional barrels of catalytic cracked.

Effects of Changing Grade Requirements Decreasing the requirement for regular from 12,000 barrels will increase profits by $5 per barrel up to a maximum decrease of 3562.5 barrels (to 8437.5 barrels); increasing this minimum requirement will decrease profits $5 per barrel, up to a maximum increase of 272.73 barrels (to 12,272.73 barrels). Changing the requirement for mid-grade will have a $3 effect for a maximum decrease of 7500 barrels (down to 0) or a maximum increase of 500 barrels (up to 8000). Changing the minimum requirement for premium would have no effect unless the increase puts the requirement above the proposed production level of 10,500 barrels.

Alternative Optimal Solutions There are Allowable Increases and Allowable Decreases of 0 for the profit coefficients of the various blends, indicating the existence of alternate optimal solutions. Thus, the crudes can be blended in other ways to make a profit of $82,500 while meeting the octane and vapor pressure requirements.

Other Linear Models

Sections 3.4.1–3.4.5 described just a few possibilities which might be solved using linear programming models. Space considerations have precluded us from presenting even more examples of linear models in this text. However, two other slightly more complex, but very important applications are presented on the accompanying CD-ROM. These models illustrate a multiperiod cash flow scheduling model (Appendix 3.2) and a model for evaluating the efficiency operations using a process known as data envelopment analysis (Appendix 3.3). Since these models illustrate new ideas and spreadsheet approaches, you are encouraged to access them from the Appendix folder on the CD-ROM.

3.5 Applications of Integer Linear Programming Models

In many real-life models, at least one of the decision variables is required to be integer-valued. If all the variables are required to be integer, the model is called an **all-integer linear programming model (AILP)** and if all are required to be binary (values of 0 or 1), the model is called a **binary integer linear programming model (BILP)**. If some of the variables are required to be either integer or binary whereas others have no such restriction, the model is called a **mixed integer linear programming model (MILP)**. In this section we present several such models including a personnel scheduling (AILP), a project selection model (BILP), a supply chain model (MILP), and an advertising model (AILP) on the accompanying CD-ROM.

We observed in Chapter 2 that to convert a linear programming model to an integer programming model in an Excel spreadsheet only involves a mouse click in the Add Constraint dialogue box of Solver. We also stated, however, that when integer variables are present, the solution time can increase dramatically and no sensitivity output is generated. Thus, rounding a linear programming solution is sometimes a preferred option.

Using Binary Variables

Appropriate use of binary variables can aid the modeler in expressing comparative relationships. To illustrate, suppose Y_1, Y_2, and Y_3 are binary variables representing whether each of three plants should be built ($Y_i = 1$) or not built ($Y_i = 0$). The following relationships can then be expressed by these variables. (You can verify these relationships by substituting all combinations of 0's and 1's into the given constraints.)

- At least two plants must be built. $Y_1 + Y_2 + Y_3 \geq 2$
- If plant 1 is built, Plant 2 must not be built. $Y_1 + Y_2 \leq 1$
- If plant 1 is built, Plant 2 must be built. $Y_1 - Y_2 \leq 0$
- One but not both of Plants 1 and 2 must be built. $Y_1 + Y_2 = 1$
- Both or neither of Plants 1 and 2 must be built. $Y_1 - Y_2 = 0$
- Plant construction cannot exceed $17 million, and the costs to build Plants 1, 2, and 3 are $5 million, $8 million, and $10 million, respectively. $5Y_1 + 8Y_2 + 10Y_3 \leq 17$

Binary variables can also be used to indicate restrictions in certain conditional situations. For example, suppose X_1 denotes the amount of a product that will be produced at Plant 1. (Note that $X_1 \geq 0$.) If Plant 1 is built, there is no other restriction on the value of X_1, but if it is not built, X_1 must be 0. This relation can be expressed by:

$$X_1 \leq MY_1$$

In this expression, M denotes an extremely large number that does not restrict the value of X_1 if $Y_1 = 1$. For example we might use 10^{20} (or 1E + 20) for M. If Plant 1 is not built ($Y_1 = 0$), the constraint becomes $X_1 \leq 0$; however, since $X_1 \geq 0$, this implies $X_1 = 0$; that is, no product will be produced at Plant 1. If Plant 1 is built ($Y_1 = 1$), then $X_1 \leq M$, which, because of the extremely large value assigned to M, effectively does restrict the value of X_1.

Now suppose that we are considering building a new plant in Chicago to produce two products, bicycles and tricycles. Suppose each bicycle requires 3 pounds of steel and each tricycle 4 pounds of steel. If the plant is built, it should have 2000 pounds of steel available per week. Thus, there will be at most 2000 pounds of steel if the plant is built but 0 pounds of steel available if it is not built. Define:

X_1 = the number of bicycles produced each week at the Chicago plant
X_2 = the number of tricycles produced each week at the Chicago plant

Now let Y_1 represent whether or not the Chicago plant is built. This situation can then be modeled as: $3X_1 + 4X_2 \leq 2000Y_1$ or, $3X_1 + 4X_2 - 2000Y_1 \leq 0$. We see that if the plant is built ($Y_1 = 1$), then the constraint is $3X_1 + 4X_2 \leq 2000$. If it is not built ($Y_1 = 0$), the constraint reduces to $3X_1 + 4X_2 \leq 0$ (which will hold only if both X_1 and X_2 are 0). That is, if the plant is not built, there is no production.

These are just some of the ideas that are modeled in the examples in this section.

3.5.1 PERSONNEL SCHEDULING MODELS

One problem that requires an integer solution is the assignment of personnel or machines to meet some minimum coverage requirements. Typically, these models have constraints that link the resources available during one period with those available for subsequent periods. The situation faced by the City of Sunset Beach is an example of one such problem.

SUNSET BEACH LIFEGUARD ASSIGNMENTS

Concepts: Integer Variables
Linking Constraints
Hidden Cells

Sunset.xls

In the summer, the City of Sunset Beach staffs lifeguard stations seven days a week. Regulations require that city employees (including lifeguards) work five days a week and be given two consecutive days off. For most city employees, these days are Saturday and Sunday, but for lifeguards, these are the two busiest days of the week.

Insurance requirements mandate that Sunset Beach provide at least one lifeguard per 8000 average daily attendance on any given day. Table 3.9 gives the average daily attendance figures and the minimum number of lifeguards required during the summer months, at Sunset Beach.

Given the current budget situation, Sunset Beach would like to determine a schedule that will employ as few lifeguards as possible.

TABLE 3.9 Average Daily Attendance and Lifeguard Requirements

Day	Average Attendance	Lifeguards Required
Sunday	58.000	8
Monday	42,000	6
Tuesday	35,000	5
Wednesday	25,000	4
Thursday	44,000	6
Friday	51,000	7
Saturday	68,000	9

SOLUTION

Sunset Beach's problem is to:

- Schedule lifeguards over five consecutive days
- Minimize the total number of lifeguards required
- Meet the minimum daily lifeguard requirements

DECISION VARIABLES

Sunset Beach must decide how many lifeguards to schedule beginning Sunday and working for five consecutive days, the number to schedule beginning Monday and working for five consecutive days, and so on:

X_1 = number of lifeguards scheduled to begin on Sunday
X_2 = number of lifeguards scheduled to begin on Monday
X_3 = number of lifeguards scheduled to begin on Tuesday
X_4 = number of lifeguards scheduled to begin on Wednesday
X_5 = number of lifeguards scheduled to begin on Thursday
X_6 = number of lifeguards scheduled to begin on Friday
X_7 = number of lifeguards scheduled to begin on Saturday

OBJECTIVE FUNCTION

The goal is to minimize the total number of lifeguards scheduled:

$$\text{MIN } X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7$$

CONSTRAINTS

For each day, at least the minimum required number of lifeguards must be on duty. Those on duty on Sunday begin their shift either on Sunday, Wednesday, Thursday, Friday, or Saturday; those on duty on Monday begin their shift either on Monday, Thursday, Friday, Saturday, or Sunday; and so on. Thus,

$$\text{(The number of lifeguards on duty Sunday)} \geq 8$$

or

$$X_1 \qquad + X_4 + X_5 + X_6 + X_7 \geq 8$$

For Monday the constraint would be:

$$X_1 + X_2 \qquad + X_5 + X_6 + X_7 \geq 6$$

THE MATHEMATICAL MODEL

The constraints for the other days are similarly derived, yielding the following model:

$$
\begin{array}{llllllllll}
\text{MIN} & X_1 & +X_2 & +X_3 & +X_4 & +X_5 & +X_6 & +X_7 & & \\
\text{ST} & & & & & & & & & \\
 & X_1 & & & +X_4 & +X_5 & +X_6 & +X_7 & \geq 8 & \text{(Sunday)} \\
 & X_1 & +X_2 & & & +X_5 & +X_6 & +X_7 & \geq 6 & \text{(Monday)} \\
 & X_1 & +X_2 & +X_3 & & & +X_6 & +X_7 & \geq 5 & \text{(Tuesday)} \\
 & X_1 & +X_2 & +X_3 & +X_4 & & & +X_7 & \geq 4 & \text{(Wednesday)} \\
 & X_1 & +X_2 & +X_3 & +X_4 & +X_5 & & & \geq 6 & \text{(Thursday)} \\
 & & X_2 & +X_3 & +X_4 & +X_5 & +X_6 & & \geq 7 & \text{(Friday)} \\
 & & & X_3 & +X_4 & +X_5 & +X_6 & +X_7 & \geq 9 & \text{(Saturday)} \\
\end{array}
$$

All variables ≥ 0 AND Integer

EXCEL INPUT/OUTPUT AND ANALYSIS

Figure 3.11 shows a spreadsheet for this model. The formula in cell B5 (=ROUNDUP(B4/8000,0) gives the number of required lifeguards on Sunday by dividing the projected attendance (B4) by the 8000 lifeguard to attendance ratio and rounding this number up. The 0 means include 0 decimal places. This formula is dragged to cells C5:H5 to obtain the daily requirements.

Sunset.xls

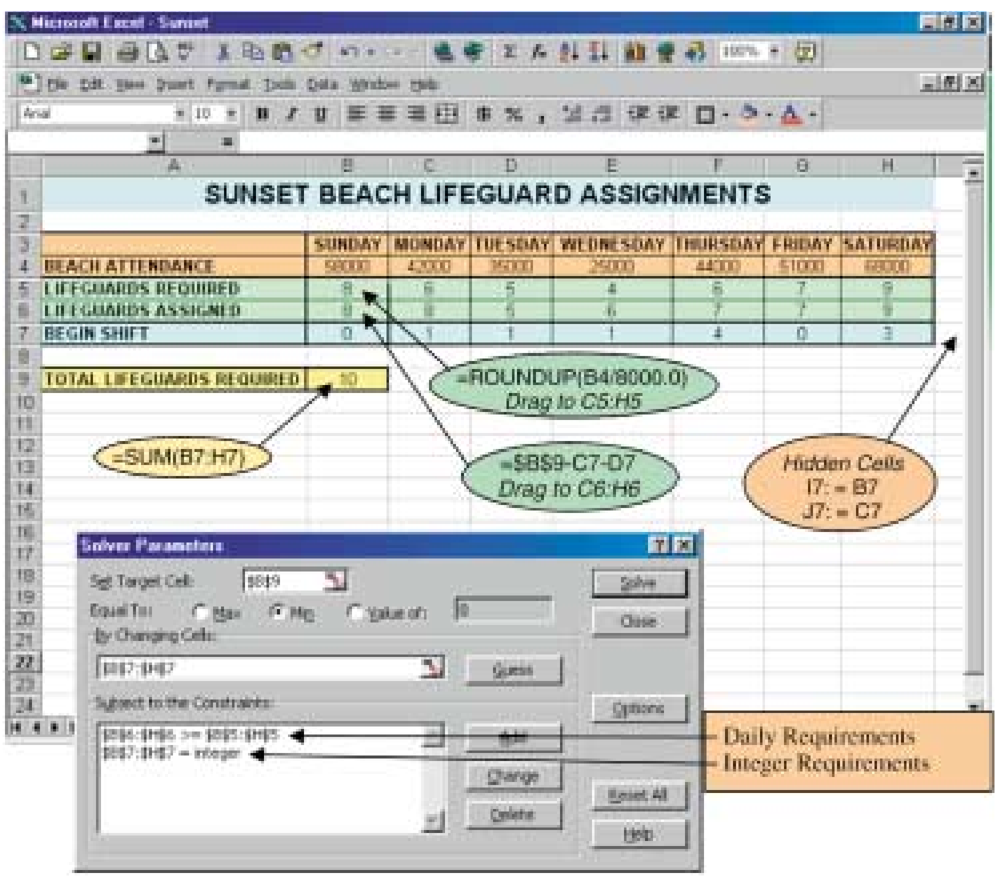


FIGURE 3.11 Optimal Spreadsheet for Sunset Beach Lifeguard Assignments

The formula in cell B6 (=$B$9-C7-D7) states that the number of lifeguards on duty on Sunday includes all lifeguards ($B$9) except those who begin their shift on Monday (C7) or Tuesday (D7). Those who begin their shift on Monday or Tuesday will finish their shifts on Friday and Saturday, respectively. So that we can drag this formula across to cells C6:H6, formulas have been assigned to (hidden) cells I7 and J7; they also give the number of lifeguards who begin their shift on Sunday and Monday.

We see that the city can get by with a total of 10 lifeguards, 1 of whom starts his shift on Monday, 1 on Tuesday, 1 on Wednesday, 4 on Thursday, and 3 on Saturday.[4] The minimum requirement for lifeguards is met on each day, with Monday and Wednesday having two extra lifeguards and Thursday one extra lifeguard assigned.

3.5.2 PROJECT SELECTION MODELS

Project selection models involve a set of "go/no-go" decisions, represented by binary decision variables, for various projects under consideration. Such models typically involve budget, space, or other restrictions, as well as a set of priorities among certain projects. For example, one might specify that project 1 may be done (or will be done) only if project 2 is done, or only if project 3 is not done, or that at least two of projects 1, 2, 3, 4, and 5 be accomplished. The situation faced by the Salem City Council is a simplified version of one such model.

SALEM CITY COUNCIL

Concepts: Binary Decision Variables
Priority Relationships

Salem.xls

At its final meeting of the fiscal year, the Salem City Council will be making plans to allocate funds remaining in this year's budget. Nine projects have been under consideration throughout the entire year.

To gauge community support for the various projects, questionnaires were randomly mailed to voters throughout the city asking them to rank the projects (9 = highest priority, 1 = lowest priority). The council tallied the scores from the 500 usable responses it received. Although the council has repeatedly maintained that it will not be bound by the results of the questionnaire, it plans to use this information while taking into account other concerns when making the budget allocations.

The estimated cost of each project, the estimated number of permanent new jobs each would create, and the questionnaire point tallies are summarized in Table 3.10.

The council's goal is to maximize the total perceived voter support (as evidenced through the questionnaires), given other constraints and concerns of the council, including the following:

- $900,000 remains in the budget.
- The council wants to create at least 10 new jobs.
- Although crime deterrence is a high priority with the public, the council feels that it must also be fair to other sectors of public service (fire and education). Accordingly, it wishes to fund at most three of the police-related projects.
- The council would like to increase the number of city emergency vehicles but feels that, in the face of other pressing issues, only one of the two emergency vehicle projects should be funded at this time. Thus, *either* the two police cars *or* the fire truck should be purchased.

[4] It turns out that there are several optimal solutions to this model, each of which requires a total of 10 lifeguards.

- The council believes that if it decides to restore funds cut from the sports programs at the schools, it should also restore funds cut from their music programs, and vice versa.
- By union contract, any additional school funding must go toward restoring previous cuts before any new school projects are undertaken. Consequently, both sports funds and music funds must be restored before new computer equipment can be purchased. Restoring sports and music funds, however, does not imply that new computers *will* be purchased, only that they *can* be.

TABLE 3.10 Project Costs, New Jobs, and Point Tallies

	Project	Cost ($1000)	New Jobs	Points
X_1	Hire seven new police officers	$400	7	4176
X_2	Modernize police headquarters	$350	0	1774
X_3	Buy two new police cars	$ 50	1	2513
X_4	Give bonuses to foot patrol officers	$100	0	1928
X_5	Buy new fire truck/support equipment	$500	2	3607
X_6	Hire assistant fire chief	$ 90	1	962
X_7	Restore cuts to sports programs	$220	8	2829
X_8	Restore cuts to school music	$150	3	1708
X_9	Buy new computers for high school	$140	2	3003

SOLUTION

The Salem City Council must choose which projects to fund. Its objective is to determine, within the constraints and concerns listed earlier, the set of projects that maximizes public support for its decisions as evidenced through the returned questionnaires.

DECISION VARIABLES

The variables, $X_1, X_2, \ldots, X_9$, are binary decision variables: $X_j = 1$ if project j is funded, and $X_j = 0$ if project j is not funded.

OBJECTIVE FUNCTION

The council's objective is to maximize the overall point score of the funded projects:

$$\text{MAXIMIZE} \quad 4176X_1 + 1774X_2 + 2513X_3 + 1928X_4 + 3607X_5 + 962X_6 + 2829X_7 + 1708X_8 + 3003X_9$$

CONSTRAINTS

Budget Constraint The maximum amount of funds to be allocated cannot exceed $900,000. Using coefficients to represent the number of thousands of dollars, this constraint can be written as:

$$400X_1 + 350X_2 + 50X_3 + 100X_4 + 500X_5 + 90X_6 + 220X_7 + 150X_8 + 140X_9 \le 900$$

Job Creation Constraint The number of new jobs created must be at least 10:

$$7X_1 + X_3 + 2X_5 + X_6 + 8X_7 + 3X_8 + 2X_9 \ge 10$$

Maximum of Three Out of Four Police Projects Constraint The number of police-related activities to be funded is at most 3:

$$X_1 + X_2 + X_3 + X_4 \leq 3$$

Mutually Exclusive Projects Constraint (Two Police Cars or a Fire Truck) Either the two police cars should be purchased or the fire truck should be purchased. This is equivalent to saying that the number of police car purchase projects plus the number of fire truck purchase projects to be funded is exactly 1:

$$X_3 + X_5 = 1$$

Corequisite Projects Constraint—Sports Funding/Music Funding If sports funds are restored music funds will be restored, and if sports funds are not restored music funds will not be restored. This constraint implies that the number of restored music fund projects funded must equal the number of sports fund projects funded; that is $X_7 = X_8$, or:

$$X_7 - X_8 = 0$$

Prerequisite Projects Constraint—Equipment vs. Sports and Music Funding Sports funding and music funding must be restored before new computer equipment can be purchased. This relationship can be expressed as two prerequisite constraints: the number of sports projects funded must be at least as great as the number of computer equipment projects funded ($X_7 \geq X_9$) AND the number of music projects funded must be at least as great as the number of computer equipment projects funded ($X_8 \geq X_9$), or:

$$X_7 - X_9 \geq 0$$
$$X_8 - X_9 \geq 0$$

Note that, taken together, these constraints mean that if $X_9 = 1$, then both X_7 and X_8 must be 1, but if either or both X_7 and $X_8 = 1$, X_9 is not required to be 1.

