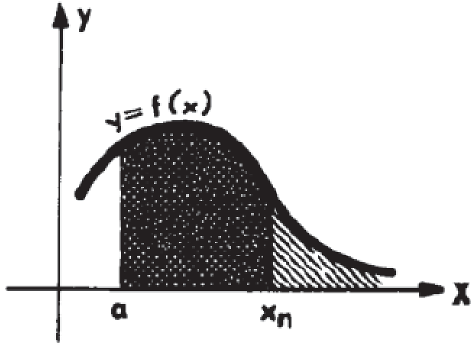
Μη-γνήσια ολοκληρώματα

Α΄ είδους



$$\int_{\alpha}^{\infty} f(x)dx = \lim_{M\to\infty}\int_{\alpha}^{M} f(x)dx$$

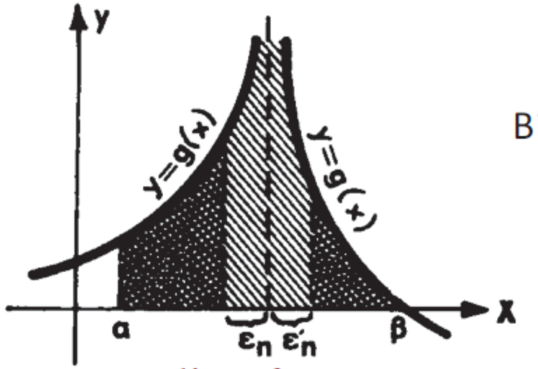
$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{\alpha} f(x)dx + \int_{\alpha}^{\infty} f(x)dx$$

υπολογίζονται ξεχωριστά

$$\int_{-\infty}^{\infty} f(x)dx = \lim_{M\to\infty}\int_{-M}^{M} f(x)dx$$

πρωτεύουσα τιμή

Β΄ είδους



Αν $\lim_{x\to c} f(x) = \pm\infty$

$$\int_{\alpha}^{c} f(x)dx = \lim_{\varepsilon\to 0}\int_{\alpha}^{c-\varepsilon} f(x)dx$$

$$\int_{\alpha}^{\beta} f(x)dx = \int_{\alpha}^{c} f(x)dx + \int_{c}^{\beta} f(x)dx$$

υπολογίζονται ξεχωριστά

$$\int_{\alpha}^{\beta} f(x)dx = \lim_{\varepsilon\to 0}\left\{\int_{\alpha}^{c-\varepsilon} f(x)dx + \int_{c-\varepsilon}^{\beta} f(x)dx\right\}$$

πρωτεύουσα τιμή

Ασκήσεις

$$I = \int_2^{+\infty} \frac{1}{x^2-1}\,dx = \lim_{A\to+\infty} \int_2^{A} \frac{1}{x^2-1}\,dx$$

$$\int_2^{A} \frac{1}{(x-1)(x+1)}\,dx = \int_2^{A} \left(\frac{1/2}{x-1} - \frac{1/2}{x+1}\right) dx =$$

$$= \frac{1}{2}\ln(x-1)\Big|_2^A - \frac{1}{2}\ln(x+1)\Big|_2^A =$$

$$= \frac{1}{2}\left[\ln(A-1) - (\ln(A+1) - \ln 3)\right] =$$

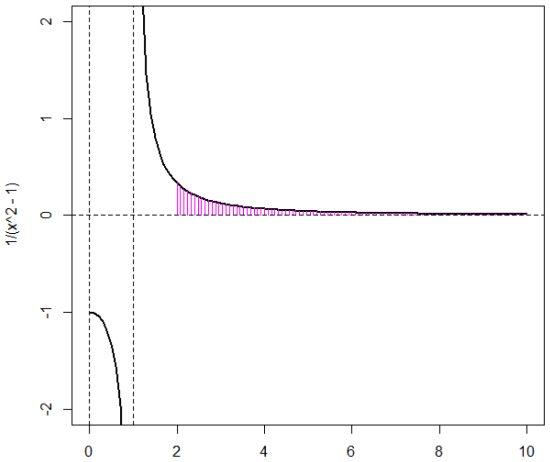
$$= \frac{1}{2}\left[\ln(A-1) - \ln(A+1) + \ln 3\right] =$$

$$= \frac{1}{2}\ln\frac{3(A-1)}{A+1}$$

$$\frac{1}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1} \Rightarrow 1 = A(x+1) + B(x-1) \Rightarrow$$

$$\Rightarrow 1 = (A+B)x + A - B \Rightarrow \begin{array}{l} A+B=0 \Rightarrow A=-B \\ A-B=1 \Rightarrow A=1/2 \end{array}$$

$$\Rightarrow \frac{1}{x^2-1} = \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$$



$$I = \frac{1}{2}\lim_{A\to+\infty} \ln\frac{3(A-1)}{A+1} = \frac{1}{2}\ln\left[3\lim_{A\to+\infty}\frac{3(A-1)}{A+1}\right]$$