MATHEMATICAL MODEL FOR THE SALEM CITY COUNCIL

The complete model for the Salem City Council, which includes the objective function, the functional and conditional constraints, and the binary restrictions, can now be stated as follows:

MAXIMIZE

$$4176X_1 + 1774X_2 + 2513X_3 + 1928X_4 + 3607X_5 + 962X_6 + 2829X_7 + 1708X_8 + 3003X_9$$

ST

$$\begin{array}{rrrrrrrrrcr}
400X_1 + & 350X_2 + & 50X_3 + & 100X_4 + & 500X_5 + & 90X_6 + & 220X_7 + & 150X_8 + & 140X_9 & \leq & 900 \\
7X_1 + & & X_3 + & & 2X_5 + & X_6 + & 8X_7 + & 3X_8 + & 2X_9 & \geq & 10 \\
X_1 + & X_2 + & X_3 + & X_4 & & & & & & \leq & 3 \\
& & X_3 + & & X_5 & & & & & = & 1 \\
& & & & & & X_7 - & X_8 & & = & 0 \\
& & & & & & X_7 - & & X_9 & \geq & 0 \\
& & & & & & & X_8 - & X_9 & \geq & 0
\end{array}$$

All X's = 0 or 1

Excel Input/Output and Analysis

Figure 3.12 shows an optimal spreadsheet for the decisions faced by the Salem City Council. The binary decision variables are in cells B4:B12, and we present the formulas for the left side of the constraints shown in cells B17:B23.

Salem.xls

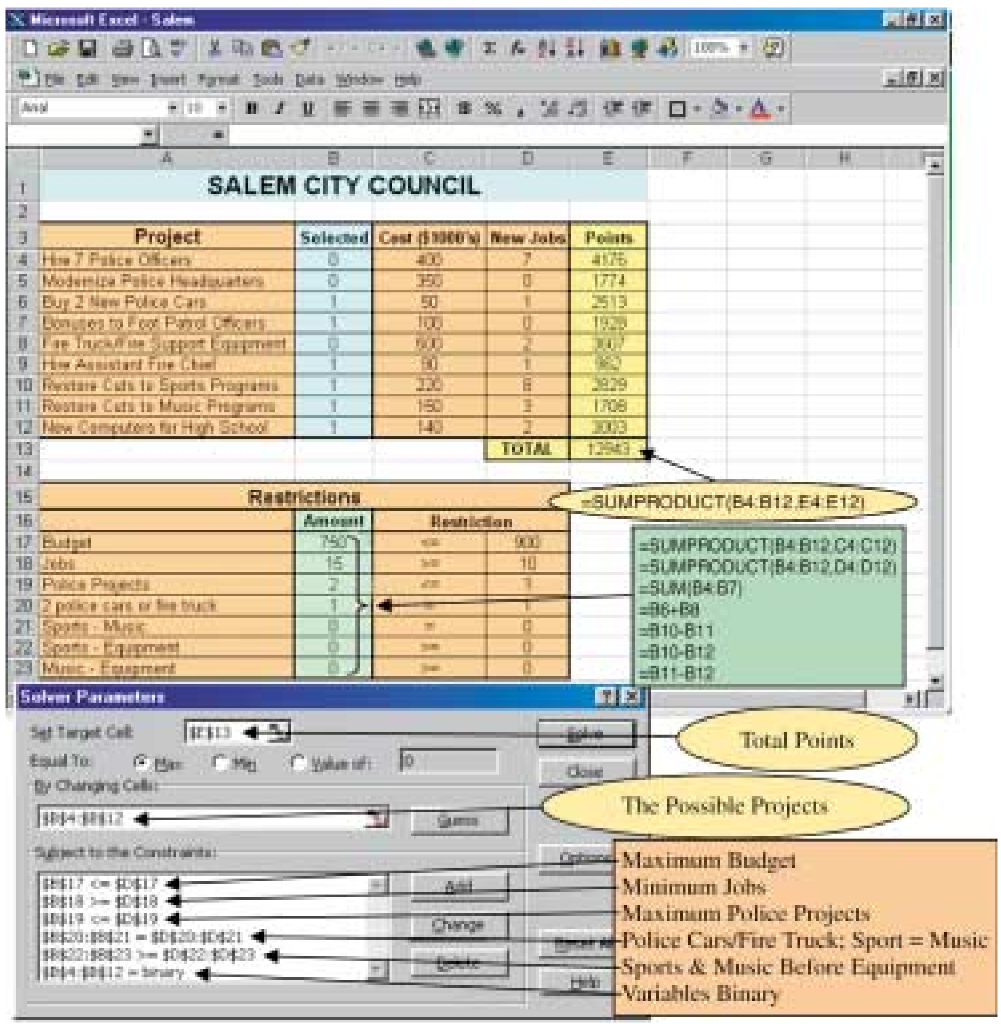


FIGURE 3.12 Optimal Spreadsheet for the Salem City Council

Much to the council's surprise, the optimal solution does not fund the two items the people most wanted—hiring seven new police officers and purchasing a new fire truck and fire support equipment! Upon further observation, the council noted that these items were also the most costly, but they were still amazed that neither would be funded given the extremely high public support for them. The solution also does not recommend funding renovations to police headquarters; instead it recommends funding all six other projects. If this recommendation is followed, the council will create five more jobs than its goal of 10 and will have a budget surplus of $1,500,000 that it can apply to next year's projects or return to the people as a tax rebate (a very popular political idea!)

3.5.3 SUPPLY CHAIN MANAGEMENT MODELS

One of the most significant managerial developments in recent years has been the emergence of supply chain management models that integrate the process of manufacturing goods and getting them to the consumer. A typical supply chain can be thought of as a decision support system that treats the acquisition of materials to produce products as well as the manufacturing, storing, and shipping of finished products as an integrated system of events rather than as stand-alone separate components of the process.

Numerous management science models discussed in this text, including manufacturing models, network models, scheduling models, forecasting models, inventory models, and queuing models, are now embraced under the ever growing set of supply chain management techniques. The overall objective of these models has been, and continues to be, to minimize total system costs while maintaining appropriate production levels and transporting needed quantities to the right locations in a timely and efficient manner. While just a few short years ago, employing efficient supply chain models gave many firms an edge in the market, today their use has nearly become essential to even compete in the marketplace.

One link in the supply chain can involve determining which plants should be made operational, which should produce specified items, and what shipping pattern should be used to distribute the finished products to retailers. Such is the situation faced by Globe Electronics, Inc.

GLOBE ELECTRONICS, INC.

Concepts: Supply Chain Management
Mixed Integer Modeling of Fixed Charges
Rounding Noninteger Solutions

Globe.xls
Globe Plant Analysis.xls

Globe Electronics, Inc. manufactures two styles of remote control cable boxes (the G50 and the H90) that various cable companies supply to their customers when cable service is established. Different companies require different models. During the late 1980s and early 1990s, due to an explosion in the demand for cable services, Globe expanded rapidly to four production facilities located in Philadelphia (the original plant), St. Louis, New Orleans, and Denver. The manufactured items are shipped from the plants to regional distribution centers located in Cincinnati, Kansas City, and San Francisco; from these locations they are distributed nationwide.

Because of a decrease in demand for cable services and technological changes in the cable industry, demand for Globe's products is currently far less than the total of the capacities at its four plants. As a result, management is contemplating closing one or more of its facilities.

Each plant has a fixed operating cost, and, because of the unique conditions at each facility, the production costs, production time per unit, and total monthly production time available vary from plant to plant, as summarized in Table 3.11.

The cable boxes are sold nationwide at the same prices: $22 for the G50, and $28 for the H90.

Current monthly demand projections at each distribution center for both products are given in Table 3.12.

To remain viable in each market, Globe must meet at least 70% of the demand for each product at each distribution center. The transportation costs between each plant and each distribution center, which are the same for either product, are shown in Table 3.13.

Globe management wants to develop an optimal distribution policy utilizing all four of its operational plants. It also wants to determine whether closing any of the production facilities will result in higher company profits.

TABLE 3.11 Production Costs, Times, Availability

Plant	Fixed Cost/ Month ($1000)	Production Cost Per Unit		Production Time (Hr./Unit)		Available Hours per Month
		G50	H90	G50	H90	
Philadelphia	40	10	14	.06	.06	640
St. Louis	35	12	12	.07	.08	960
New Orleans	20	8	10	.09	.07	480
Denver	30	13	15	.05	.09	640

TABLE 3.12 Monthly Demand Projections

	Demand		
	Cincinnati	Kansas City	San Francisco
G50	2000	3000	5000
H90	5000	6000	7000

TABLE 3.13 Transportation Costs per 100 Units

	To		
	Cincinnati	Kansas City	San Francisco
From			
Philadelphia	$200	$300	$500
St. Louis	$100	$100	$400
New Orleans	$200	$200	$300
Denver	$300	$100	$100

SOLUTION

The situation facing Globe Electronics is the portion of the supply chain that involves the manufacture and delivery of finished products to various distribution centers. Prior to this, Globe would be involved with ordering raw materials and scheduling personnel in the production process. Subsequent links would involve the storage process at the distribution centers and the sale and dissemination of the completed goods to retail establishments. Specifically for this portion of the model, Globe is seeking to:

- Determine the number of G50 and H90 cable boxes to be produced at each plant
- Determine a shipping pattern from the plants to the distribution centers
- Maximize net total monthly profit
- Not exceed the production capacities at any plant
- Ensure that each distribution center receives between 70% and 100% of its monthly demand projections

The model Globe uses to solve for the optimal solution with all four plants operating is developed in the following section.

DECISION VARIABLES

Management must decide the total number of G50 and H90 cable boxes to produce monthly at each plant, the total number to be shipped to each distribution center, and the shipping pattern of product from the plants to the distribution centers. The entries in the following two matrices designate the decision variables for this model.

Shipment of G50 Cable Boxes:

	Cincinnati	Kansas City	San Francisco	Total Produced
Philadelphia	G_{11}	G_{12}	G_{13}	G_P
St. Louis	G_{21}	G_{22}	G_{23}	G_{SL}
New Orleans	G_{31}	G_{32}	G_{33}	G_{NO}
Denver	G_{41}	G_{42}	G_{43}	G_D
Total Received	G_C	G_{KC}	G_{SF}	G

Shipment of H90 Cable Boxes:

	Cincinnati	Kansas City	San Francisco	Total Produced
Philadelphia	H_{11}	H_{12}	H_{13}	H_P
St. Louis	H_{21}	H_{22}	H_{23}	H_{SL}
New Orleans	H_{31}	H_{32}	H_{33}	H_{NO}
Denver	H_{41}	H_{42}	H_{43}	H_D
Total Received	H_C	H_{KC}	H_{SF}	H

OBJECTIVE FUNCTION

The *gross profit* (exclusive of fixed plant costs) is given by \$22 (Total G50's Produced) + \$28 (Total H90's Produced) − (Total Production Cost) − (Total Transportation Costs). Thus, the objective function is:

$$\begin{aligned}\text{MAX } & 22G + 28H - 10G_P - 12G_{SL} - 8G_{NO} - 13G_D - 14H_P - 12H_{SL} - \\ & 10H_{NO} - 15H_D - 2G_{11} - 3G_{12} - 5G_{13} - 1G_{21} - 1G_{22} - 4G_{23} - 2G_{31} - \\ & 2G_{32} - 3G_{33} - 3G_{41} - 1G_{42} - 1G_{43} - 2H_{11} - 3H_{12} - 5H_{13} - 1H_{21} - \\ & 1H_{22} - 4H_{23} - 2H_{31} - 2H_{32} - 3H_{33} - 3H_{41} - 1H_{42} - 1H_{43}\end{aligned}$$

From this quantity we would subtract the total monthly fixed costs for the four plants of \$125,000.

CONSTRAINTS

This model contains summation constraints for the total amounts of G50 and H90 cable boxes produced at each plant and the total number shipped to each distribution center, production time limits at the plants, and shipping limits to the distribution plants.

1. *Summation Constraints for Total Production*

	Total G50's Produced	***Total H90's Produced***
Philadelphia:	$G_{11} + G_{12} + G_{13} = G_P$	$H_{11} + H_{12} + H_{13} = H_P$
St. Louis:	$G_{21} + G_{22} + G_{23} = G_{SL}$	$H_{21} + H_{22} + H_{23} = H_{SL}$
New Orelans:	$G_{31} + G_{32} + G_{33} = G_{NO}$	$H_{31} + H_{32} + H_{33} = H_{NO}$
Denver:	$G_{41} + G_{42} + G_{43} = G_D$	$H_{41} + H_{42} + H_{43} = H_D$
TOTAL:	$G_P + G_{SL} + G_{NO} + G_D = G$	$H_P + H_{SL} + H_{NO} + H_D = H$

2. *Summation Constraints for Total Shipments*

	Total G50's Shipped	***Total H90's Shipped***
Cincinnati:	$G_{11} + G_{21} + G_{31} + G_{41} = G_C$	$H_{11} + H_{21} + H_{31} + H_{41} = H_C$
Kansas City:	$G_{12} + G_{22} + G_{32} + G_{42} = G_{KC}$	$H_{12} + H_{22} + H_{32} + H_{42} = H_{KC}$
San Francisco:	$G_{13} + G_{23} + G_{33} + G_{43} = G_{SF}$	$H_{13} + H_{23} + H_{33} + H_{43} = H_{SF}$

3. *Production Time Limits at Each Plant*

Philadelphia:	$.06G_P + .06H_P \leq 640$
St. Louis:	$.07G_{SL} + .08H_{SL} \leq 960$
New Orleans:	$.09G_{NO} + .07H_{NO} \leq 480$
Denver:	$.05G_D + .09H_D \leq 640$

4. *Minimum Amount Shipped to Each Distribution Center ≥70% (Total Demand); Maximum Amount Shipped to Each Distribution Center ≤(Total Demand)*

	Minimum Shipment	***Maximum Shipment***
Cincinnati:	$G_C \geq 1400$	$G_C \leq 2000$
	$H_C \geq 3500$	$H_C \leq 5000$
Kansas City:	$G_{KC} \geq 2100$	$G_{KC} \leq 3000$
	$H_{KC} \geq 4200$	$H_{KC} \leq 6000$
San Francisco:	$G_{SF} \geq 3500$	$G_{SF} \leq 5000$
	$H_{SF} \geq 4900$	$H_{SF} \leq 7000$

5. *Nonnegativity*

$$\text{All G's and H's} \geq 0$$

Theoretically, we should also require that the variables be integers, but we will ignore this restriction and round if necessary. This will substantially reduce the solution time. The rounding could result in a slightly less than optimal result, or it might slightly violate one of the constraints. But in the context of this problem, such minor violations would probably be acceptable.

EXCEL INPUT/OUTPUT AND ANALYSIS

Figure 3.13 shows a spreadsheet and the resulting solution for this model. In this figure cells F5:F9 and F14:F18 contain the row sums giving the total G50 and H90 production at the plants. Cells C9:E9 and C18:E18 contain the column sums giving the total shipments to the distribution centers. Hidden cells C29:E29 and C30:E30 contain formulas giving 70% of the G50 and H90 demand at the distribution centers respectively. The objective function formula in cell K19 is the total revenue of G50's + the total revenue of H90's less the total production costs of G50's, the total production costs of H90's, the total shipping costs of G50's, the total shipping costs of H90's, and the fixed costs of operating each plant.

Globe.xls

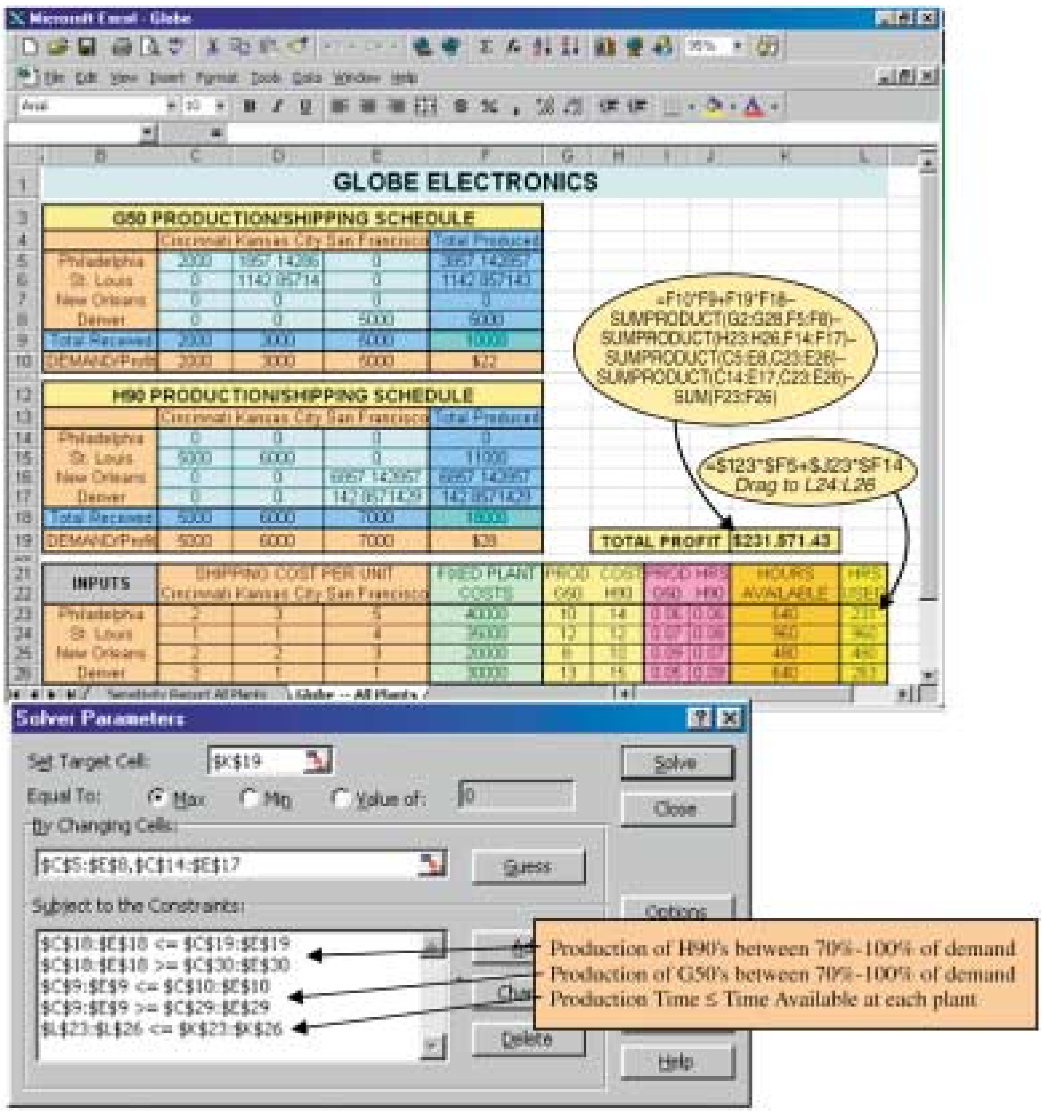


FIGURE 3.13
Optimal Spreadsheet for Globe Electronics with All Plants Operational

When the model is solved, we see that the optimal solution contains noninteger values in cells D5, D6, E16, and E17. But if these values are simply rounded down, the result is a feasible solution with a net monthly profit reduced to \$231,550. Because this is so close to the optimal value for the linear programming model of \$231,571.43 shown in cell K19, while it may not be the exact optimal integer solution, it is at least very close to it!

The preceding solution assumes that all plants are operational. However, because of the large fixed cost component at each plant, this may not be the best overall solution. As part of the supply chain model, Globe should consider which plants it wishes to keep operational.

Using Binary Variables to Model Fixed Charge Components

Whether or not each plant remains operational can be expressed by using the following binary variables.

$$
\begin{aligned}
Y_P &= \text{number of operational Philadelphia plants}\\
Y_{SL} &= \text{number of operational St. Louis plants}\\
Y_{NO} &= \text{number of operational New Orleans plants}\\
Y_D &= \text{number of operational Denver plants}
\end{aligned}
$$

The fixed operating costs can be accounted for in the objective function by subtracting from the previous objective function the expression: $40{,}000Y_P + 35{,}000Y_{SL} + 20{,}000Y_{NO} + 30{,}000Y_D$.

The production constraints are modified as follows:

$$
\begin{aligned}
.06G_P &+ .06H_P \le 640Y_P\\
.07G_{SL} &+ .08H_{SL} \le 960Y_{SL}\\
.09G_{NO} &+ .07H_{NO} \le 480Y_{NO}\\
.05G_D &+ .09H_D \le 640Y_D
\end{aligned}
$$

Thus if, for instance, the Philadelphia plant is closed ($Y_P = 0$), the first constraint will force total production at the Philadelphia plant to be 0. This in turn implies that all shipments from the Philadelphia plant would also be 0.

The revised spreadsheet model is shown in Figure 3.14. Note that the binary decision variables are in cells A5:A8. These values are copied to cells A14:A17 and to cells A23:A26. Hidden cell N23 contains the formula K23*A5, which is dragged to N24:N26. These cells give the actual production hour availability depending on whether or not the corresponding plant is operational. This allows for easy modification of the last entry in the Solver dialogue box.

Although we require binary variables, we will not require that the shipping variables be integers. If you try it, you will see that this would increase the solution time from a couple of seconds to many minutes, if not hours!

From Figure 3.17 we see that this supply chain model is optimized by closing the Philadelphia plant, running the other three plants at capacity, and scheduling monthly production according to quantities (rounded down) shown in the spreadsheet.[5] The rounded down solution gives a net monthly profit of \$266,083 (again very close to the linear programming value of \$266,114.91). This is \$266,083 − \$231,550 = \$34,533 per month greater than the optimal monthly profit with all four plants operational, resulting in an annual increase in profit of 12(\$34,533) = \$414,396!

A MANAGEMENT REPORT

The results from this supply chain model and the one used to solve the problem with no plant closures can form the basis for a management report. One item of interest to management may be a breakdown of the distribution of costs and production time under each plan so that Globe can determine where the major costs lie. In addition, management may wish to explore further options. These issues are addressed in the memorandum to Globe Electronics on the following page.

OTHER INTEGER MODELS

Numerous other situations lend themselves to integer programming formulations. One such application, dealing with the selection of advertising media, is discussed in Appendix 3.4 in the Appendix folder on the accompanying CD-ROM. Since several formulation and spreadsheet concepts are illustrated by this example, the reader is encouraged to read and study this model.

[5] Note that the entry in cell C5 is 9.6E-09 = .0000000096. This is roundoff error; it should be 0.

Globe Plant Analysis.xls

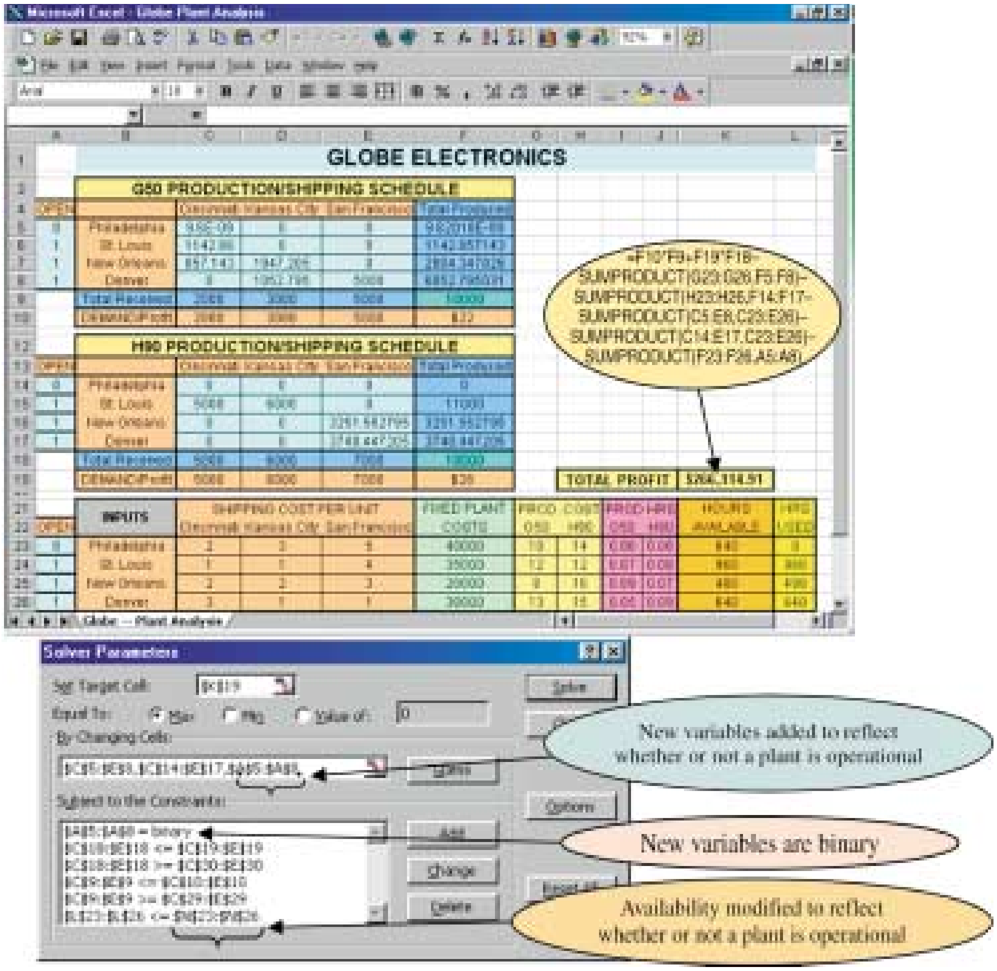


FIGURE 3.14 Optimal Spreadsheet for Globe Electronics Allowing for Closure of Plants

·SCG·
STUDENT CONSULTING GROUP

MEMORANDUM

To: Carol Copley, Vice President
Globe Electronics, Inc.

From: Student Consulting Group

Subj: Recommendation for Monthly Operations

We have been asked to evaluate plant production of the G50 and H90 cable boxes manufactured at the Philadelphia, St. Louis, New Orleans, and Denver plants. Recent product demand projections for the coming year from the Cincinnati, Kansas City, and San Francisco distribution centers have dropped to such a point that a substantial amount of unused production time is available at the plants. Given the large fixed plant operating costs, we have been asked to evaluate the feasibility and the potential cost savings of closing one or more of the plants.

In our analysis we assumed that the demand forecast for the upcoming year, as shown in Table I, is an accurate reflection of future sales.

Table I Monthly Demand Forecasts for the Next Fiscal Year

	Cincinnati	Kansas City	San Francisco
G50	2000	3000	5000
H90	5000	6000	7000

Based on production time data and the information in Table I, we developed profit maximization models for your situation. These models assume a $22 and $28 selling price for G50 and H90 models, respectively, and take into account the following:

1. The fixed operating cost at each plant
2. The variable production costs associated with each product at each plant
3. The unit transportation costs of shipping cable boxes from the plants to the distribution centers
4. Production not exceeding demand

Management's imposed condition that at least 70% of the demand for each product be supplied to each distribution center did not turn out to be a limiting factor in the analysis.

OPTIMAL PRODUCTION SCHEDULES

Given the current situation at the four plants in operation, Table II gives a production schedule that should maximize Globe's total net monthly profit.

Table II Production/Transportation Schedule: All Plants Operational

Plant	Product	Amount	Cincinnati	Kansas City	San Francisco
Philadelphia	G50	3857	2000	1857	
	H90	0			
St. Louis	G50	1143		1143	
	H90	11000	5000	6000	
New Orleans	G50	0			
	H90	6857			6857
Denver	G50	5000			5000
	H90	143			143
Total	G50	10000	2000	3000	5000
	H90	18000	5000	6000	7000

Notice that, although this production plan meets the full demand at the distribution centers, no G50 models are produced in New Orleans, no G90 models are produced in Philadelphia, and only 143 H90 models are produced monthly at the Denver plant. As a result, there is considerable excess capacity at both the Philadelphia and Denver plants.

Under this plan, as shown in Table III, Globe will be utilizing only 1934 production hours, or 71% of available production capacity. Observe from Table IV that the Philadelphia plant will be *un*profitable, while the Denver plant will be only marginally profitable.

TABLE III Distribution of Production Time (Hours) All Plants Operational

Plant	G50	H90	Total	Total Capacity	Excess Capacity
Philadelphia	231	0	231	640	409
St. Louis	80	880	960	960	0
New Orleans	0	480	480	480	0
Denver	250	13	263	640	377
Total	561	1373	1934	2720	786

TABLE IV Distribution of Monthly Revenues and Costs (All Plants Operational)

Plant	Revenue Sales	Costs Production	Trans-portation	Operations	Total Cost	Net Profit
Philadelphia	$ 84,854	$ 38,570	$ 9,571	$ 40,000	$ 88,141	($ 3,287)
St. Louis	$333,146	$145,716	$12,143	$ 35,000	$192.859	$140,287
New Orleans	$191,996	$ 68,570	$20,571	$ 20,000	$109,141	$ 82,855
Denver	$114,004	$ 67,145	$ 5,143	$ 30,000	$102,288	$ 11,716
Total	$724,000	$320,001	$47,428	$125,000	$492,429	$231,571

Plant Closings Tables V, VI, and VII give a production and distribution schedule that would result from closing the Philadelphia plant. We estimate that, by closing the Philadelphia plant, Globe can achieve an approximate 15% increase in profit, from $231,571 to $266,115 per month. This is an annual increase in profit of $414,528.

TABLE V Production/Transportation Schedule (Philadelphia Plant Closed)

Plant	Product	Amount	Cincinnati	Kansas City	San Francisco
St. Louis	G50	1143		1143	
	H90	11000	5000	6000	
New Orleans	G50	2804	2000	804	
	H90	3252			3252
Denver	G50	6053		1053	5000
	H90	3748			3748
Total	G50	10000	2000	3000	5000
	H90	18000	5000	6000	7000

TABLE VI Distribution of Production Time (Hours) (Philadelphia Plant Closed)

Plant	G50	H90	Total	Total Capacity	Excess Capacity
St. Louis	80	880	960	960	0
New Orleans	252	228	480	480	0
Denver	303	337	640	640	0
Total	635	1445	2080	2080	0

TABLE VII Distribution of Monthly Revenues and Costs (Philadelphia Plant Closed)

Plant	Revenue Sales	Production	Costs Trans-portation	Operations	Total Cost	Net Profit
St. Louis	$333,146	$145,716	$12,143	$35,000	$192,859	$140,287
New Orleans	$152,744	$ 54,952	$15,364	$20,000	$ 90,316	$ 62,428
Denver	$238,110	$134,909	$ 9,801	$30,000	$174,710	$ 63,400
Total	$724,000	$335,577	$37,308	$85,000	$457,885	$266,115

Based on this analysis, it would seem prudent to close the Philadelphia plant. The remaining three plants will then be fully utilized, and all projected demand will be met.

Because Globe Electronics has its roots in the Philadelphia area, however, the company may wish to examine additional alternatives. We conducted another analysis to determine the most profitable production schedule while keeping the Philadelphia plant operational.

The best production plan in this case is to close the Denver plant and execute the manufacturing and distribution plan detailed in Table VIII. Net profit under this plan would be $256,667 per month, approximately 4% less than if the Philadelphia plant were closed. Although this amounts to annual profit that is approximately $113,000 less than if the firm closes the Philadelphia plant, it is still approximately an 11% (or about a $300,000) annual increase over the best schedule with all plants operational.

Note that under this plan:

- The Philadelphia plant produces only G50 models.
- The St. Louis plant produces only H90 models.
- The New Orleans plant ships only to San Francisco.

TABLE VIII Production/Transportation Schedule (Denver Plant Closed)

Plant	Product	Amount	Cincinnati	Kansas City	San Francisco
Philadelphia	G50	9333	2000	3000	4333
	H90	0			
St. Louis	G50	0			
	H90	12000	5000	6000	1000
New Orleans	G50	667			667
	H90	6000			6000
Total	G50	10000	2000	3000	5000
	H90	18000	5000	6000	7000

Such a pattern may have additional benefits or detractions of which we are unaware and which were not considered in our analysis. Barring such additional costs or cost savings, Tables IX and X detail the production time and cost distribution for this schedule. Note that this schedule also has the benefit of the availability of some excess capacity (at the Philadelphia plant) to cover any unanticipated surges in demand.

TABLE IX Distribution of Production Time (Hours) (Denver Plant Closed)

Plant	G50	H90	Total	Total Capacity	Excess Capacity
Philadelphia	560	0	560	640	80
St. Louis	0	960	960	960	0
New Orleans	60	420	480	480	0
Total	620	1380	2000	2080	80

TABLE X Distribution of Monthly Revenues and Costs (Denver Plant Closed)

		Costs				
Plant	**Revenue Sales**	**Production**	**Trans-portation**	**Operations**	**Total Cost**	**Net Profit**
Philadelphia	$205,326	$ 93,330	$34,665	$40,000	$167,995	$ 37,331
St. Louis	$336,000	$144,000	$15,000	$35,000	$194,000	$142,000
New Orleans	$182,674	$ 65,336	$20,000	$20,000	$105,336	$ 77,338
Total	$724,000	$302,666	$69,665	$95,000	$467,331	$256,669

SUMMARY AND RECOMMENDATION The results of our analysis are as follows:

Options for Globe Electronics, Inc.

Option	*Annual Profit*
Close the Philadelphia plant	$3,193,380
Close the Denver plant	$3,080,028
All plants operational	$2,778,852

Although we have detailed the cost and transportation distribution of each plan, management must decide whether extenuating circumstances would make one of the less profitable plans more acceptable. Factors such as the impact on the community of plant closings, the costs (not included in this report) of actually closing a facility, and the benefits of a structured distribution pattern or available excess capacity to meet demand fluctuations should all be considered before a final decision is made.

Should management require further study on any of these points, we would be happy to assist in the analysis.

3.6 Summary

Linear and integer programming models have been applied successfully in a wide variety of business and government applications, some of which are cited in the first section of this chapter. We have given an outline and numerous hints on how to build successful mathematical models and how to convert these models into good, easy to understand spreadsheet models. We have applied these concepts to simplified applications taken from a variety of business and government sectors. In the process, we offered a thorough analysis of output results and illustrated many of the pitfalls and anomalies that can occur in both the modeling and solution phases. These include how to detect and resolve situations involving unboundedness, infeasibility, and multiple optimal solutions; when and how to use summation variables and constraints; and how to use integer and binary variables to appropriately model particular situations. We have also introduced the concepts of data envelopment analysis (on the CD-ROM) and supply chain management, both of which are important topics in today's business climate.

Not all mathematical models can be modeled by a linear objective function and linear constraints. Chapter 13 on the accompanying CD-ROM discusses the topics of goal programming (which can involve repeated solving of linear programs), dynamic programming (which involves making a sequence of interrelated decisions), and the general nonlinear model.

On the CD-ROM

• Excel spreadsheets for linear programming models	**Galaxy Expansion.xls**
	Jones Investment.xls
Infeasibility→	**St. Joseph.xls**
Alternate Optimal Solutions→	**St. Joseph (Revised).xls**
Unbounded Solution→	**Euromerica Liquors.xls**
	Euromerica Liquors (Revised).xls
	United Oil.xls
	Powers.xls
	Sir Loin.xls
	Sir Loin Composite.xls
• Excel spreadsheets for integer linear programming models	**Sunset.xls**
	Vertex Software.xls
	Salem.xls
	Globe.xls
	Globe Plant Expansion.xls
• Duality	**Supplement CD2**
• The Simplex Method	**Supplement CD3**
• Algorithms for Solving Integer Models	**Supplement CD4**
• Problem Motivations	**Problem Motivations**
• Additional "Real Life" Applications	**Appendix 3.1**
• A Multiperiod Cash Flow Scheduling Model	**Appendix 3.2**
• Data Envelopment Analysis	**Appendix 3.3**
• An Integer Programming Advertising Model	**Appendix 3.4**
• Problems 41–50	**Additional Problems/Cases**
• Cases 4–6	**Additional Problems/Cases**

Problems

Problems 1–27 can be formulated as linear programming models.