(γιατί η συνάρτηση ln είναι συνεχής)

$$\frac{A-1}{A+1} = \frac{A(1-1/A)}{A(1+1/A)} \Rightarrow \lim_{A\to+\infty}\frac{A-1}{A+1} = \lim_{A\to+\infty}\frac{1-1/A}{1+1/A} = 1$$

$$\Rightarrow \boxed{I = \frac{1}{2}\ln 3}$$

Ασκήσεις

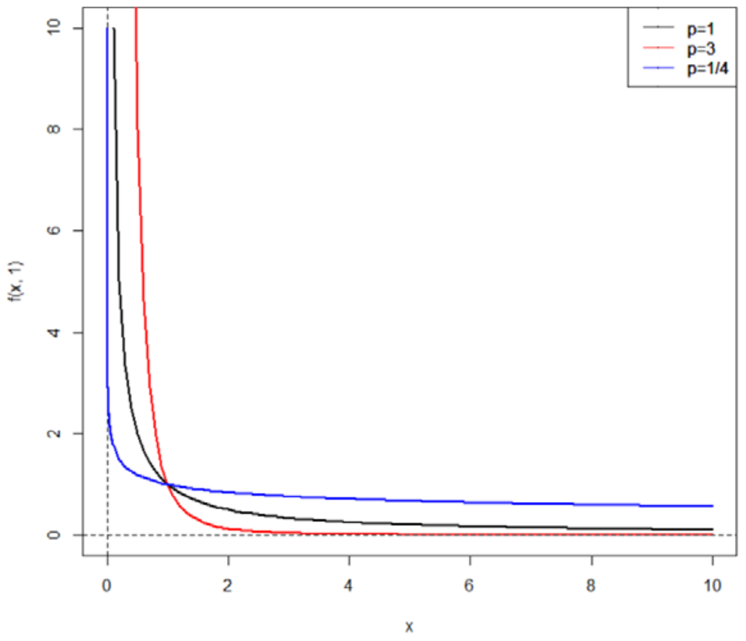
$$I=\int_a^{+\infty}\frac{1}{x^p}\,dx \quad (a>0)\ (p>0)$$

$$\int_a^{+\infty}\frac{1}{x^p}\,dx=\lim_{A\to+\infty}\int_a^A\frac{1}{x^p}\,dx$$

α) $p=1$: $I=\int_a^{+\infty}\frac{1}{x}\,dx=\lim_{A\to+\infty}\int_a^A\frac{1}{x}\,dx$

$$\int_a^A\frac{1}{x}\,dx=\left[\ln x\right]_a^A=\ln A-\ln a$$

$\Rightarrow I=\lim_{A\to+\infty}\ln A-\ln a=+\infty$ αποκλίνει

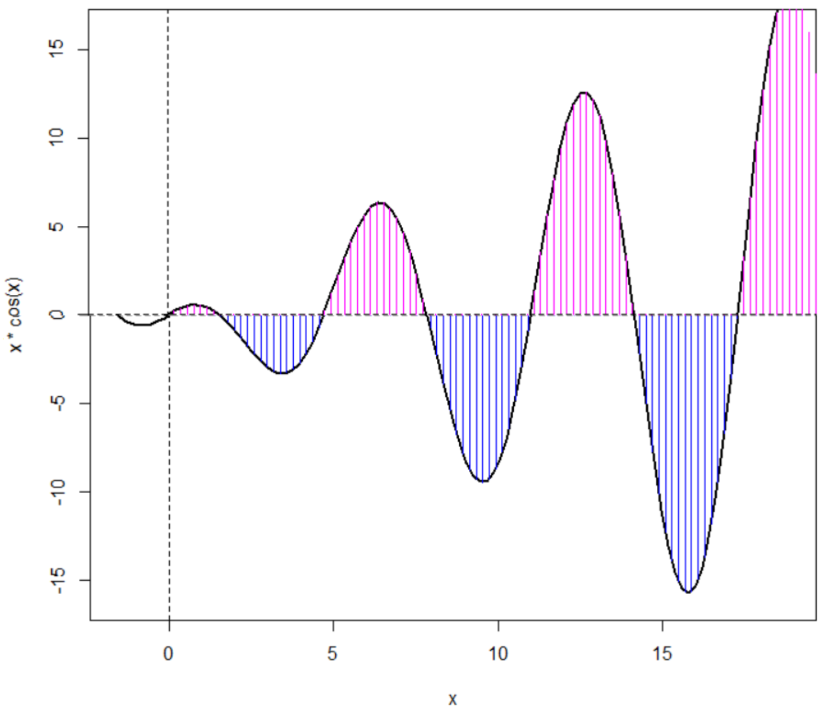


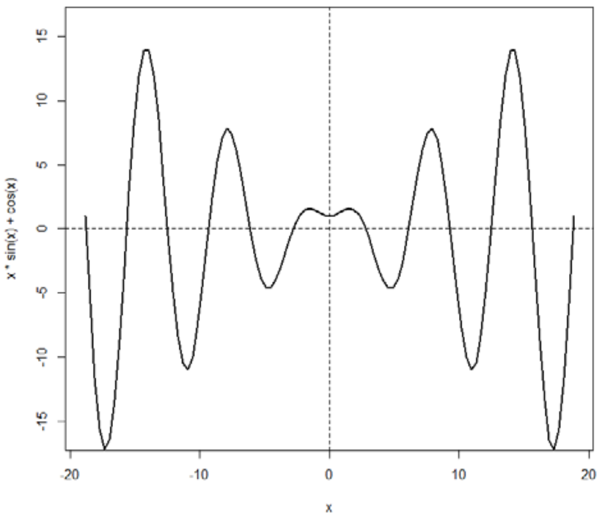
β) $(p\neq 1)$ $\int_a^A x^{-p}\,dx=\left[\frac{x^{-p+1}}{-p+1}\right]_a^A=$

$$=\frac{A^{-p+1}-a^{-p+1}}{-p+1}$$

$\Rightarrow I=\lim_{A\to+\infty}\frac{A^{-p+1}-a^{-p+1}}{1-p}=$ $\nearrow$ $\frac{a^{1-p}}{1-p}$, $p>1$; $\searrow$ αποκλ., $p<1$