Problems 28–40 can be formulated as integer linear programming models.

1. PRODUCTION SCHEDULING. Coolbike Industries manufactures boys and girls bicycles in both 20-inch and 26-inch models. Each week it must produce at least 200 girl models and 200 boys models. The following table gives the unit profit and the number of minutes required for production and assembly for each model.

Bicycle	Unit Profit	Production Minutes	Assembly Minutes
20-inch girls	$27	12	6
20-inch boys	$32	12	9
26-inch girls	$38	9	12
26-inch boys	$51	9	18

The production and assembly areas run two (eight-hour) shifts per day, five days per week. This week there are 500 tires available for 20-inch models and 800 tires available for 26-inch models. Determine Coolbike's optimal schedule for the week. What profit will it realize for the week?

2. APPLIANCE PRODUCTION. Kemper Manufacturing can produce five major appliances—stoves, washers, electric dryers, gas dryers, and refrigerators. All products go through three processes—molding/pressing, assembly, and packaging. Each week there are 4800 minutes available for molding/pressing, 3000 available for packaging, 1200 for stove assembly, 1200 for refrigerator assembly, and 2400 that can be used for assembling washers and dryers. The following table gives the unit molding/pressing, assembly, and packing times (in minutes) as well as the unit profits.

	Molding/ Pressing	Assembly	Packaging	Unit Profit
Stove	5.5	4.5	4.0	$110
Washer	5.2	4.5	3.0	$ 90
Electric Dryer	5.0	4.0	2.5	$ 75
Gas Dryer	5.1	3.0	2.0	$ 80
Refrigerators	7.5	9.0	4.0	$130

a. What weekly production schedule do you recommend? What is the significance of the fractional values?

b. Suppose the following additional conditions applied:
- The number of washers should equal the combined number of dryers.
- The number of electric dryers should not exceed the number of gas dryers by more than 100 per week.
- The number of gas dryers should not exceed the number of electric dryers by more than 100 per week.

Now what weekly production schedule do you recommend?

3. MANUFACTURING. Kelly Industries manufactures two different structural support products used in the construction of large boats and ships. The two products, the Z345 and the W250, are produced from specially treated zinc and iron and are produced in both standard and industrial grades. Kelly nets a profit of $400 on each standard Z345 and $500 on each standard W250. Industrial models net a 40% premium.

Each week, up to 2500 pounds of zinc and 2800 pounds of iron can be treated and made available for production. The following table gives the per unit requirements (in pounds) for each model.

	Z345		W250	
	Standard	Industrial	Standard	Industrial
Zinc	25	46	16	34
Iron	50	30	28	12

Kelly has a contract to supply a combined total of at least 20 standard or industrial Z345 supports to Calton Shipbuilders each week. Company policy mandates that at least 50% of the production must be industrial models and that neither Z345 models nor W250 models can account for more than 75% of weekly production. By adhering to this policy, Kelly feels, it can sell all the product it manufactures.

a. Determine a weekly production plan for Kelly Industries. What interpretation can you give to the fractional values that are part of the optimal production quantities?

b. What proportion of the production are W250 models? What does that tell you about how the profit will be affected if the 75% limit is loosened or eliminated?

c. State whether you should buy *additional* shipments of zinc, should they become available at the following premiums above zinc's normal cost.
   i. 100 pounds for $1500
   ii. 100 pounds for $2600
   iii. 800 pounds for $10,000

4. FINANCIAL INVESTMENT. The Investment Club at Bell Labs has solicited and obtained $50,000 from its members. Collectively, the members have selected the three stocks, two bond funds, and a tax-deferred annuity shown in the following table as possible investments.

Investment	Risk	Projected Annual Return
Stock—EAL	High	15%
Stock—BRU	Moderate	12%
Stock—TAT	Low	9%
Bonds—long term		11%
Bonds—short term		8%
Tax-deferred annuity		6%

The club members have decided on the following strategies for investment:

- All $50,000 is to be invested.
- At least $10,000 is to be invested in the tax-deferred annuity.
- At least 25% of the funds invested in stocks are to be in the low-risk stock (TAT).
- At least as much is to be invested in bonds as stocks.
- No more than $12,500 of the total investment is to be placed in investments with projected annual returns of less than 10%.

a. Formulate and solve a linear program that will maximize the total projected annual return subject to the conditions set forth by the Investment Club members.
b. What is the projected rate of return of this portfolio? What rate of return should investors expect on any additional funds received, given the restrictions of the club? Explain why this rate would hold for all additional investment dollars.
c. For which investment possibilities are the estimates for the projected annual return most sensitive in determining the optimal solution?
d. Give an interpretation of the shadow prices for the right-hand side of each constraint.

5. PRODUCTION. Minnesota Fabrics produces three sizes of comforters (full, queen, and king size) that it markets to major retail establishments throughout the country. Due to contracts with these establishments, Minnesota Fabrics must produce at least 120 of each size comforter daily. It pays $0.50 per pound for stuffing and $0.20 per square foot for quilted fabric used in the production of the comforters. It can obtain up to 2700 pounds of stuffing and 48,000 square feet of quilted fabric from its suppliers.

Labor is considered a fixed cost for Minnesota Fabrics. It has enough labor to provide 50 hours of cutting time and 200 hours of sewing time daily. The following table gives the unit material and labor required as well as the selling price to the retail stores for each size comforter.

	Stuffing (pounds)	Quilted Fabric (sq. ft.)	Cutting Time (minutes)	Sewing Time (minutes)	Selling Price
Full	3	55	3	5	$19
Queen	4	75	5	6	$26
King	6	95	6	8	$32

a. Determine the daily production schedule that maximizes total daily gross profit (= selling price − material costs.). How much of the available daily material and labor resources would be used by this production schedule?
b. What is the lowest *selling price* for queen size comforters that Minnesota Fabrics could charge while maintaining the optimal production schedule recommended in part a?
c. Suppose Minnesota Fabrics could obtain additional stuffing *or* quilted fabric from supplementary suppliers. What is the most it should be willing to pay for:
   i. An extra pound of stuffing? Within what limits is this valid?
   ii. An extra square foot of quilted fabric? Within what limits is this valid?
   iii. An extra *minute* of cutting time? Within what limits is this valid?
   iv. An extra minute of sewing time? Within what limits is this valid?
d. Suppose the requirement to produce at least 120 king size comforters were relaxed. How would this affect the optimal daily profit?

6. HIGH PROTEIN/LOW CARBOHYDRATE DIET. One of the current diets that seems to produce substantial weight loss in some persons is the high protein/low carbohydrate diet advocated by such authors as Atkins in his book, *Diet Revolution*, and Eades and Eades in their book, *Protein Power*. Although many nutritionists are concerned about the side effects of these diets, others feel the potential risks are outweighed (no pun intended) by the weight loss.

The following table gives the grams of fat, carbohydrates, and protein as well as the calorie count in four potential foods: steak (8 oz. portion), cheese (1 oz.), apples (1 medium), and whole milk (8 oz.). Jim Blount, a 45-year-old male, has done the calculations suggested in these books and has determined that he should have a calorie intake of between 1800 and 2000 calories and protein intake of at least 100 grams, but he should not consume more than 45 grams of carbohydrates daily. For breakfast Jim had two eggs and three strips of bacon, one piece of buttered high protein toast, and water. This breakfast contained 390 calories, 15 grams of carbohydrates, 20 grams of protein, and 29 grams of fat. Although these diets do not limit fat intake, Jim wishes to minimize his total fat consumed by constructing a diet for lunch and dinner consisting of only the four foods listed in the table. What do you recommend?

	Calories	Fat (grams)	Protein (grams)	Carbohydrates (grams)
Steak (8 oz.)	692	51	57	0
Cheese (1 oz.)	110	9	6	1
Apple	81	1	1	22
Milk	150	8	8	12

7. COMPUTER PRODUCTION. MVC Enterprises can manufacture four different computer models; the Student, Plus, Net, and Pro models. The following gives the configurations of each model:

	Student	Plus	Net	Pro
Processor	Celeron	Pentium	Celeron	Pentium
Hard Drive	20 gb	20 gb	20 gb	30 gb
Floppy Drives	1	1	2	1
Zip Drive	YES	YES	NO	YES
Audio/Video	CD R/W	DVD	DVD + CD R/W	DVD + CD R/W
Monitor	15″	15″	17″	17″
Case	Tower	Mini-Tower	Mini-Tower	Tower
Production Time (hrs.)	.4	.5	.6	.8
Unit Profit	$70	$80	$130	$150

- MVC must satisfy a contract that produces a minimum of 100 Net models per week.
- MVC employs 25 workers, each of whom averages 30 production hours each per week.
- The following resources are available weekly:

Processors	*Hard Drives*	*Other Drives*
Celeron—700	20 gb—800	Floppy—1600
Pentium—550	30 gb—950	Zip—1000
Audiovisual	*Monitors*	*Cases*
CD R/W—1600	15″—850	Mini-Tower—1250
DVD—900	17″—800	Tower—750

a. Determine the optimal weekly production schedule for MVC. What is the optimal weekly profit?
b. What is the minimum price that would justify producing the Plus model? Explain.
c. If MVC could purchase additional 17″ monitors for $15 more than what they are currently paying for them, should they do this? Explain.
d. Suppose an additional worker could be hired for $1000 per week over the existing weekly worker salary. (*Recall that workers average 30 hours per week*.) Should MVC do this? Explain.

8. PRODUCTION. Pacific Aerospace is one of four subcontractors producing computer-controlled electrical switching assemblies for the proposed NASA space station. Pacific has the capability to produce three types of systems in-house: the Pacific Aerospace Delta, Omega, and Theta. NASA needs hundreds of all three systems and will purchase whatever Pacific chooses to produce.

All assemblies contain the tiny modified X70686 computer chip that cost Pacific $500 to manufacture. Seven such chips are available daily. Other materials needed for the manufacture of the assembly cost $200 for the Delta, $400 for the Omega, and $300 for the Theta, but they are not considered in short enough supply to restrict production.

Each assembly must pass through a production center; it is then subjected to rigorous testing and quality control checks. The following table gives the relevant data for each assembly.

	Delta	Omega	Theta	Daily Availability
Contract price	$1500	$1800	$1400	
X70686 Chip	$ 500	$ 500	$ 500	7
Other material/labor	$ 200	$ 400	$ 300	–
Net profit	$ 800	$ 900	$ 600	
Production (hrs.)	2	1	1	8
Quality checks (hrs.)	$1\frac{1}{3}$	$2\frac{2}{3}$	$1\frac{1}{3}$	8

a. Formulate and solve for the optimal daily production schedule. Note that no Omega systems would be produced. Why not?
b. What is the minimum contract price that would initiate production of the Omega systems?
c. What is the minimum X70686 availability for which the solution in (a) remains optimal?
d. Suppose you have the option of improving the profit by instituting one of the following options. Which would be of most value to Pacific Aerospace?
   i. Receiving, on a daily basis, six additional X70686 chips for $3100.
   ii. Utilizing three extra production hours daily at a cost of $525 ($175/hr.)
   iii. Utilizing one additional quality check hour daily at a cost of $200 ($200/hr.)

9. SURVEY SAMPLING. Gladstone and Associates is conducting a survey of 2000 investors for the financial advising firm of William and Ryde to determine satisfaction with their services. The investors are to be divided into four groups:

Group I: Large investors with William and Ryde
Group II: Small investors with William and Ryde
Group III: Large investors with other firms
Group IV: Small investors with other firms

The groups are further subdivided into those that will be contacted by telephone and those that will be visited in person. Due to the different times involved in soliciting information from the various groups, the estimated cost of taking a survey depends on the group and method of survey collection. These are detailed in the following table.

	Survey Costs	
Group	Telephone	Personal
I	$15	$35
II	$12	$30
III	$20	$50
IV	$18	$40

Determine the number of investors that should be surveyed from each group by telephone and in person to minimize Gladstone and Associates' overall total estimated cost if:

- At least half of those surveyed invest with William and Ryde.
- At least one-fourth are surveyed in person.
- At least one-half of the large William and Ryde investors surveyed are contacted in person.
- At most 40% of those surveyed are small investors.
- At least 10% and no more than 50% of the investors surveyed are from each group.
- At most 25% of the small investors surveyed are contacted in person.

10. DIET PROBLEM. Grant Winfield is a 71-year-old grandfather who likes to mix breakfast cereals together for taste and as a means of getting at least 50% of the recommended daily allowances (RDA) of five different vitamins and minerals. Concerned about his sugar intake, he wishes his mixture to yield the lowest possible amount of sugar. For taste, each of the cereals listed in the following table must make up at least 10% of the total mixture. The table shows the amounts of the vitamins, minerals, and sugar contained in one ounce together with 1/2 cup of skim milk.

Percentage of RDA per Ounce with 1/2 Cup Skim Milk

	Vitamins					Sugars
	A	C	D	B6	Iron	(Grams)
Multigrain Cheerios	30	25	25	25	45	12
Grape Nuts	30	2	25	25	45	9
Product 19	20	100	25	100	100	9
Frosted Bran	20	25	25	25	25	15

a. Formulate and solve for the number of ounces in each cereal that should be mixed together in order to minimize total sugar intake while providing at least 50% of the RDA for each of Vitamins A, C, D, B6, and iron. How much sugar would be consumed in the process?

b. How much total cereal does Grant need to eat to achieve the minimum 50% RDA in all five categories? How much milk does he consume in doing this?

c. Determine the shadow prices for this problem. Interpret the shadow prices and the corresponding ranges of feasibility.

d. If Grant eliminates the restriction that each cereal must account for at least 10% of the mixture, then, by inspection, why wouldn't any Frosted Bran be included in the mix? Verify this conclusion by deleting these constraints from the original formulation and re-solving.

11. VENTURE CAPITAL. Delta Venture Capital Group is considering whether to invest in Maytime Products, a new company that is planning to compete in the small kitchen appliance market. Maytime has three products in the design and test phase: (1) a unique refrigerator/oven that can be programmed in the morning to cook foods (like chicken) but keeps the food refrigerated until the cooking process begins; (2) a French fry maker that can make long thick French fries or small thin shoestring fries; and (3) a French toast maker that cooks French toast of any size evenly on the top and bottom simultaneously.

Delta's primary concern is how much money it must invest with Maytime before it will show a profit. The company will need an immediate initial investment of $2,000,000 to secure a plant and cover overhead costs. This investment must be paid back with initial profits. The following table gives the anticipated selling price, the variable cost per unit manufactured, and the initial demand for each product obtained through market research. Delta would have to commit to the $2,000,000 and then would like to minimize its total variable cost outlay until Maytime turns a profit (i.e., until Maytime can cover the initial $2,000,000 investment with profits (=selling price − variable cost) from its sales.)

Product	Selling Price	Variable Cost	Initial Demand
Refrigerator/Oven	$240	$140	5000
French Fry Maker	$ 85	$ 50	4000
French Toast Maker	$ 63	$ 36	2300

Maytime will initially produce no more than 15,000 units of any of the items but will meet anticipated initial demand. What production quantities for the products will minimize the total variable cost of the items produced while attaining a "profit" of $2,000,000?

12. MANUFACTURING. Bard's Pewter Company (BPC) manufactures pewter plates, mugs, and steins that include the campus name and logo for sale in campus book stores. The time required for each item to go through the two stages of production (molding and finishing) and the corresponding unit profits are given in the following table.

	Molding Time (min.)	Finishing Time (min.)	Unit Profit
Plates	2	8	$2.50
Mugs	3	12	$3.25
Steins	6	14	$3.90

BPC employs 12 workers, each of whom works 8 hours per day; 4 are assigned to the molding operation, the others to the finishing operation. BPC's marketing department has recommended the following:

- At least 150 mugs should be produced daily.
- The number of steins produced can be at most twice the combined total number of plates and mugs produced.
- Plates can account for no more than 30% of the total daily unit production.

a. If management accepts the marketing department's recommendations in full, determine an optimal daily production schedule for BPC.

b. Suppose the 12 workers could be reassigned optimally to the two operations. By how much would the daily profit change?

13. MORTGAGE INVESTMENT. Tritech Mortgage specializes in making first, second, and even third trust deed loans on residential properties and first trust deeds on commercial properties. Any funds not invested in mortgages are invested in an interest-bearing savings account. The following table gives the rate of return and the company's risk level for each possible type of loan.

Loan Type	Rate of Return	Risk
First Trust Deeds	7.75%	4
Second Trust Deeds	11.25%	6
Third Trust Deeds	14.25%	9
Commercial Trust Deeds	8.75%	3
Savings Account	4.45%	0

Tritech wishes to invest $68,000,000 in available funding so that:

- Yearly return is maximized.
- At least $5,000,000 is to be available in a savings account for emergencies.
- At least 80% of the money invested in trust deeds should be in residential properties.
- At least 60% of the money invested in residential properties should be in first trust deeds.
- The average risk should not exceed 5.

a. What distribution of funding do you recommend? What is the rate of return on this distribution of funds?

b. Suppose the rate of return on first trust deeds increases. What is the maximum rate of return so that your recommendation in part (a) remains optimal? What would be the overall rate of return on the investment if this rate were increased to its maximum limit?

14. PRODUCTION INVENTORY. The Mobile Cabinet Company produces cabinets used in mobile and motor homes. Cabinets produced for motor homes are smaller and made from less expensive materials than are those for mobile homes. The home office in Ames, Iowa, has just distributed to its individual manufacturing centers the production quotas required during the upcoming summer quarter. The scheduled production requirements for the Lexington, Kentucky, plant are given in the following table.

Production Requirements—Mobile Cabinet Company

	July	August	September
Motor home	250	250	150
Mobile home	100	300	400

Each motor home cabinet requires three man-hours to produce, whereas each mobile home cabinet requires five man-hours. Labor rates normally average $18 per hour. During July and August, however, when Mobile employs many part-time workers, labor rates average only $14 and $16 per hour, respectively. A total of 2100 man-hours are available in July, 1500 in August, and 1200 in September. During any given month, management at the Lexington plant can schedule up to 50% additional man-hours, using overtime at the standard rate of time and a half. Material costs for motor home cabinets are $146; for mobile home cabinets they are $210.

The Lexington plant expects to have 25 motor home and 20 mobile home assembled cabinets in stock at the beginning of July. The home office wants the Lexington plant to have at least 10 motor home and 25 mobile cabinet assemblies in stock at the beginning of October to cover possible shortages in production from other plants.

The Lexington plant has storage facilities capable of holding up to 300 cabinets in any one month. The costs for storing motor home and mobile home cabinets from one month to the next are estimated at $6 and $9 per cabinet, respectively. Devise a monthly production schedule that will minimize the costs at the Lexington plant over the quarter.

Hint: Define variables so that you can fill in the following charts.

Quarterly Production Schedule of Motor Home Cabinets

	Regular Time	Overtime
July		
August		
September		

Quarterly Production Schedule of Mobile Home Cabinets

	Regular Time	Overtime
July		
August		
September		

Quarterly Storage Schedule

	Motor Home	Mobile Home
July		
August		
September		

15. AGRICULTURE. BP Farms is a 300-acre farm located near Lawrence, Kansas, owned and operated exclusively by Bill Pashley. For the upcoming growing season, Bill will grow wheat, corn, oats, and soybeans. The following table gives relevant data concerning expected crop yields, labor required, expected preharvested expenses, and water required (in addition to the forecasted rain). Also included is the price per bushel Bill expects to receive when the crops are harvested.

Crop	Yield (bu./acre)	Labor (hr./acre)	Expenses ($/acre)	Water (acre-ft./acre)	Price ($/bu.)
Wheat	210	4	$50	2	$3.20
Corn	300	5	$75	6	$2.55
Oats	180	3	$30	1	$1.45
Soybeans	240	10	$60	4	$3.10

Bill wishes to produce at least 30,000 bushels of wheat and 30,000 bushels of corn, but no more than 25,000 bushels of oats. He has $25,000 to invest in his crops, and he plans to work up to 12 hours per day during the 150-day season. He also does not wish to exceed the base water supply of 1200 acre-feet allocated to him by the Kansas Agriculture Authority.

a. Formulate the problem for BP Farms as a linear program and solve for the optimal number of acres of each crop Bill should plant in order to maximize his total expected return from the harvested crops.
b. If the selling price of oats remains $1.45 a bushel, to what level must the yield increase before oats should be planted? If the yield for oats remains 180 bushels per acre, to what level would the price of oats have to rise before oats should be planted?
c. If there were no constraint on the minimum production of corn, would corn be planted? How much would the profit decrease if corn were not grown?
d. La Mancha Realty owns an adjacent 40-acre parcel, which it is willing to lease to Bill for the season for $2000. Should Bill lease this property? Why or why not?

16. SUPPLY CHAIN MANAGEMENT. Lion Golf Supplies operates three production plants in Sarasota, Florida; Louisville, Kentucky; and Carson, California. The plant in Sarasota can produce the high-end "professional" line of golf clubs and the more moderate "deluxe" line. The plant in Louisville can produce the deluxe line and basic "weekender" line, while the one in Carson can produce all three models. The amount of steel, aluminum, and wood required to make a set of each line of clubs (including waste), the monthly availability of these resources at each of three plants, and the gross profit per set are given in the following table.

	Steel	Aluminum	Wood	Gross Profit
Professional	3.2 lbs.	5.0 lbs.	5.2 lbs.	$250
Deluxe	3.6 lbs.	4.0 lbs.	4.8 lbs.	$175
Weekender	2.8 lbs.	4.5 lbs.	4.4 lbs.	$200
Available Monthly—Sarasota	5000 lbs.	7000 lbs.	10000 lbs.	
Available Monthly—Louisville	9000 lbs.	13000 lbs.	18000 lbs.	
Available Monthly—Carson	14000 lbs.	18000 lbs.	20000 lbs.	

Lion has three major distribution centers in Anaheim, California, Dallas, Texas, and Toledo, Ohio. The projected monthly demand and the unit transportation costs for each line between the manufacturing centers and distribution centers are given in the following table. Lion must ship between 80% and 100% of the demand for each line to each distribution center.

Professional	Sarasota	Louisville	Carson	Total Demand
Anaheim	$45		$9	600
Dallas	$32		$40	400
Toledo	$30		$50	200
Deluxe	**Sarasota**	**Louisville**	**Carson**	**Total Demand**
Anaheim	$40	$34	$6	800
Dallas	$28	$18	$35	1000
Toledo	$25	$10	$40	1100
Weekender	**Sarasota**	**Louisville**	**Carson**	**Total Demand**
Anaheim		$30	$5	800
Dallas		$15	$30	1500
Toledo		$9	$36	1000

Determine an optimal production/shipping pattern for Lion Golf Supplies.

17. SUPPLY CHAIN MANAGEMENT. Consider the Lion Golf Supplies model of problem 16.

a. Suppose that the following table gives the fixed monthly operating cost of each of the production plants.

Plant	Cost
Sarasota	$250,000
Louisville	$350,000
Carson	$500,000

Assuming that between 80% and 100% of the demand for each line must be filled at each distribution center, what recommendation would you now make concerning which plants should be operational and the production and shipping distribution pattern at each operational plant?

b. Suppose that *in addition to the fixed plant operating* expenses, each distribution center has fixed monthly operating expenses as shown in the following table.

Distribution Center	Cost
Anaheim	$ 50,000
Dallas	$100,000
Toledo	$ 90,000

Assuming that between 80% and 100% of the demand for each line must be met at each distribution center that is operational, what recommendation would you now make concerning which plants should be operational and the production and shipping distribution pattern at each operational plant?

18. PORTFOLIO ANALYSIS. Sarah Williams has $100,000 to allocate to the investments listed in the following table. Bill Wallace, her investment counselor, has prepared the following estimates for the potential annual return on each investment.

Investment	Expected Return	Minimum Return	Maximum Return
Bonanza Gold (high-risk stock)	15%	−50%	100%
Cascade Telephone (low-risk stock)	9%	3%	12%
Money market account	7%	6%	9%
Two-year Treasury bonds	8%	8%	8%

Sarah wishes to invest her money in such a way as to maximize her expected annual return based on Bill Wallace's projections, with the following restrictions:

- At most $50,000 of her investment should be in stocks.
- At least $60,000 of her investment should have the potential of earning a 9% or greater annual return.
- At least $70,000 should be liquid during the year; this implies that at most $30,000 can be in two-year Treasury bonds.
- The minimum overall annual return should be at least 4%.
- All $100,000 is to be invested.

Assume that the investments will perform independently of one another so that the returns on the investment opportunities are uncorrelated. Formulate and solve a linear program for Sarah.

19. BLENDING—OIL REFINING. California Oil Company (Caloco) produces two grades of unleaded gasoline (regular and premium) from three raw crudes (Pacific, Gulf, and Middle East). The current octane rating, the availability (in barrels), and the cost per barrel for a given production period are given in the following table.

Crude	Octane	Availability	Cost
Pacific	85	3000 barrels	$14.28/barrel
Gulf	87	2000 barrels	$15.12/barrel
Middle East	95	8000 barrels	$19.74/barrel

For this period, Caloco has contracts calling for a minimum of 200,000 gallons of regular and 100,000 gallons of premium gasoline, and it has a refining capacity of 400,000 total gallons. (A barrel is 42 gallons.) Caloco sells regular gasoline to retailers for $0.52 and premium gasoline for $0.60 per gallon.

To be classified as "regular," the refined gas must have an octane rating of 87 or more; premium must have an octane rating of 91 or more. Assume that the octane rating of any mixture is the weighted octane rating of its components.

a. Solve for the optimal amount of each crude to blend into each gasoline during this production period.

b. Suppose Caloco could obtain an additional 50,000 gallons in refining capacity for the period by putting other projects on hold. Putting these projects on hold is estimated to cost Caloco $5000 in contract penalties. Should the company absorb these fees and secure this extra 50,000-gallon refining capacity?

c. Given your answer to part (a), calculate the amount Caloco would spend purchasing Middle East oil for the period. Suppose Middle East distributors currently have a glut of crude and are in need of some hard currency. They are willing to enter into a contract with Caloco to sell it all 8000 barrels at $16.80 a barrel. Would the Middle East distributors receive more cash from Caloco under this arrangement? Would it be profitable to Caloco to accept this offer? Discuss the ramifications of this action for domestic oil producers.

20. PERSONNEL EVALUATION. At Nevada State University, the process for determining whether or not a professor receives tenure is based on a combination of qualitative evaluations and a quantitative formula derived by using linear programming. The process works as follows.

In an Annual Personnel File (APF), the professor submits evidence of his or her (1) teaching effectiveness, (2) research performance, (3) other professional activities, and (4) on-campus professional service. A personnel committee of three evaluators (who are full professors) independently evaluate the professor's file and assign a numerical rating between 0 and 100 to each of the four categories. For each category, the scores from the three evaluators are averaged together to give a single score for that category.

To determine the maximum overall score for the professor, a linear program is used for selecting the best weights (percentages) to assign to each category, satisfying the following university criteria.

- Teaching must be weighted at least as heavily as any other category.
- Research must be weighted at least 25%.
- Teaching plus research must be weighted at least 75%.
- Teaching plus research must be weighted no more than 90%.
- Service is to be weighted at least as heavily as professional activities.
- Professional activities must be weighted at least 5%.
- The total of the weights must be 100%.

Professor Anna Sung is up for tenure. To receive tenure, she must receive a weighted total score of at least 85. The three personnel committee members evaluated Anna as follows:

Committee	Teaching	Research	Professional	Service
Ron	90	60	90	80
Mabel	75	60	95	95
Nick	90	75	85	95

Will Professor Sung be awarded tenure?

21. BOAT BUILDERS/DISTRIBUTION. California Catamarans builds the Matey-20 catamaran boat in three locations: San Diego, Santa Ana, and San Jose. It ships the boats to its company-owned dealerships in Newport Beach (NB), Long Beach (LB), Ventura (VEN), San Luis Obispo (SLO), and San Francisco (SF). Production costs and capacities vary from plant to plant, as do shipping costs from the manufacturing plants to the dealerships. The following tables give costs, capacities, and demands for August.

Costs

		Shipping Cost to				
Plant	Production Cost	NB	LB	VEN	SLO	SF
San Diego	$1065	$200	$220	$280	$325	$500
Santa Ana	$1005	$125	$125	$280	$350	$400
San Jose	$ 975	$390	$365	$300	$250	$100

Plant Capacities and August Demand

Plant Capacity		August Demand	
San Diego	38	Newport Beach	42
Santa Ana	45	Long Beach	33
San Jose	58	Ventura	14
		San Luis Obispo	10
		San Francisco	22

Develop a production and shipping schedule for the Matey-20 catamaran for this period that minimizes the total production and shipping costs.

22. TRIM LOSS. The Cleveland Sprinkler Company buys 3/4-inch Schedule 40 PVC pipes, which come in 10-foot lengths, and cuts them into the 30-inch, 42-inch, and 56-inch lengths it requires for its projects. The following table gives the number of pieces of each on hand and the current requirements for each of the three lengths. Any cut of less than 30 inches is considered waste (trim loss) and is discarded. The company would like to purchase enough pipe to satisfy its requirements while minimizing its total trim losses.

	30″	42″	56″
Current inventory	0	400	150
Required	1500	900	750

Hint: The 10-foot (120 inches) pipes can be cut into several variations (e.g., four 30-inch lengths, two 30-inch lengths, and one 42-inch length with 18 inches of trim loss; one 30-inch length and two 42-inch lengths with 6 inches of trim loss; etc). The decision variables are the number of pipes cut into each of these configurations.

23. MARKETING. DAQ Electronics sells nearly 200 consumer electronics products to the general public under its own brand name, including computers, stereos, CD-ROMs, and radar detectors. DAQ's annual budget for advertising in television, radio, newspapers, and its own circulars is $700,000. This year DAQ wishes to spend at least half of its budget in television and radio, and it does not wish to spend more than $300,000 in any one advertising medium.

The company measures its success in exposure units, which are estimates of the audience reached per advertising dollar spent. The following table gives the relevant exposure unit estimates for the total public in general and for the various target populations DAQ wishes to reach.

Exposure Units

Medium	Total	Yuppie	College	Audiophile
Television	28	10	5	5
Radio	18	7	2	8
Newspapers	20	8	3	6
Circular	15	4	1	9
Minimum exposure		2,500,000	1,200,000	1,800,000

a. Comment on the fact that a person may fit into more than one category. Does this violate any linear programming assumption?
b. Ignore the possible conflict in part (a) and formulate and solve a linear program that seeks to maximize the total overall exposure given the $700,000 budget and the restrictions imposed.
c. How would deleting the constraint that a total of at least $350,000 must be spent on television and radio affect the results?

24. RETAILING. Bullox Department Store is ordering suits for its spring season. It orders four styles of suits. Three are "off-the-rack suits": (1) polyester blend suits, (2) pure wool suits, and (3) pure cotton suits. The fourth style is an imported line of fine suits of various fabrics. Studies have given Bullox a good estimate of the amount of hours required of its sales staff to sell each suit. In addition, the suits require differing amounts of advertising dollars and floor space during the season. The following table gives the unit profit per suit as well as the estimates for salesperson-hours, advertising dollars, and floor space required for their sale.

Suit	Unit Profit	Salesperson Hours	Advertising Dollars	Display Space (sq. ft.)
Polyester	$35	0.4	$2	1.00
Wool	$47	0.5	$4	1.50
Cotton	$30	0.3	$3	1.25
Import	$90	1.0	$9	3.00

Bullox expects its spring season to last 90 days. The store is open an average of 10 hours a day, 7 days a week; an average of two salespersons will be in the suit department. The floor space allocated to the suit department is a rectangular area of 300 feet by 60 feet. The total advertising budget for the suits is $15,000.

a. Formulate the problem to determine how many of each type of suit to purchase for the season in order to maximize profits and solve as a linear program.

b. For polyester suits, what would be the effect on the optimal solution of
   i. overestimating their unit profit by $1; by $2?
   ii. underestimating their unit profit by $1; by $2?
c. Show whether each of the following strategies, individually, would be profitable for Bullox:
   i. utilizing 400 adjacent square feet of space that had been used by women's sportswear. This space has been projected to net Bullox only $750 over the next 90 days.
   ii. spending an additional $400 on advertising.
   iii. hiring an additional salesperson for the 26 total Saturdays and Sundays of the season. This will cost Bullox $3600 in salaries, commissions, and benefits but will add 260 salesperson-hours to the suit department for the 90-day season.
d. Suppose we added a constraint restricting the total number of suits purchased to no more than 5000 for the season. How would the optimal solution be affected?

25. ZOO DESIGN. The San Diego Wild Animal Park has won countless awards for its design and concepts and its record of successfully breeding many endangered species. Now an investment group wishes to bring a similar attraction to the Orlando, Florida, area. The group has secured and plans to develop a 350-acre parcel of land not too far from Disneyworld, Universal Studios, and other central Florida attractions.

The animal park can be thought of as being divided into seven general areas:

- Zoo habitat attractions
- Show areas where animal shows will be seen throughout the day
- Restaurant areas
- Retail establishments
- Maintenance areas
- "Green" areas—consisting of parks and other required green spaces
- Walkways and service roads that intermingle throughout the park

The following is a list of zoning agency and other conditions that must be met by zoo planners:

- Each acre devoted to habitat areas is expected to generate $1000 per hour in gross profit to the park and is to be surrounded by .03 acre of green area. At least 40% of the park will consist of habitat areas (not including the required green areas).
- Each acre devoted to show areas is expected to generate $900 per hour in gross profit to the park and is to be surrounded by .40 acre of green areas. At least 5% of the park will consist of show areas (not including the required green areas).
- Combined, the habitat and show areas (excluding corresponding green areas) should not account for more than 70% of the park. Also, the show areas (including corresponding green areas) should not represent more than 20% of the combined acreage for habitat and show areas (including their corresponding green areas).
- At least 25% of the park that is not dedicated to habitat and show areas (not including their required green areas) should be green areas.
- Maintenance facility space is required as follows: .01 acre for each acre of habitat, .10 acre for every acre of shows, .08 acre for every acre of restaurants, .06 acre for every acre of retail establishments, .02 acre for every acre of green space, and .04 acre for every acre of walkways/roads.
- Restaurants will average .25 acre. Each must be surrounded by .15 acre of green space. It is estimated that each restaurant will generate $800 per hour in gross profit. The park should contain between 20 and 30 restaurants.
- Retail stores will average .20 acre and will be surrounded by .10 acre of green areas. Each store will generate approximately $750 per hour in gross profit. The park should contain between 15 and 25 stores, but there should be at least as many restaurants as retail stores.
- At least 10 acres of the park should be walkways and service roads. Adjoining each walkway and service road must be green areas equal to 25% of the corresponding walkway/service area.
- At least 100 acres of the animal park should be park areas, which are green areas not required by the habitat and show areas, restaurant and retail establishments, and walkways/pathways.
- The park will be open 10 hours per day 365 days per year. It has fixed daily operating expenses of $2,000,000.

Create an optimal design for the park that will maximize total hourly gross profit. The design should indicate the number of acres devoted to zoo habitat, show attractions, maintenance, walkways, and park areas. It should also detail the number and acreage required for restaurant and retail store areas and summarize the total green space in the animal park. What would be the annual *net* profit of this design?

26. HEALTH FOODS. Health Valley Foods produces three types of health food bars in two-ounce sizes: the Go Bar, the Power Bar, and the Energy Bar. The Energy Bar also comes in an 8-ounce size. The three main ingredients in each bar are a protein concentrate, a sugar substitute, and carob. The recipes for each bar in terms of percentage of ingredients (by weight) and the daily availabilities of each of the ingredients are as follows.

Bar	% Protein Concentrate	% Sugar Substitute	% Carob
Go	20	60	20
Power	50	30	20
Energy	30	40	30
Daily availability	600 lbs.	1000 lbs.	800 lbs.

The following costs are incurred in the production of the health food bars:

	Costs
Labor and packaging (2-oz. bars)	$0.03/bar
Labor and packaging (8-oz. bars)	$0.05/bar
Protein concentrate	$3.20/lb.
Sugar substitute	$1.40/lb.
Carob	$2.60/lb.

Health Valley's wholesale selling prices to health food stores are $0.68, $0.84, and $0.76, respectively, for 2-ounce sizes of the Go Bar, the Power Bar, and the Energy Bar, and $3.00 for the 8-ounce Energy Bar. The company has facilities for producing up to 25,000 2-ounce bars and 2000 8-ounce bars daily. It manufactures at least 2500 of each of the 2-ounce bars daily. No 2-ounce bar is to account for more than 50% of the total production of 2-ounce bars, and the total production (by weight) of Energy Bars is not to exceed more than 50% of the total production (by weight).

Determine an optimal daily production schedule of health food bars for Health Valley Foods.