Ασκήσεις





$$I = \int_0^{+\infty} x\,\text{συν}x\,dx = \lim_{A\to+\infty}\int_0^A x\,\text{συν}x\,dx$$

$$\int_0^A x\,\text{συν}x\,dx = \int_0^A x(\text{ημ}x)'\,dx =$$

$$= [x\,\text{ημ}x]_0^A - \int_0^A \text{ημ}x\,dx =$$

$$= A\,\text{ημ}A + [\text{συν}x]_0^A = A\,\text{ημ}A + \text{συν}A - 1$$

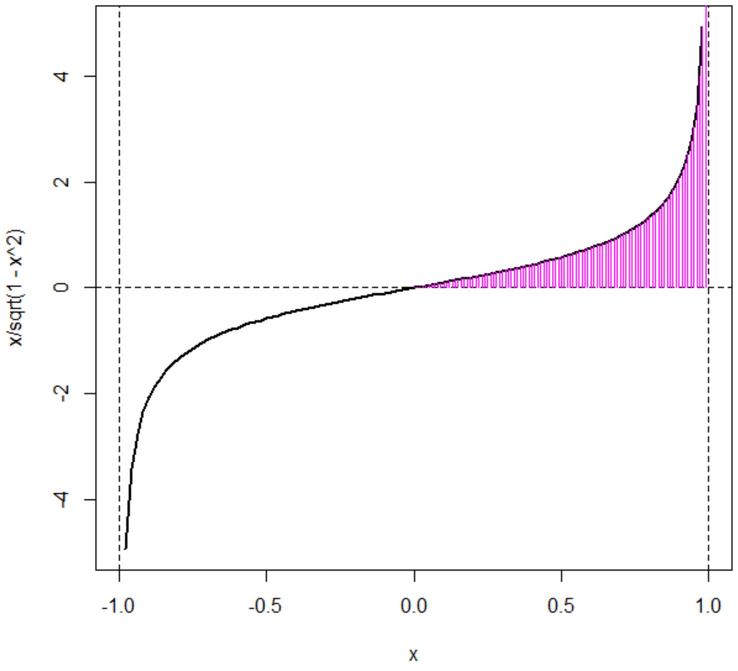
$$\Rightarrow I = \lim_{A\to+\infty}(A\,\text{ημ}A + \text{συν}A) - 1$$

Είναι γνωστό ότι $|\text{ημ}A| < 1$ και $|\text{συν}A| < 1$

Αλλά $\nexists$ το $\lim_{A\to+\infty}(A\,\text{ημ}A + \text{συν}A)$

Άρα το γενικευμένο ολοκλήρωμα δεν συγκλίνει

Ασκήσεις




```
> f<-function(x){x/sqrt(1-x^2)}
> integrate(f,lower=0,upper=1)
1 with absolute error < 5e-06
```

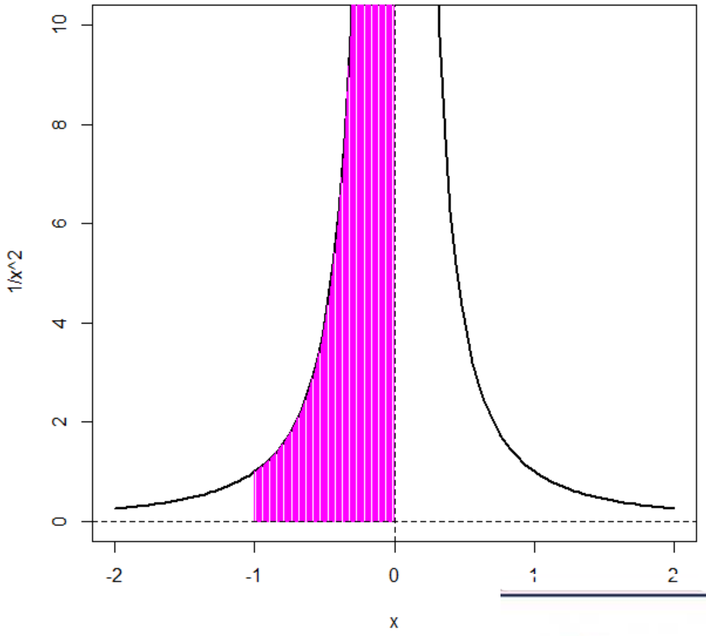
$$I=\int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx=\lim_{\varepsilon\to 0^+}\int_0^{1-\varepsilon}\frac{x}{\sqrt{1-x^2}}\,dx$$

$$=-\frac{1}{2}\int_0^{1-\varepsilon}(1-x^2)^{-1/2}\,d(1-x^2)=-\frac{1}{2}\left[\frac{(1-x^2)^{1/2}}{\frac{1}{2}}\right]_0^{1-\varepsilon}=$$

$$=-\left(1-(1-\varepsilon)^2\right)^{1/2}+1$$

$$\Rightarrow I=\lim_{\varepsilon\to 0^+}-\left(1-(1-\varepsilon)^2\right)^{1/2}+1=1$$

Ασκήσεις



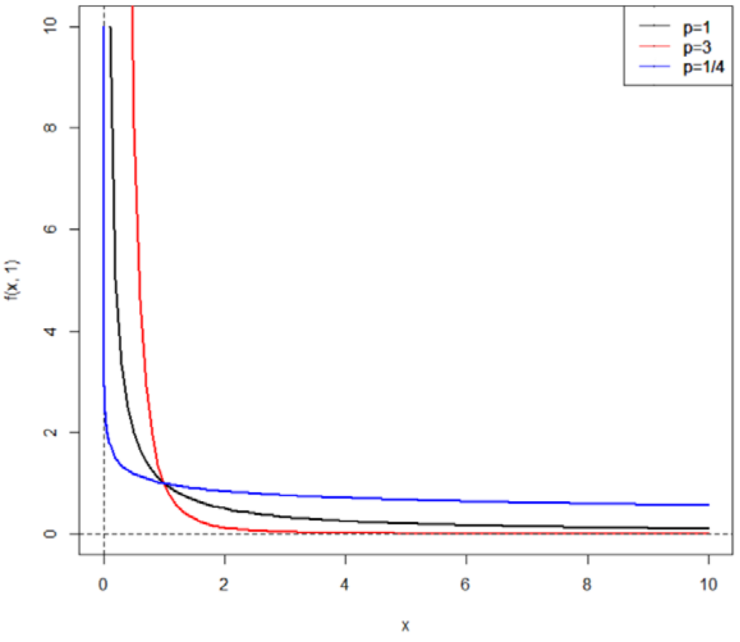

```
> f<-function(x){1/x^2}
> integrate(f,lower=-1,upper=0)
Error in integrate(f, lower = -1, upper = 0) :
  the integral is probably divergent
```

$$I=\int_{-1}^{0}\frac{1}{x^2}dx=\lim_{\varepsilon\to 0^+}\int_{-1}^{-\varepsilon}\frac{1}{x^2}dx$$

$$\int_{-1}^{-\varepsilon}x^{-2}dx=\left[\frac{x^{-1}}{-1}\right]_{-1}^{-\varepsilon}=-\left[(-\varepsilon)^{-1}+1\right]$$

$$I=\lim_{\varepsilon\to 0^+}\varepsilon^{-1}+1=+\infty$$ (αποκλίνει)

Ασκήσεις



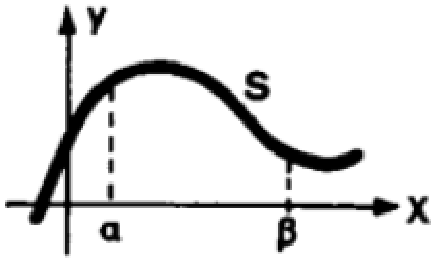
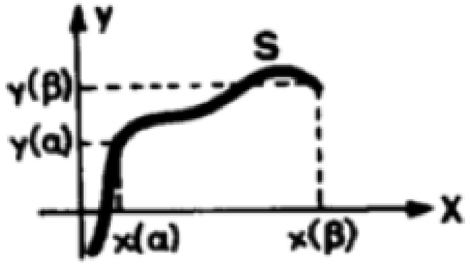
$$I_B = \int_0^a \frac{1}{x^p}\,dx \quad (a>0),\ (p>0)$$

p=1: $\int_0^a \frac{1}{x}\,dx = \lim_{\varepsilon \to 0^+} \int_\varepsilon^a x^{-1}\,dx =$

$$= \lim_{\varepsilon \to 0^+} (\ln a - \ln \varepsilon) = +\infty \ (\text{αποκλίνει})$$

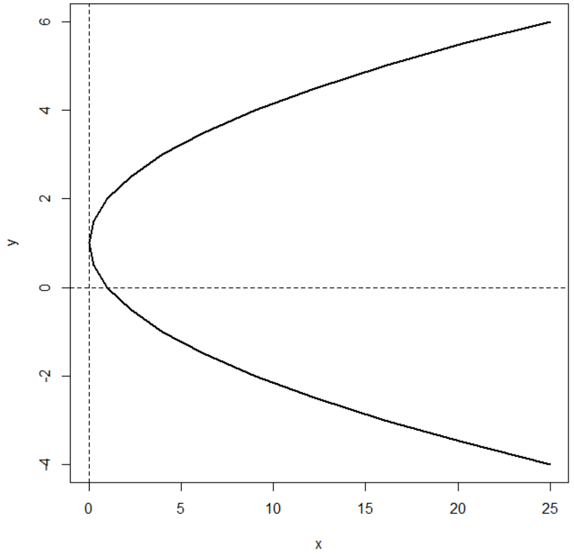
p≠1: $\int_0^a \frac{1}{x^p}\,dx = \lim_{\varepsilon \to 0^+} \int_\varepsilon^a x^{-p}\,dx =$

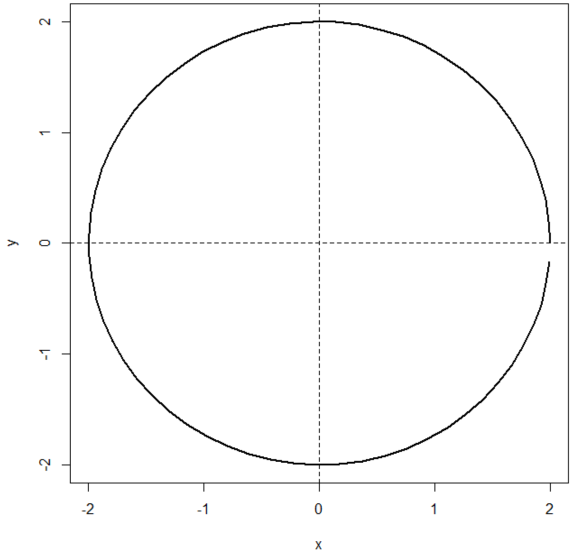
$$= \lim_{\varepsilon \to 0^+} \frac{a^{-p+1} - \varepsilon^{-p+1}}{-p+1} = \begin{cases} \text{αποκλίνει}, & p>1 \\ \frac{a^{1-p}}{1-p}, & 0<p<1 \end{cases}$$

Εφαρμογή	Σχήμα	Τύπος
Μήκος καμπύλης y=f(x) από x=α μέχρι x=β		$s=\int_{\alpha}^{\beta}\sqrt{1+(y')^2}\,dx$ όπου $y'=\dfrac{dy}{dx}$
Μήκος καμπύλης x=x(t), y=y(t) από t=α μέχρι t=β		$s=\int_{\alpha}^{\beta}\sqrt{\dot{x}^2+\dot{y}^2}\,dt$ όπου $\dot{x}=\dfrac{dx}{dt}$, $\dot{y}=\dfrac{dy}{dt}$