27. ADVERTISING. JL Foods is planning to increase its advertising campaign from $1.4 million to $2 million based, in part, on the introduction of a new product. JL Taco Sauce, to accompany its traditional products, JL Ketchup and JL Spaghetti Sauce. In the past, JL Foods promoted its two products individually, splitting its advertising budget equally between ketchup and spaghetti sauce.

From past experience, the marketing department estimates that each dollar spent advertising *only* ketchup increases ketchup sales by four bottles and each dollar spent advertising *only* spaghetti sauce increases its sales by 3.2 bottles. Since JL makes $0.30 per bottle of ketchup and $0.35 per bottle of spaghetti sauce sold (excluding the sunk cost of the given advertising budget), this amounts to a return of $1.20 (=4 × $0.30) per advertising dollar on ketchup and $1.12 (=3.2 × $0.35) per advertising dollar on spaghetti sauce. Because taco sauce is a new product, its initial return is projected to be only $0.10 per bottle, but each advertising dollar spent solely on taco sauce is estimated to increase sales by 11 bottles. The company also projects that sales of each product would increase by another 1.4 bottles for each dollar spent on joint advertising of the three products.

JL wishes to maximize its increase in profits this year from advertising while also "building for the future" by adhering to the following guidelines for this year's advertising spending:

- A maximum of $2 million total advertising
- At most $400,000 on joint advertising
- At least $100,000 on joint advertising
- At least $1 million promoting taco sauce, either individually or through joint advertising
- At least $250,000 promoting ketchup only
- At least $250,000 promoting spaghetti sauce only
- At least $750,000 promoting taco sauce only
- At least as much spent this year as last year promoting ketchup, either individually or by joint advertising
- At least as much spent this year as last year promoting spaghetti sauce, either individually or by joint advertising
- At least 7.5 million total bottles of product sold

a. Determine the optimal allocation of advertising dollars among the four advertising possibilities (advertising for each product individually and joint advertising). Give the total return per advertising dollar of this solution and express this as a percentage of the $2 million advertising budget.

b. What is the return on additional advertising dollars?

c. Suppose the constraint requiring that at least $750,000 be spent promoting only taco sauce were lowered to $700,000. How much would the profit increase?

28. RESTAURANT CREW ASSIGNMENT. Burger Boy Restaurant is open from 8:00 A.M. to 10:00 P.M. daily. In addition to the hours of business, a crew of workers must arrive one hour early to help set up the restaurant for the day's operations, and another crew of workers must stay one hour after 10:00 P.M. to clean up after closing.

Burger Boy operates with nine different shifts:

Shift	Type	Daily Salary
1. 7AM–9AM	Part-time	$15
2. 7AM–11AM	Part-time	$25
3. 7AM–3PM	Full-time	$52
4. 11AM–3PM	Part-time	$22
5. 11AM–7PM	Full-time	$54
6. 3PM–7PM	Part-time	$24
7. 3PM–11PM	Full-time	$55
8. 7PM–11PM	Part-time	$23
9. 9PM–11PM	Part-time	$16

A needs assessment study has been completed, which divided the workday at Burger Boy into eight 2-hour blocks. The number of employees needed for each block is as follows:

Time Block	Employees Needed
7AM–9AM	8
9AM–11AM	10
11AM–1PM	22
1PM–3PM	15
3PM–5PM	10
5PM–7PM	20
7PM–9PM	16
9PM–11PM	8

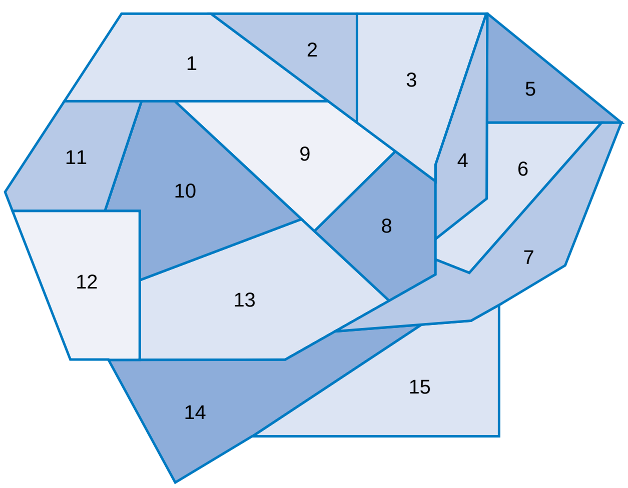
Burger Boy wants at least 40% of all employees at the peak time periods of 11:00 A.M. to 1:00 P.M. and 5:00 P.M. to 7:00 P.M. to be full-time employees. At least two full-time employees must be on duty when the restaurant opens at 7:00 A.M. and when it closes at 11:00 P.M.

- Formulate and solve a model Burger Boy can use to determine how many employees it should hire for each of its nine shifts to minimize its overall daily employee costs.

29. LAW ENFORCEMENT. The police department of the city of Flint, Michigan, has divided the city into 15 patrol sectors, such that the response time of a patrol unit (squad car) will be less than three minutes between any two points within the sector.

Until recently, 15 units, one located in each sector, patrolled the streets of Flint from 7:00 P.M. to 3:00 A.M. However, severe budget cuts have forced the city to eliminate some patrols. The chief of police has mandated that each sector be covered by at least one unit located either within the sector or in an adjacent sector.

The accompanying figure depicts the 15 patrol sectors of Flint, Michigan. Formulate and solve a binary model that will determine the minimum number of units required to implement the chief's policy.



Police Patrol Sectors Flint, Michigan

30. POLLUTION CONTROL. General Motors has received orders from the City of Los Angeles for 30 experimental cars, 20 experimental vans, and 10 experimental buses that meet clean air standards due to take effect in three years. The vehicles can be manufactured in any of four plants located in Michigan, Tennessee, Texas, and California. Due to differences in wage rates, availability of resources, and transportation costs, the unit cost of production of each of these vehicles varies from location to location. In addition, there is a fixed cost for producing any experimental vehicles at each location. These costs (in $1000's) are summarized in the following table.

	Cars	Vans	Buses	Fixed Cost
Michigan	15	20	40	150
Tennessee	15	28	29	170
Texas	10	24	50	125
California	14	15	25	500

Using an integer model with 12 integer variables (representing the number of each model produced at each plant) and four binary variables (indicating whether or not a particular plant is to be used for production of the experimental vehicles), determine how many experimental vehicles should be produced at each plant? What are the total production costs and fixed costs of this plan?

31. REAL ESTATE. Atlantic Standard Homes is developing 20 acres in a new community in the Florida Keys. There are four models it can build on each lot, and Atlantic Standard must satisfy three requirements: at least 40 are to be one story; at least 50 are to have three or more bedrooms; and there are to be at least 10 of each model. Atlantic Standard estimates the following gross profits:

Model	Lot Size (acre)	Stories	Bedrooms	Profit
Tropic	.20	1	2	$40,000
Sea Breeze	.27	1	3	$50,000
Orleans	.22	2	3	$60,000
Grand Key	.35	2	4	$80,000

a. Formulate the problem as an integer linear programming model and solve for Atlantic Standard's optimal production of homes in this community.

b. If the variables had not been restricted to be integers, the optimal linear programming solution gives $X_1 = 30$, $X_2 = 10$, $X_3 = 35.45$, $X_4 = 10$. Round this solution to an integer point. Is is feasible? How much lower is the optimal profit of the rounded solution than the optimal integer solution found in part (a)?

c. Assume that a minimum of 12 homes must be built for at least three of the four models. Using four additional binary variables and five additional constraints, modify the model to reflect this new condition and solve for the new optimal distribution of homes for the Atlantic Standard project.

32. VANPOOLING. Logitech, a rapidly growing high-tech company located in suburban Boston, Massachusetts, has been encouraging its employees to carpool. These efforts have met with only moderate success, and now the company is setting aside up to $250,000 to purchase small- and medium-size vans and minibuses to establish a van-pool program that will transport employees between various pickup points and company headquarters. Four models of vans and two models of minibuses are under consideration, as detailed in the following table.

	Maker	Cost	Passenger Capacity	Annual Maintenance
Vans				
Nissan	Japan	$26,000	7	$ 5000
Toyota	Japan	$30,000	8	$ 3500
Plymouth	U.S.	$24,000	9	$ 6000
Ford (Stretch)	U.S.	$32,000	11	$ 8000
Minibuses				
Mitsubishi	Japan	$50,000	20	$ 7000
General Motors	U.S.	$60,000	24	$11,000

a. Formulate and solve a vehicle purchase model for Logitech that will maximize the total passenger capacity of the fleet given that:
   - Up to $250,000 will be spent on vehicles.
   - Annual maintenance cost should not exceed $50,000.
   - Total number of vehicles purchased should not exceed eight.
   - At least one minibus should be purchased.
   - At least three vans should be purchased.
   - At least half the vehicles should be made in the United States.

b. Determine the optimal solution if the amount Logitech committed for vehicle purchase were: (i) $253,900; (ii) $254,000; (iii) $249,900; (iv) $259,900; (v) $260,000. Comment.

c. Characterize the problem if the amount Logitech committed to the program were $100,000.

33. MERCHANDISING. Office Warehouse has been downsizing its operations. It is in the process of moving to a much smaller location and reducing the number of different computer products it carries. Coming under scrutiny are 10 products Office Warehouse has carried for the past year. For each of these products, Office Warehouse has estimated the floor space required for effective display, the capital required to restock if the product line is retained, and the short-term loss that Office Warehouse will incur if the corresponding product is eliminated (through liquidation sales, etc.).

Product Line	Manufacturer	Cost of Liquidation	Capital to Restock	Floor Space (ft^2)
Notebook computer	Toshiba	$10,000	$15,000	50
Notebook computer	Compaq	$ 8,000	$12,000	60
PC	Compaq	$20,000	$25,000	200
PC	Packard Bell	$12,000	$22,000	200
MacIntosh computer	Apple	$25,000	$20,000	145
Monitor	Packard Bell	$ 4,000	$12,000	85
Monitor	Sony	$15,000	$13,000	50
Printer	Apple	$ 5,000	$14,000	100
Printer	HP	$18,000	$25,000	150
Printer	Epson	$ 6,000	$10,000	125

Office Warehouse wishes to minimize the loss due to liquidation of product lines subject to the following conditions:

- At least four of these product lines will be eliminated.
- The remaining products will occupy no more than 600 square feet of floor space.
- If one product line from a particular manufacturer is eliminated, all products from that manufacturer will be eliminated. (This affects Compaq, Packard Bell, and Apple.)
- At least two computer models (notebook, PC, or MacIntosh), at least one monitor model, and at least one printer model will continue to be carried by Office Warehouse.
- At most $75,000 is to be spent on restocking product lines.
- If the Toshiba notebook computer is retained, the Epson line of printers will also be retained.

Solve for the optimal policy for Office Warehouse.

34. SOFTWARE DEVELOPMENT. The Korvex Corporation is a company concerned with developing CD-ROM software applications that it sells to major computer manufacturers to include as "packaged items" when consumers purchase systems with CD-ROM drives. The company is currently evaluating the feasibility of developing six new applications. Specific information concerning each of these applications is summarized in the following table.

Application	Projected Development Cost	Programmers Required	Projected Present Worth Net Profit
1	$ 400,000	6	$2,000,000
2	$1,100,000	18	$3,600,000
3	$ 940,000	20	$4,000,000
4	$ 760,000	16	$3,000,000
5	$1,260,000	28	$4,400,000
6	$1,800,000	34	$6,200,000

Korvex has a staff of 60 programmers and has allocated $3.5 million for development of new applications.

a. Formulate and solve a binary integer linear programming model for the situation faced by the Korvex Corporation.

b. Assume also that the following additional conditions hold:
   - It is anticipated that those interested in application 4 will also be interested in application 5, and vice versa. Thus, if either application 4 or application 5 is developed, the other must also be developed.
   - The underlying concepts of application 2 make sense only if application 1 is included in the package. Thus, application 2 will be developed only if application 1 is developed.
   - Applications 3 and 6 have similar themes; thus, if application 3 is developed, application 6 will not be developed, and vice versa.

- To ensure quality products, Korvex does not wish to expand its product line too rapidly. Accordingly, it wishes to develop at most three of the potential application products at this time.

Incorporate these constraints into the model developed for part (a), and determine the optimal choice of applications Korvex should develop.

35. ACCOUNTING/PERSONNEL HIRING. Jones. Jimenez, and Sihota (JJS) is expanding its tax service business into the San Antonio area. The company wishes to be able to service at least 100 personal and 25 corporate accounts per week.

JJS plans to hire three levels of employees: CPAs, experienced accountants without a CPA, and junior accountants. The following table gives the weekly salary level as well as the projection of the expected number of accounts that can be serviced weekly by each level of employee.

Employee	Total Number of Accounts	Maximum Number of Corporate Accounts	Weekly Salary
CPAs	6	3	$1200
Experienced accountant	6	1	$ 900
Junior accountant	4	0	$ 600

JJS wishes to staff its San Antonio office so that at least two-thirds of all its employees will be either CPAs or experienced accountants. Determine the number of employees from each experience level the firm should hire for its San Antonio office to minimize its total weekly payroll.

36. ADVERTISING. Century Productions is in the process of promotion planning for its new comedy motion picture. *Three Is a Crowd*, through television, radio, and newspaper advertisements. The following table details the marketing department's estimate of the cost and the total audience reached per exposure in each medium.

	TV	Radio	Newspaper
Cost per exposure	$4,000	$500	$1,000
Audience reached per exposure	500,000	50,000	200,000

The marketing department does not wish to place more than 250 ads in any one medium.

a. What media mix should Century use if it wishes to reach the maximum total audience with an advertising budget of $500,000?

b. What media mix should Century use if it wishes to reach an audience of 30 million at minimum total cost?

c. The cost of producing the television advertisement is $500,000; the radio spot costs $50,000 to write and produce; and the newspaper ad costs $100,000 for design, graphics, and copy. If the total promotional budget is not to exceed $1 million (including the cost of producing the television or radio spot or the newspaper advertisement), use a mixed integer model to determine the production and media mix Century Productions should use.

37. MANUFACTURING. Floyd's Fabrication has just received an order from Gimbal Plumbing Fixtures for 100,000 specially designed three-inch-diameter casings to be delivered in one week. The contract price was negotiated up front; hence, Floyd's maximum profit will be obtained when its costs are minimized.

Floyd's has three production facilities capable of producing the casings. Production costs do not vary between locations, but the changeover (setup) costs do vary, as does the cost of transporting the finished items to Gimbal. The following table details these costs.

Location	Changeover Cost	Transportation Cost (per 1000)	Maximum Weekly Production
Springfield	$1200	$224	65,000
Oak Ridge	$1100	$280	50,000
Westchester	$1000	$245	55,000

Formulate and solve this problem as a mixed integer linear programming model.

38. PERSONAL FINANCE. After many years of earning extremely low bank interest rates, Shelley Mednick has decided to give the stock market a try. This is her first time investing, and she wants to be extra cautious. She has heard that a new stock offering from TCS, a telecommunication company, is being sold at $55 per share (including commissions) and is projected to sell at $68 per share in a year. She is also considering a mutual fund, MFI, which one financial newsletter predicts will yield a 9% return over the next year.

For this first venture into the market, Shelley has set extremely modest goals. She wants to invest just enough so that the expected return on her investment will be $250. Furthermore, since she has more confidence in the performance of the mutual fund than the stock, she has set the restriction that the maximum amount invested in TCS is not to exceed 40% of her total investment, or $750, whichever is smaller.

What combination of shares of TCS and investment in MFI is necessary for Shelley to meet her goal of a projected $250 gain for the year. (*Note*: The number of shares of TCS must be integer-valued.)

39. TRUCKING. The We-Haul Company is about to lease 5000 new trucks for its California operations. The specifications of each truck under consideration are as follows.

Truck	Country	Capacity	Capital Outlay	Monthly Lease
Ford	U.S.	1 ton	$2000	$500
Chevrolet	U.S.	1 ton	$1000	$600
Dodge	U.S.	3/4 ton	$5000	$300
Mack	U.S.	5 tons	$9000	$900
Nissan	Japan	1/2 ton	$2000	$200
Toyota	Japan	3/4 ton	$ 0	$400

We-Haul has decided that, for public relations reasons, given the current "Buy American" atmosphere, it will lease at least 60% or 3000 of the trucks from American manufacturers. Each truck requires an initial capital outlay as well as monthly lease payments. We-Haul feels that it can support a total monthly lease payment of at most $2,750,000. Its fleet requirements mandate at least a 10,000-ton total payload capacity for the 5000 trucks leased. Determine the number of each truck We-Haul should lease to minimize its total initial capital outlay.

40. K OUT OF N CONSTRAINTS PUBLIC POLICY. You can model the situation in which only K out of N constraints must hold, by doing the following. First, choose a cell to hold M, a very, very high value—usually 1E10 which is 10,000,000,000 will do, but you may wish to make it larger. Then do the following

1. For *each* of the N constraints, define a binary variable Y_i to be 0 if constraint i *does* hold and 1 if constraint i *does not* hold.
2. Modify the right-hand sides of each N constraint as follows:
   - If constraint i is a "≤" constraint: Add the term $M*Y_i$ to the right-hand side.
   - If constraint i is a "≥" constraint: Subtract the term $M*Y_i$ from the right-hand side.
   - If constraint i is an "=" constraint:
     - Change the constraint to two constraints, one with a "≤" sign, the other with a "≥" sign.
     - Add the term $M*Y_i$ to the right-hand side of the new "≤" constraint.
     - Subtract the term $M*Y_i$ from the right-hand side of the new "≥" constraint.
3. Add the constraint $Y_1 + Y_2 + \ldots + Y_N \le K - N$.

Now apply this technique to the Salem City Council model in Section 3.5.3, suppose that the council would like to convey to the public that it is fiscally responsible, concerned about safety, interested in job growth, and sensitive to Salem's educational needs.

To show it is fiscally responsible, the council would like to:

- Carry over at least $250,000 to next year's budget. That is, it would like year-end spending to be at most $900,000 − $250,000 = $650,000.

To show concern for public safety, the council would like to:

- Fund at least 3 of the 6 police and fire projects.
- Add the 7 new police officers.

To show interest in job growth, the council would like to:

- Create at least 15 new full-time jobs, not just 10.

To demonstrate sensitivity to education, the council would like to:

- Fund all three educational projects.

The council members realize that there are not enough resources to meet all five of these objectives, but they feel the voters would look favorably upon them if at least three of these 5 objectives are met, in addition to meeting the other constraints in the model.