Γραφική Παράσταση στην R: για παραμετρικές συναρτήσεις

```
> dat <- data.frame(t=seq(-5, 5, 0.5))
> x<-function(t) t^2
> y<-function(t) t+1
> dat$x<-x(dat$t)
> dat$y<-y(dat$t)
> with(dat, plot(x,y, type="l",lwd=2))
> dat
      t     x    y
1  -5.0 25.00 -4.0
2  -4.5 20.25 -3.5
3  -4.0 16.00 -3.0
4  -3.5 12.25 -2.5
5  -3.0  9.00 -2.0
6  -2.5  6.25 -1.5
7  -2.0  4.00 -1.0
8  -1.5  2.25 -0.5
9  -1.0  1.00  0.0
10 -0.5  0.25  0.5
11  0.0  0.00  1.0
12  0.5  0.25  1.5
13  1.0  1.00  2.0
14  1.5  2.25  2.5
15  2.0  4.00  3.0
16  2.5  6.25  3.5
```



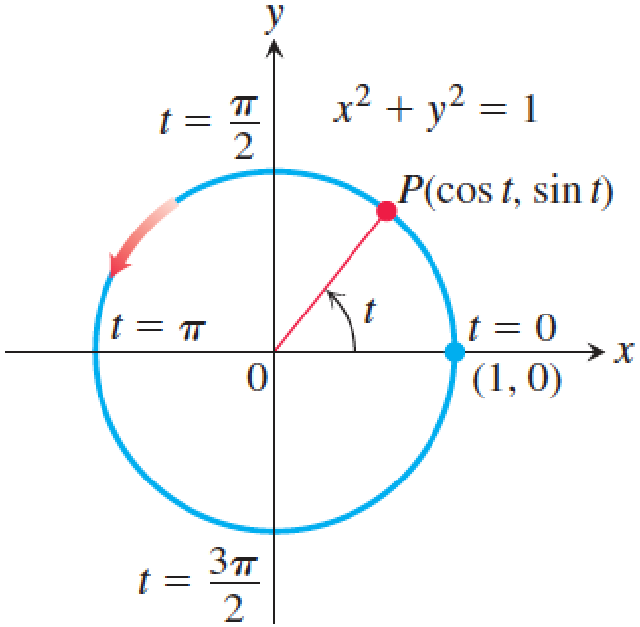



```
> r<-2
> dat <- data.frame(t=seq(0, 2*pi, 0.1))
> x<-function(t) r*cos(t)
> y<-function(t) r*sin(t)
> dat$x<-x(dat$t)
> dat$y<-y(dat$t)
> with(dat, plot(x,y, type="l",lwd=2))
> dat
     t          x          y
1  0.0 2.00000000 0.00000000
2  0.1 1.99000833 0.19966683
3  0.2 1.96013316 0.39733866
4  0.3 1.91067298 0.59104041
5  0.4 1.84212199 0.77883668
6  0.5 1.75516512 0.95885108
7  0.6 1.65067123 1.12928495
8  0.7 1.52968437 1.28843537
9  0.8 1.39341342 1.43471218
10 0.9 1.24321994 1.56665382
11 1.0 1.08060461 1.68294197
12 1.1 0.90719224 1.78241472
13 1.2 0.72471551 1.86407817
14 1.3 0.53499766 1.92711637
15 1.4 0.33993429 1.97089946
16 1.5 0.14147440 1.99498997
```

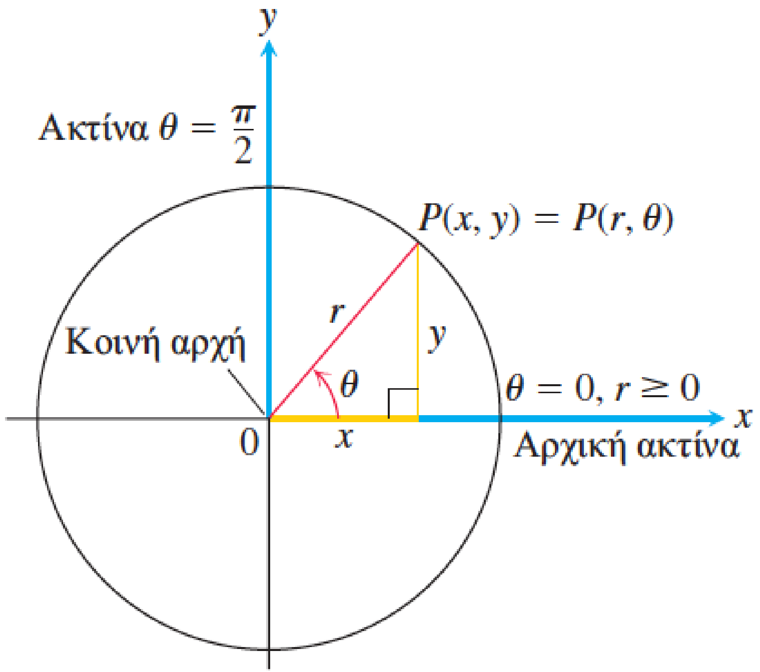
Αλλαγή συστήματος συντεταγμένων: Πολικές Συντεταγμένες (στο επίπεδο)

Equations Relating Polar and Cartesian Coordinates

$$x = r\cos\theta, \qquad y = r\sin\theta, \qquad r^2 = x^2 + y^2, \qquad \tan\theta = \frac{y}{x}$$



ΣΧΗΜΑ 11.4 Οι εξισώσεις $x = \cos t$ και $y = \sin t$ περιγράφουν κίνηση επί του κύκλου $x^2 + y^2 = 1$. Το βέλος δείχνει τη φορά αύξησης του t (φορά διαγραφής του κύκλου). (Παράδειγμα 3.)