Which projects do you recommend that the Salem City Council accomplish so that:

- The original set of conditions discussed in Section 3.5.3 is still met.
- At least 3 of the 5 new objectives are met.
- The total overall point score of funded projects is maximized.

PROBLEMS 41–50 ARE ON THE CD

CASE STUDIES

CASE 1: Calgary Desk Company

It is August and the Calgary Desk Company (CALDESCO) of Calgary, Alberta, is about to plan the production schedule for its entire line of desks for September. CALDESCO is a well-established manufacturer. Due to an internal policy of production quotas (which will be detailed later), it has been able to sell all desks manufactured in a particular month. This, in turn, has given the company reliable estimates of the unit profit contributed by each desk model and style.

The Desks

CALDESCO manufactures a student size desk (24 in. × 42 in.), a standard size desk (30 in. × 60 in.), and an executive size desk (42 in. × 72 in.) in each of the three lines: (1) economy, (2) basic pine, and (3) hand-crafted pine.

The economy line uses aluminum for the drawers and base and a simulated pine-laminated 1-inch particle board top. Although the basic pine desk use $1\frac{1}{2}$-inch pine sheets instead of particle board, they are manufactured on the same production line as the tops of the economy line models. Because its drawers and base are made of wood, however, a different production line is required for this process.

Hand-crafted desks have solid pine tops that are constructed by craftsmen independent of any production line. This desk line uses the same drawers and base (and hence the same production line for this process) as the basic pine desk line. Hand-crafted desks are assembled and finished by hand.

Production

Production Line 1 is used to manufacture the aluminum drawers and base for the economy models; production line 2 is used to manufacture the tops for the economy and basic models. There are two production lines 3, which are used to manufacture drawers and bases for the basic and hand-crafted lines. (Two lines are necessary to meet production targets.)

The production times available on the three production lines are summarized on the Excel spreadsheet below. The time requirements (in minutes) per desk for the three different types of production lines, the finishing and assembly times, and the time required to hand-craft certain models are also summarized on the spreadsheet.

Labor

CALDESCO currently employs a workforce of 30 craftsmen, but due to vacations, illnesses, etc., CALDESCO expects to have only an average of 80% of its craftsmen available throughout the month. Each available craftsman works 160 hours per month. The expected total labor availability, which is also given on the spreadsheet, is:

$$(.80) * (30 \text{ craftsmen}) * (160 \text{ hours/craftsmen}) * (60 \text{ minutes/hour}) = 230{,}400 \text{ worker-minutes.}$$

Each craftsman in CALDESCO's shop is capable of doing all the tasks required to make any model desk; including running of the manufacturing lines, assembling the product, or performing the detailed operations necessary to produce the hand-crafted models.

Two craftsmen are required for each production line, but only a single craftsman is needed for hand crafting and a single craftsman is needed for assembly and finishing. Thus, the total amount of man-minutes required to produce a desk = 2 × (the total production line time) + (hand-crafting time) + (assembly/finishing time).

Materials Requirements

As detailed earlier, the economy desks use aluminum and laminated particle board, whereas the basic and hand-

	A	B	C	D	E	F	G	H	I	J	K	L
1	CALDESCO--SEPTEMBER											
2												
3	PROFIT, ORDERS, MATERIALS (SQ.FT.), PRODUCTION TIME (MIN) PER DESK											
4												
5	LINE	SIZE	PROFIT	SEPT. ORDERS	ALUMINUM	PARTICLE BOARD	PINE SHEETS	LINE 1 TIME	LINE 2 TIME	LINE 3 TIME	ASSEM./ FINISHING	HAND-CRAFTING
6	ECONOMY	STUDENT	25	750	14	8		1.5	1		10	
7		STANDARD	30	900	24	15		2.0	1		11	
8		EXECUTIVE	40	100	30	24		2.5	1		12	
9	BASIC	STUDENT	50	400			22		1	3	15	
10		STANDARD	80	800			40		1	4	18	
11		EXECUTIVE	125	100			55		1	5	20	
12	HAND-	STUDENT	100	25			25			3	20	50
13	CRAFTED	STANDARD	250	150			45			4	25	60
14		EXECUTIVE	350	50			60			5	30	70
15												
16												
17	RESOURCE AVAILABILITY FOR SEPTEMBER											
18												
19	LABOR(MAN-MINUTES)			230400								
20	ALUMINUM (SQ.FT.)			50000								
21	PARTICLE BOARD(SQ.FT.)			30000								
22	PINE SHEETS(SQ.FT.)			200000								
23	PRODUCTION LINE 1 (MIN.)			9600								
24	PRODUCTION LINE 2 (MIN.)			9600								
25	PRODUCTION LINE 3 (MIN.)			19200								
26												
27	PRODUCTION QUOTAS (OF TOTAL PRODUCTION)											
28												
29		MIN %	MAX %									
30	ECONOMY	25	50									
31	BASIC	35	55									
32	HAND-CR.	15	25									
33	STUDENT	20	40									
34	STANDARD	40	65									
35	EXECUTIVE	10	25									

Production Process

crafted models use real pine. The amounts of aluminum, particle board, and $1\frac{1}{2}$-inch thick pine sheets (in square feet) required to produce each style of desk are summarized on the spreadsheet along with the September availability of aluminum, particle board, and pine sheets.

Company Policy/Quotas

CALDESCO has been able to sell all the desks it produces and to maintain its profit margins in part by adhering to a set of in-house quotas. These maximum and minimum quotas for desk production are given on the spreadsheet.

CALDESCO will meet all outstanding orders for September. These are also summarized on the spreadsheet.

Profit Contribution

The unit profits, which have been determined for each style of desk, are also summarized on the spreadsheet.

The Report

Prepare a report recommending a production schedule to CALDESCO for September. In your report, analyze your results, detail the amount of each resource needed if your recommendation is implemented, and discuss any real-life factors that might be considered that have not been addressed in this problem summary nor listed on the spreadsheet. Discuss some appropriate "what-if" analyses including

- An analysis of the viability of instituting a bonus plan costing about $35,000 per month that is anticipated to reduce absenteeism from 20% to 15%.
- An analysis of the possibility of purchasing a new production line 2 for $400,000 and hiring 10 new workers at $30,000 each per year (assume the 20% absentee rate). Assume that September is a typical month and that the resulting increased production per month would be matched each month. Specifically determine:
  - How long it would be before these changes became profitable (i.e., until these fixed costs were paid off).
  - After the purchase of the line was paid off, how much additional profit you would expect to earn each month. (Don't forget the added cost of new workers.)
- Other observations and scenarios you feel might be reasonable.

Your report should give a complete description/analysis of your final recommendation complete with tables, charts, graphs, and so on. The complete model and the computer printouts are to be included in appendices.

Note: The Excel file giving the spreadsheet is CALGDESK.XLS in the Excel files folder on the CD-ROM.

CASE 2: Lake Saddleback Development Corporation

Lake Saddleback Development Corporation (LSDC) is developing a planned community of homes and condominiums around a section of Lake Saddleback, Texas. The idea is to develop 300 acres of land it owns on and near the lake in such a way that it maximizes its profits from the development while offering an appropriate variety of different home plans in different products. In addition, the corporation wishes to analyze the feasibility of developing a 10-acre sports/recreational complex.

LSDC is building four products: (1) the Grand Estate Series; (2) the Glen Wood Collection; (3) the Lakeview Patio Homes; and (4) the Country Condominiums. Within each product are three to four floor plans of various styles, as described in the following list.

Lot Sizes

Lots for all models include the land on which the house resides, the garage (which is not considered part of the advertised square footage of the house), and yard space. It excludes outside parking and space for parks, roads, undeveloped landscape, and so on.

All models in the Grand Estate series are built on one-half acre lots, and 50 half-acre lots on the lake are to be used exclusively by the Grand Estate Series homes. The selling price of these exclusive homes will be an additional 30% plus $50,000 more than the models not on the lake. (For example, the $700,000 Trump model would sell for $960,000 if on the lake.) Each of the Grand Estate series plans must have at least eight units on the lake.

Some Grand Cypress models (in the Glen Wood Homes series) may be built on "premium" quarter-acre lots. In addition, some Bayview models (in the Lakeview Patio Homes series) may be built on "premium" one-sixth acre lots. No more than 25% of the total Grand Cypress models and 25% of the total Bayview models may be built on the premium lots.

Lot sizes for the Country Condominiums are fixed at 1500 square feet.

The minimum standard lot for homes in the Glen Wood and Lakeview series homes (except for the premium models) is $\frac{1}{10}$ of an acre. Lot sizes for certain models can be higher if the following calculation exceeds $\frac{1}{10}$ acre.

$$\text{Lot Size} = (\text{Ground Area of House}) + (\text{Yard Size}) + (\text{Garage Size})$$

Ground Area The ground area of any single-story house is the advertised square footage of the house. The ground area for two-story homes is 75% of the advertised square footage.

Plan	Selling Price	Size (sq. ft.)	Bedrooms	Bathrooms	Stories	Garage Size
Grand Estates						
The Trump*	$700,000	4000	5 + den	4	2	3 car
The Vanderbilt*	$680,000	3600	4 + den	3	2	3 car
The Hughes*	$650,000	3000	4	3	1	3 car
The Jackson*	$590,000	2600	3	3	1	3 car
Glen Wood Collection						
Grand Cypress*	$420,000	2800	4 + den	3	2	3 car
Lazy Oak	$380,000	2400	4	3	2	2 car
Wind Row	$320,000	2200	3	3	2	2 car
Orangewood	$280,000	1800	3	$2\frac{1}{2}$	1	2 car
Lakeview Patio Homes						
Bayview*	$300,000	2000	4	$2\frac{1}{2}$	2	2 car
Shoreline	$270,000	1800	3 + den	$2\frac{1}{2}$	2	2 car
Docks Edge	$240,000	1500	3	$2\frac{1}{2}$	1	2 car
Golden Pier	$200,000	1200	2	2	1	2 car
Country Condominiums						
Country Stream	$220,000	1600	3	2	2	—
Weeping Willow	$160,000	1200	2	2	1	—
Picket Fence	$140,000	1000	2	$1\frac{1}{2}$	1	—

*Some of these models may occupy larger premium or lakeside lots with higher selling prices.

Yard Size For homes in the Glen Wood series, yard sizes are 1200 square feet for single-story homes, and the same as the ground area of the house for two-story homes. For homes in the Lakeview Patio Home series, yard sizes are 900 square feet for single-story homes. For two-story homes in this series, the yard size is 600 square feet + 50% of the ground area of the house.

Garage Size Two-car garages occupy 500 square feet of ground space, and three-car garages occupy 750 square feet of ground space. Note that there are no garages for the Country Condominium models.

Parking

Current code requires one parking space per bedroom for each unit built. For example, outside parking space for two cars would be required for a four-bedroom house with a two-car garage. Each outside parking space will occupy 200 square feet of space. No more than 15 acres of the project may be used for *outside* parking. All parking for the Country Condominiums is outside.

Roads/Greenbelts, Etc.

A total of 1000 square feet per house is to be set aside for the building of roads, greenbelts, and small parks to add both to the aesthetics and necessities of the project.

Variety

Throughout the entire project, the following maximum and minimum percentages have been established by the marketing department (*Note*: Condominiums are included in the following figures.)

	Maximum	*Mininum*
Two-bedroom homes	25%	15%
Three-bedroom homes	40%	25%
Four-bedroom homes	40%	25%
Five-bedroom homes	15%	5%

In addition, none of the four products (Grand Estate, Glen Wood, Lakeview, and Country) is to make up more than 35% or less than 15% of the units built in the development. Furthermore, within each product, each plan must occupy between 20% and 35% of the total units of that product. For appearances' sake, no more than 70% of the single-family homes (all homes except the Country Condominiums) may be two-story homes.

Affordable Housing

In the affluent Lake Saddleback area, any house priced at $200,000 or below is considered "affordable" housing. The federal government requires at least 15% of the project to be designated affordable housing.

Profit

LSDC has determined the following percentages of the sales prices to be net profits:

Grand Estates	22%
Glen Wood*	18%
Lakeview*	20%
Country Condominiums	25%

*There is a 20% premium added to the selling price for the premium Grand Cypress and the premium Bayview models. Two-thirds of this premium can be considered additional profit on these models.

Objectives

1. LSDC needs to determine the number of units of each plan of each product to build in order to maximize its profit.
2. If LSDC builds a 10-acre sports/recreation complex on the property, this would:
   - Decrease the usable area to build houses by 10 acres—all other constraints still apply.
   - Cost LSDC $8,000,000 to build.
   - Enhance the value of all houses so that LSDC would raise the selling prices of the homes by the following amounts:
     —Grand Estates (not on lake)—add 5% (e.g., add $35,000 to profit for Trumps, etc.)
     —Grand Estates (on lake)—add another $40,000 (e.g., profits increase by $40,000)
     —Glen Wood and Lakeview Patio Homes (nonpremium except Golden Pier)
       —Add 3% (e.g., add $12,600 to profit of Grand Cypress)
       —Premium Models—add a flat $16,000
       —Golden Pier—$0 (no change, so that it can still qualify as affordable housing)
     —Country Condominiums—add a flat $10,000

All of the selling price increases would be added to the original gross profits to determine the new gross profit (prior to subtracting the $8,000,000 for the complex).

The Report

Prepare a detailed report analyzing this project and make suggestions for the number of each type of each unit to be built. Give and support your recommendations on whether or not to build the sports/recreation complex. Do appropriate "what-if" analyses and give a summary of your final recommendations. (*Hint*: You may wish to solve as a linear program and round.)

CASE 3: Pentagonal Pictures, Inc.

Pentagonal Pictures produces motion pictures in Hollywood and distributes them nationwide. Currently, it is considering 10 possible films; these include dramas, comedies, and action adventures. The success of each film depends somewhat on both the strength of the subject matter and the appeal of the cast. Estimating the cost of a film and its potential box office draw is inexact at best; still, the studio must rely on its experts' opinions to help it evaluate which projects to undertake.

The following table lists the films currently under consideration by Pentagonal Pictures, including the projected cost and box office gross receipts.

			No-Name Cast		Big Star Cast	
Film	Rating	Type	Cost	Box Office Gross	Cost	Box Office Gross
Two-Edged Sword	PG-13	Action	$ 5M	$ 8M	$10M	$15M
Lady in Waiting	R	Drama	$12M	$20M	$25M	$35M
Yesterday	PG	Drama	$ 8M	$10M	$12M	$26M
Golly Gee	PG	Comedy	$ 7M	$12M	$15M	$26M
Why I Cry	PG-13	Drama	$15M	$30M	$30M	$45M
Captain Kid	PG	Comedy	$10M	$20M	$17M	$28M
Oh Yes!	R	Comedy	$ 4M	$ 7M	$ 8M	$12M
Nitty Gritty	PG	Comedy	$11M	$15M	$14M	$20M
The Crash	R	Action	$20M	$28M	$40M	$65M
Bombs Away	R	Action	$25M	$37M	$50M	$80M

In addition to these production costs, each movie will have a $1 million advertising budget, which will increase to $3 million if the movie is to have a "big star" cast. Assume that the studio receives 80% of a film's gross receipts. The company would like to maximize its net profit (gross profit − production costs − advertising costs) for the year.

Pentagonal has a production budget of $100 million and an advertising budget of $15 million. In addition, it would like to adhere to the following restrictions:

1. At least half the films produced should have a rating of PG or PG-13.

2. At least two comedies are to be produced.
3. If *The Crash* is produced, *Bombs Away* will not be.
4. At least one drama is to be produced.
5. At least two films should have big-name casts.
6. At least two PG films should be produced.
7. At least one action movie with a big-name cast should be produced.

Prepare a report for Pentagonal Pictures that recommends which films should be produced and with which casts. Detail how the budgets will be spent. Include in your report a sensitivity analysis that considers how varying budgets for both production costs and advertising (while spending at most a total of $115 million) would affect your recommendation. Finally, discuss the effects of Pentagonal's seven restrictions and report the effect of requiring only six of the seven, five of the seven, and four of the seven to hold.

CASES 4–6 ARE ON THE CD

Chapter 3 Extra Problems/Cases

41. FOOD SERVICE. Jami Gourmen operates a food truck that primarily services workers at construction and industrial sites. Jami is particularly popular because she only uses fresh ingredients purchased each morning from a local distributor. These include:

3 8-pound Swift turkey breasts (@ $20 each)	$ 60
3 12-pound Butchers roast beefs (@ $42 each)	$126
3 10-pound Hormel honey cured hams (@ $30 each)	$ 90
3 8-pound Alpine Swiss cheeses (@ $18 each)	$ 54
300 sourdough rolls	$ 60
miscellaneous condiments	$ 30

In addition to this $420 in fixed daily food costs, Jami incurs $280 in other daily costs, including gas, truck payments, insurance, and wages for an assistant.

Jami slices each of the meats and cheeses into one-ounce portions first thing in the morning; then she makes sandwiches from these ingredients, wraps them in plastic wrap, and stores them on the truck. She has space to store up to 300 sandwiches and has no problem selling all the sandwiches she makes. The following table gives the ounces of each ingredient in each of the five sandwiches she sells as well as her current selling prices.

Sandwich	Price	Turkey	Beef	Ham	Cheese
Turkey De-Lite	$2.75	4	0	0	1
Beef Boy	$3.50	0	4	0	1
Hungry Ham	$3.25	0	0	4	2
Club	$4.00	2	2	2	2
All Meat	$4.25	3	3	3	0

a. Formulate and solve for the optimal number of each type of sandwich to make daily. Given that she operates 200 days per year, what does Jami *net* annually from making and selling sandwiches?

b. What is the shadow price and the range of feasibility for cheese? Give a precise interpretation.

c. Jami is considering buying another bulk package of one of the meats or cheese. If only one additional bulk package is purchased, which would be the most profitable to Jami?

42. APPAREL INDUSTRY. Exclaim! Jeans is setting up a production schedule for the coming week. Exclaim! can make four jean products: men's and women's jackets and pants. Although it can make different sizes of each, the variation in material usage and labor between sizes is negligible. Each jacket and pair of pants goes through cutting and stitching operations before being boxed. The following table gives the profit, denim, cutting time, stitching time, and boxing time required *per* 100 *items*, as well as the total resource availabilities during the week.

Item	Profit	Denim (yd.)	Cutting (hr.)	Stitching (hr.)	Boxing (hr.)
Men's jackets	$2,000	150	3	4.0	.75
Women's jackets	$2,800	125	4	3.0	.75
Men's pants	$1,200	200	2	2.0	.50
Women's pants	$1,500	150	2	2.5	.50
Available this week		2500	36	36.0	8

a. Develop and solve a linear programming model. for Exclaim! Jeans which will maximize its profit for the week.

b. Suppose that, in addition to the existing restrictions, management wishes to produce at least 500 of each item. Add these constraints to your linear program and re-solve the problem. What is the result? To what do you attribute this result?

c. Suppose the minimum production for each item is 300. What is the optimal solution?

d. Suppose a constraint was added that requires at least 50% of the items manufactured to be women's items. How would this affect the optimal solution? Suppose instead that the added constraint requires that at least 50% of the items manufactured are men's items. How would this affect the optimal solution?

43. BANK LOAN POLICIES. Montana State Savings Bank is currently scheduling $10 million in deposits. First trust deeds yield 9%, second trust deeds 10.5%, automobile loans 12.25%, and business loans 11.75%. In addition, Montana State Savings Bank can invest in risk-free securities yielding 6.75%. Regulatory commissions of the state and federal governments require the following:

- At most one-third of deposits must be in risk-free securities.
- Home loans (first and second trust deed) cannot exceed the amount in risk-free securities.
- Business loans may not account for more than 49% of the total loans and trust deed investments.
- Automobile loans may not exceed 50% of the home loans (first and second trust deeds).

How should Montana State Bank invest the $10 million in deposits?

44. LABOR FORCE REDUCTIONS. Reductions in the defense budget are causing problems for Williams, Osborne, and Evans (WOE), a leading supplier of C^3I systems. WOE is faced with the need to downsize its labor force, while, at the same time, reduce waste and improve its competitive position. Its problem is to develop a mix of labor skills and functions which will not only be adequate to perform ongoing work but also meet certain headcount or cost reduction goals.

The following table summarizes the levels and types of people currently at WOE, along with other relevant data.

Manpower Characteristic Summary

Grade	Title	Weekly Salary	Job Category	Time Charges Direct	Time Charges Overhead	Current Headcount
100	Operations manager	$1600	Management	20%	80%	40
100	Department manager	$1200	Management	30%	70%	200
100	Section head	$1000	Management	80%	20%	900
100	Engineer	$ 800	Technical	100%	0%	6000
101	Technician	$ 600	Technical	100%	0%	3000
102	Business support	$ 500	Administration	30%	70%	150
103	Secretary	$ 350	Clerical	30%	70%	900

A number of goals have been established for the downsizing effort. These goals, which impact management's flexibility in achieving its objectives, are as follows:

- The combined total of operations managers and department managers should be reduced by 50%.
- The section head level of management should be eliminated.
- Operations managers are to number no more than 20% of the department managers.
- The ratio of technical personnel to management personnel must be at least 20:1.
- Between 5% and 10% of the total headcount must be clerical.
- Administration is to make up only 1% to 2% of the total headcount.
- Overhead charges must be between 5% and 10% of direct charges.
- Direct labor costs must equal $4.8 million weekly.
- At least six operations managers are needed to lead the diverse technical areas.
- Department manager direct labor charges should not make up more than 10% of the technical labor charges.
- The ratio of engineers to technicians should not exceed 4:1.
- To assure a balanced workforce, the percentage decrease or increase in headcount devoted to any grade level must not exceed 20%. (For example, there are currently 7140 grade-level 100s; this is 7140/11,190 = 63.8% of the total headcount. After downsizing, this percentage could increase or decrease by 0.2(63.8%) = 12.76%. Thus, the number of grade-level 100 personnel after downsizing must be between 51.04% and 76.56% of the new total headcount.)

What staff reduction plan minimizes WOE's total weekly payroll?

45. EDUCATION. The School of Business at Nebraska State University has received authorization from the university president to hire up to 20 new full-time faculty members and spend up to $1,275,000 a year in new salaries. The school is seeking professors at all levels: assistant, associate, and full professor.

The dean of the school would prefer not to hire full professors. Accordingly, while the school may not seek faculty for all 20 positions, the dean has authorized that at least 50% of the new faculty who are hired should be assistant professors, and at least 70% of the new faculty hired should be below the rank of full professor. The respective departments within the school have convinced the dean, however, that at least three full professors should be hired to provide expertise in the areas of telecommunications, ethics, and international management.

The following table gives the average salaries and years of experience for professors in each of the three ranks. The school is interested in hiring the most qualified mix of faculty (as measured by the total combined years of experience) that meets the university's limitations and the dean's authorizations.

Rank	Average Salary	Average Years of Experience
Assistant	$55,003	2
Associate	$69,885	7
Full	$93,471	14

Formulate and solve an integer linear programming model for the problem faced by the School of Business.

46. MANPOWER SCHEDULING. Guardmaster Services provides 24-hour security services for the 65-story Union Tower Building in New York City. The number of security officers required varies with the time of day. Peak demand for security officers occurs at the beginning of the workday, at lunch time, and at the end of the workday, while fewer officers are needed in the dead of night. A needs assessment of the minimum staffing requirements for Union Tower is given in the table at the bottom of this page.

Time Block	Mid–5AM	5AM–7AM	7AM–9AM	9AM–11AM	11AM–2PM	2PM–4PM	4PM–7PM	7PM–Mid
Guards Needed	5	8	12	10	15	9	12	7

Guardmaster schedules overlapping shifts so that it is not caught in total transition during any period of the day. Guards are scheduled for 8-hour shifts. There are eight such shifts, spread three hours apart, beginning at midnight (midnight to 8 A.M., 3 A.M. to 11 A.M., etc.)

a. Formulate and solve a *linear* program that will use the minimum number of security guards while meeting the minimum security requirements. (*Hint*: Break up the day into one-hour time blocks. Eliminate any obviously redundant constraints. There are eight nonredundant constraints.)
b. Interpret the shadow prices and ranges of feasibility. What is the effect of increasing the minimum number of officers required (i) from midnight to 5 A.M. to 7? (ii) from 9 A.M. to 11 A.M. to 12? (iii) from 11 A.M. to 2 P.M. to 17?
c. Suppose all officers whose shifts begin at midnight or 3 A.M. get a $5 per day bonus. Using the sensitivity output only, can you conclude whether the optimal solution will change?
d. Solve part (a) as an integer linear program. Did you get the same results? Do parts (b) and (c) make sense?

47. PRODUCTION. The R&D department of Little Trykes, Inc. has developed six new prototype tricycle models that can go into production in the coming year. The amount of plastic and the number of big and small wheels for each model, the monthly availabilities, the fixed cost of beginning production, and the unit profits excluding fixed costs are given in the following table.

	Unit Profit	Small Wheels	Big Wheels	Plastic (lb.)	Setup Costs
Liltryke	$1.50	3	0	.8	$16,500
Pinktryke	$2.00	1	2	1.2	$18,000
Herotryke	$2.25	2	1	1.5	$17,500
Robinhood	$2.75	2	1	2.1	$18,000
Jeeptryke	$3.00	2	1	1.8	$20,000
Monster	$3.50	0	3	3.0	$17,000
Available Monthly		10,000	8000	9000	

a. Formulate a MILP model for this problem to determine how many units of each product should be produced if Little Trykes wishes to keep the amount of money spent on new setups to a maximum of $70,000. (*Hint: Convert all production data to yearly figures.*)
b. Assume that the maximum expenditure of $70,000 for new setups is just one part of five company goals. The other four are:
   - If the Herotryke is produced, the Robinhood will not be produced.
   - At least four new models are to be produced.
   - If the Jeeptryke is produced, the Monster will also be produced.
   - At least 1500 pounds of plastic should be left over each month for use in the company's other products.

   Management wishes to meet at least four of these five goals. Formulate a MILP model for this problem and solve for the optimal production schedule.

48. PRODUCT DEVELOPMENT. The electric car division at Detroit Motor Company has promised that at least three new models will be delivered and produced within the next four years. Accordingly, the company has committed up to 50 engineers this year, 60 next year, and 75 in the third year to this division. In addition, a support staff of up to 8 this year, 12 the next, and 20 the third year will be made available to the division.

Currently, six models are under consideration, code-named the Alpha, Beta, Delta, Gamma, Kappa, and Sigma, respectively. The projected number of engineering and support staff hours required each year for the development of each model and the expected net present worth profits of each potential electric car line are given in the table at the bottom of the page.

Detroit has decided that if the Alpha model is produced the Beta model should not be produced, and if the Sigma model is produced the Alpha model should be produced. If engineers work an average of 2500 hours a year, and support staff an average of 2000 hours per year, develop and solve a model to determine which models the electric car division at Detroit should produce.

	Year 1		Year 2		Year 3		Present Net
	Engineer	Staff	Engineer	Staff	Engineer	Staff	Worth ($M's)
Alpha	15,000	5000	40,000	5000	40,000	8000	12
Beta	18,000	5000	25,000	4000	30,000	7000	11
Delta	19,000	3000	19,000	3000	19,000	3000	9
Gamma	20,000	4000	25,000	6000	30,000	7500	15
Kappa	8,000	2000	12,000	2000	18,000	3000	7
Sigma	22,000	5000	27,000	7000	32,000	8000	20

49. EVALUATION OF COLLEGE CAMPUSES. Following the ascent of native son Dick Cheney to the vice presidency of the United States, the state of Wyoming found increasing public interest in its attractions and experienced an upswing in population. Many of its new citizens were college-age individuals interested in an in-state education. Thus, in addition to its well-established university in Laramie, the University of Wyoming, the state began a three-campus Wyoming State University system with branches in Cody, Casper, and the state capital of Cheyenne. After a few years of operation Wyoming's governor has asked for an assessment of how efficiently each campus is operating.

A state committee of educational experts has chosen to measure the average college grade point average, the graduation rate, and the percent who find employment within three months of graduation as output measures to be weighed against entering SAT scores, the faculty to student ratio, and the university budget as inputs at each of the three campuses. The following table summarizes these values for each of the three campuses. Using a data envelopment analysis approach, which campuses seem to be inefficient?

Campus	Avg. SAT Score	Fac/ Stu Ratio	Budget ($M)	GPA	Grad. Rate	Percent Employ-ment
Cody	920	.068	$20.3	2.63	.43	.78
Casper	960	.059	$24.6	2.57	.55	.79
Cheyenne	1000	.061	$35.2	2.81	.54	.83

50. EVALUATION OF RETAIL STORES. Nottingham Enterprises owns five retail stores in the northern Ohio area. Two are large discount stores similar to Wal-Mart stores, two are department stores similar to Macy's department stores, and one is an upscale establishment similar to a Nieman-Marcus store. Management wishes measure how efficiently each store is operating based on annual sales (in $millions) in four departments (men's clothing, women's clothing, cosmetics, and jewelry) compared to the average family income of its credit card customers, the number of employees, and the overall store size of each of the five retail stores. The data are given in the following table.

- Determine which stores appear to be operating inefficiently and which are efficient.

Store	Income	# Employ.	Size Sq. Ft.	Men's Clothes	Women's Clothes	Cosmetics	Jewelry
Discount 1	32987	275	23876	9.6	20.3	16.3	4.1
Discount 2	32987	215	28755	10.3	17.9	15.5	4.6
Dept. Store 1	54321	185	19000	14.5	55.2	27.4	22.3
Dept. Store 2	54321	180	18750	15.2	44.8	26.8	28.4
Upscale Store	99765	85	11000	12.5	45.9	19.9	35.1

CASE 4: Horn Shoe Company[1]

The Horn Shoe Company, a firm primarily producing women's shoes in several factories throughout the country, is adding temporary workers to produce its new Fall line of shoes. The company projects that it will need 200,000 additional shoes for May, 300,000 for June, 270,000 for July, and 150,000 for August.

Horn can hire both experienced and novice workers on a temporary basis. After one month, novice workers are then classified as apprentices; after two months, they are considered "experienced" workers. Cost and productivity estimates for temporary workers are as follows:

	Novice	Apprentice	Experienced
Costs			
Hiring	$1000	—	$1500
Training	$ 800/month	$ 400/month	$ 0/month
Salary	$1600/month	$2400/month	$3000/month
Termination	$ 250	$ 500	$ 700
Productivity			
Shoes	400/month	600/month	800/month

Temporary workers are hired at the beginning of a month, and when they are terminated, this occurs at the end of a month. All temporary workers must be terminated by the end of August. Prepare a report recommend-

[1] Based on a case developed by Dr. Zvi Goldstein, California State University, Fullerton.

ing an optimal hiring/termination policy over the four-month period which minimizes Horn's total costs while meeting the additional production requirements. Include an analysis of the effects of changes on training and termination costs and on required production quotas.

Formulation Hint: Define the following variables for *each month*:

- Number of novices hired for the month
- Number of experienced workers hired for the month
- Total number of apprentices during the month
- Total number of experienced workers during the month
- Total number of novices terminated at the end of the month
- Total number of apprentices terminated at the end of the month
- Total number of experienced workers terminated at the end of the month

Be sure to include constraints for each month expressing:

(Total experienced workers in month i)
= (Total experienced workers in month i–l)
+ (Number of experienced workers hired in month i)
− (Number of experienced workers terminated in month i–l)
+ (Number of apprentices in month i–l)
− (Number of apprentices terminated in month i–l)

and

(Total Number of apprentices in month i)
= (Total novices in month i–l)
− (Number of novices terminated in month i–l)

CASE 5: Todd & Taylor

Jerry Todd, a managing partner of the investment firm of Todd and Taylor, is designing a portfolio for Greg Edmonds. Greg has $500,000 cash to invest, and Jerry has identified 12 different investments, falling into four broad categories that both Jerry and Greg feel would be potential candidates for the portfolio. In addition, Todd has learned that two investment partnerships will be open to investment six months from now and can be considered to be potential investments at that time.

The table at the top of the next page lists the investments and their important characteristics. The expected annual after-tax returns accounts for all commissions and service charges. Note that Beekman Corporation stock and Beekman Corporation bonds are two separate investments, whereas Calton REIT is a single investment, a stock that is also a real estate investment.

Greg will hold any money invested in the one-year CD for the entire year. All other investments made at the beginning of the year will be sold at the end of the first six months at which time it is assumed he will have earned 50% of the annual return. The initial amounts in these investments plus the returns will then be available for investments during the second six months. Returns on all investments (except the one-year CD) made during the second six months should again earn 50% of the annual return. Jerry wishes to determine the investments Greg should make during the first six months of the year and the investments he should make during the second six months of the year that will maximize the total value of the portfolio at the end of the year. However, the investments should be made subject to a number of concerns Greg has raised regarding his portfolio including:

1. Throughout the year the average risk factor must be no greater than 55.
2. Throughout the year the average liquidity factor must be at least 85.
3. At least $10,000 is to be invested in the Beekman Corporation.
4. At least 10% but not more than 50% of the non-"money" portion of the portfolio should be invested in each category of investment during each six-month period.
5. With the exception of the money category investments, no more than 20% of the portfolio ($100,000) should be in any one investment.
6. At least $25,000 should be kept in the money market fund throughout the year.
7. A minimum investment of $125,000 should be in bonds throughout the year.
8. Throughout the year at most 40% of the total portfolio in investments with expected annual after-tax returns of less than 10% are to have risk factors exceeding 25.
9. Throughout the year at least one-half of the portfolio must be totally liquid (i.e., have a liquidity factor of 100).

Prepare a report to assist Jerry Todd in developing a portfolio for Greg Edmonds. Include in the report the following analyses:

- The expected after-tax return on the investment plan
- By how much each of Greg's restrictions were met, including the determination of the overall risk and liquidity factors
- The expected after-tax return for additional investment above $500,000
- The most sensitive after-tax return estimates that could affect the optimal solution
- The effect of a relaxation in the minimum dollar amount to be placed in the money market fund

Investments Currently Available

Category	Investment	Estimated Annual After-Tax Return	Liquidity Factor	Risk Factor
Stocks				
	Beekman Croporation	8.5%	100	62
	Taco Grande	10.0%	100	71
	Calton REIT	10.5%	100	78
	Qube Electronics	12.0%	100	95
Bonds				
	Berlin Power	5.8%	95	19
	Beekman Corporation	6.4%	92	33
	Metropolitan Transit	7.2%	79	23
Real Estate				
	Socal Apartment Part.	9.0%	0	50
	Calton REIT	(See above)		
Money				
	T-Bill Account	4.6%	80	0
	Money Market Fund	5.2%	100	10
	Six Month CD	7.2%	0	0
	One Year CD	7.8%	0	0

Investments Available in Six Months

Category	Investment	Estimated Annual After-Tax Return	Liquidity Factor	Risk Factor
Investment Partnerships				
	Abid.com	9.5%	20	68
	Parkstone Medical	12.0%	40	79

Case 6: Sun World Citrus

Many citrus processors located in southern California have found it increasingly less profitable to operate in the area and have simply ceased operation. This has increased demand for citrus products from those remaining businesses in the region. Sun World Citrus, located in the Coachella Valley, approximately 100 miles from Los Angeles, is one citrus processor remaining in business.

Sun World currently operates a plant with two production lines. Each line requires four workers and is capable of processing 100,000 boxes of oranges annually. There are also two supervisors at the plant. Workers earn \$20,000 per year, while each supervisor earns \$40,000 annually. Fixed yearly operating expenses amount to about \$100,000 per year.

Sun World sells a 60-count box of oranges for \$6.20 per box. It costs Sun World approximately two cents per orange to grow, pick, and transport it to the plant. Since demand has been about 200,000 boxes annually (the plant's capacity), no changes have been made in quite some time. However, a recent study done for Sun World indicates that demand will increase to 500,000 boxes within a year.

The company is evaluating three alternatives to meet the projected increase in demand: (1) modernizing its current equipment; (2) expanding the current plant by adding another (modern) production line to its two lines; and (3) purchasing and building another plant approximately 25 miles from the existing facility.

The following table details some of the data management is considering.

Option	Additional Workers	Added Supervisors	Additional Yearly Fixed Cost*	Additional Yearly Capacity
Modernize equipment	2	0	\$ 30,000	25,000/ machine
Plant expansion	5	1	\$ 50,000	150,000
New plant	10	2	\$500,000	280,000

*Includes the cost of financing each improvement.

Management wishes to add no more than 15 new workers or two new supervisors and would like to know which projects to undertake to meet at least 100,000 of the anticipated 300,000 box increase in yearly demand.

Prepare a report for Sun World Citrus that recommends a course of action that maximizes net additional profit. Discuss the ramifications of your recommendation.