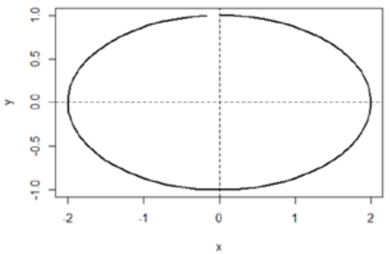


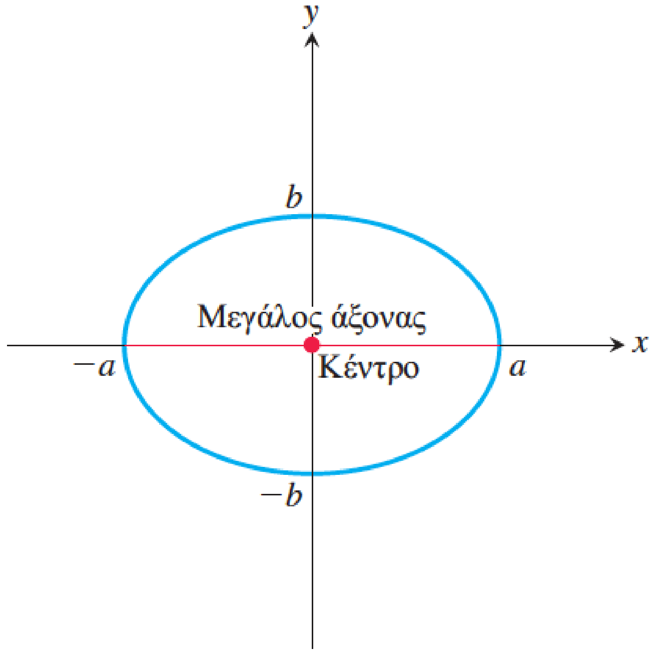
ΣΧΗΜΑ 11.26 Ο συνήθης τρόπος συσχέτισης πολικών και καρτεσιανών συντεταγμένων.

Ασκήσεις

$$(x, y) = (f(t), g(t))$$

$$x = a \sin t \quad y = b \cos t,\ a > b \text{ and } 0 \le t \le 2\pi$$





Γραφική παράσταση της έλλειψης

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,\ a > b,$$

όπου ο μεγάλος άξονας είναι οριζόντιος.

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = a^2 \cos^2 t + b^2 \sin^2 t$$

$$= a^2 - (a^2 - b^2) \sin^2 t$$

$$= a^2 [1 - e^2 \sin^2 t]$$

$e = \sqrt{1 - \frac{b^2}{a^2}}$ (eccentricity, not the number 2.71828 . . .)

$$P = 4a \int_0^{\pi/2} \sqrt{1 - e^2 \sin^2 t}\, dt.$$

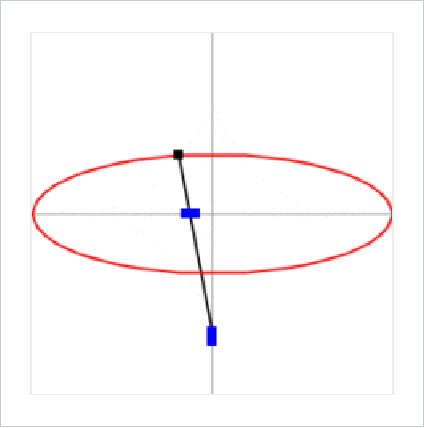
```
> a<-1; b<-1;              # a>b
> e<-sqrt(1-b^2/a^2)
> f<-function(x){sqrt(1-e^2*sin(x)^2)}
> integrate(f,lower=0,upper=pi/2)$value
[1] 1.570796
> 4*a*integrate(f,lower=0,upper=pi/2)$value
[1] 6.283185
> 2*pi
[1] 6.283185
```

```
> a<-2; b<-1;
> e<-sqrt(1-b^2/a^2)
> integrate(f,lower=0,upper=pi/2)$value
[1] 1.211056
> 4*a*integrate(f,lower=0,upper=pi/2)$value
[1] 9.688448
```

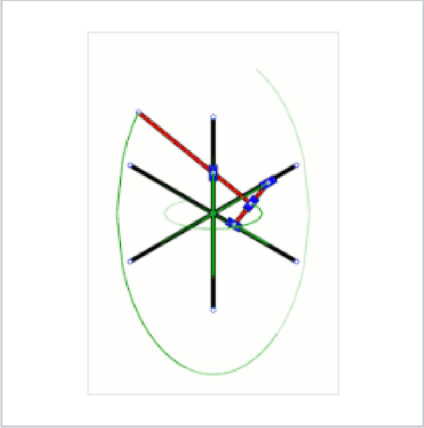
Trammel of Archimedes

Ελλειψογράφος του Αρχιμήδη

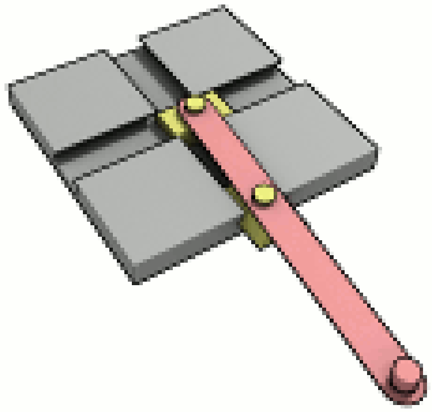
From Wikipedia, the free encyclopedia



Trammel of Archimedes as ellipsograph



Trammel of Archimedes with three sliders



Let C be the outer end of the rod, and A, B be the pivots of the sliders. Let AB and BC be the distances from A to B and B to C, respectively. Let us assume that sliders A and B move along the y and x coordinate axes, respectively. When the rod makes an angle θ with the x-axis, the coordinates of point C are given by

$$x = (AB + BC)\cos\theta$$
$$y = BC\sin\theta$$

These are in the form of the standard parametric equations for an ellipse in canonical position. The further equation

$$\frac{x^2}{(AB+BC)^2} + \frac{y^2}{(BC)^2} = 1$$

is immediate as well.

Vatican

Bramante staircase, built in 1505 and Momo 1932

St. Vitus Cathedral

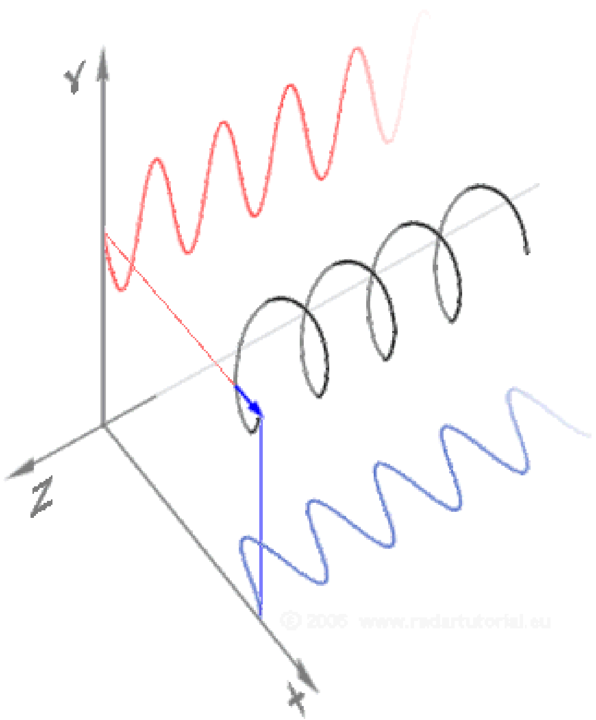
This nearly 100 metres high tower offers a unique view of Prague. The climb of more than 280 steps also presents the view of the bells of the Cathedral.

Archimedes

Helix The equations

$$x = a\cos t, \quad y = a\sin t, \quad z = bt \quad (a > 0, b > 0)$$

describe a so-called *helix* as a *right screw*. If the observer is looking into the positive direction of the z-axis, which is at the same time the axis of the screw, then the screw climbs in a counter-clockwise direction. A helix winding itself in the opposite orientation is called a *left screw*.

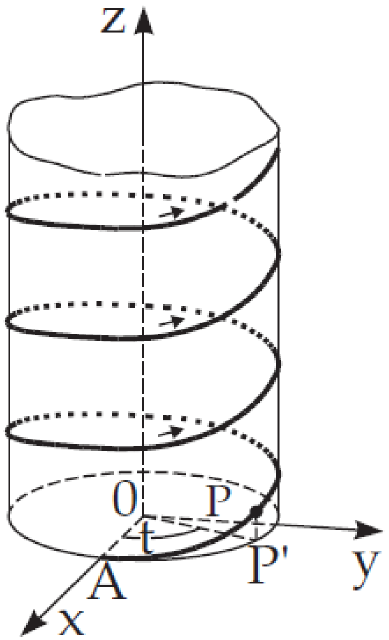


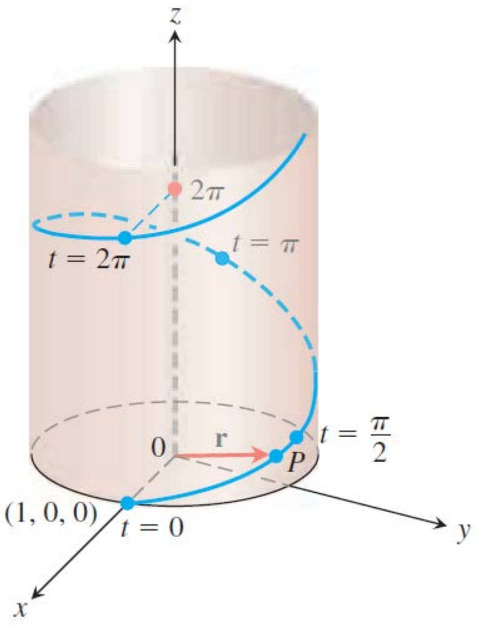
α=b=1

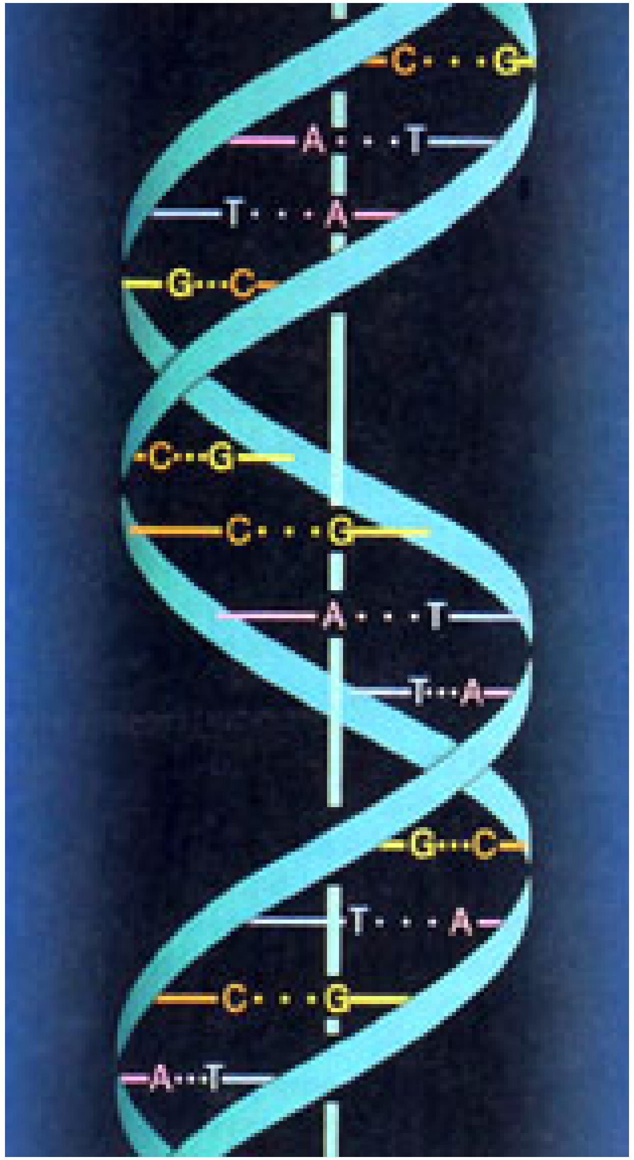
$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt.$$

$$L = \int_0^{2\pi} \sqrt{(-\sin t)^2 + (\cos t)^2 + (1)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{2}\, dt = 2\pi\sqrt{2}$$

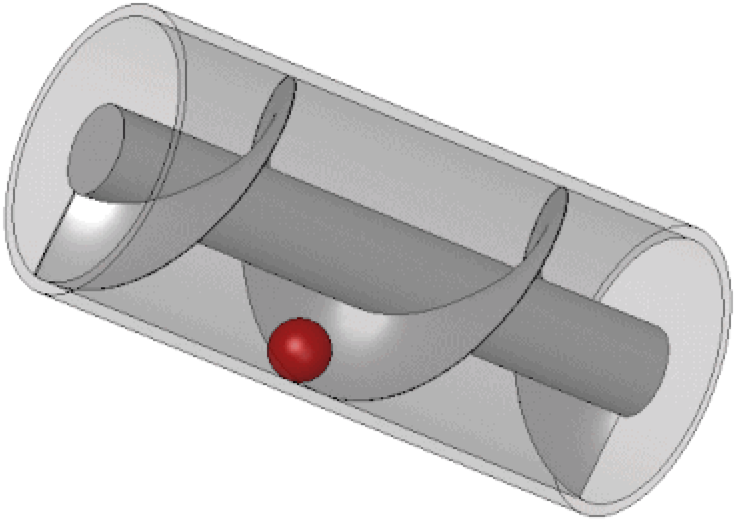






Watson and Crick with their model of DNA (1953)

Αντλία νερού στην Αίγυπτο το 1950
που χρησιμοποιεί τον κοχλία του Αρχιμήδους



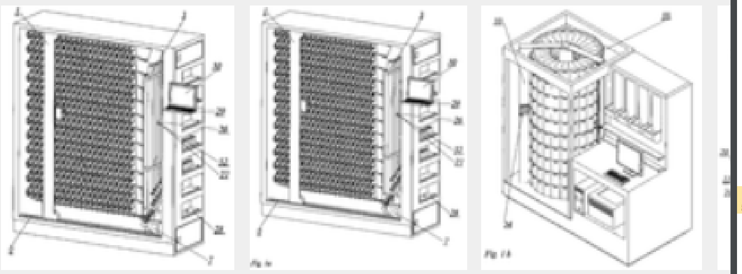
Σύγχρονη μορφή κοχλία του Αρχιμήδη που αντικατέστησε ανεμόμυλους και χρησιμοποιούνται για την αποξήρανση [πόλντερ]() στην [Ολλανδία]()

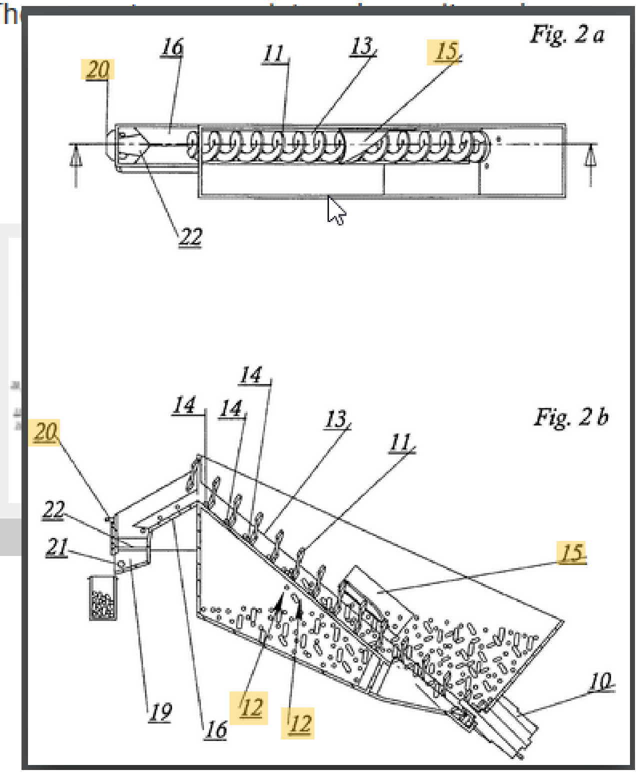
Pharmaceutical dispensing system for medicament and pre-packaged medication

Abstract

This invention focuses on resolving outstanding problems of pharmaceutical dispensing systems. A need for dispensing of both medicaments and pre-packed medications has been addressed. This invention comprises an accurate counting system for multiple concurrently dispensed medicaments. Accurate, high speed medicament dispensing with little or no jam ups is achieved. Self recovery is provided to prevent medicament over counts. Machine self learn modes replace manual intervention by pharmacy personnel. Concurrent multi-tasking is provided and a quick action robotic arm further expedites the dispensing process. A touch screen computer is provided for coordinating the individual modules and overall control. Th
restricts access to unauthorized persons.

Images (9)





Classifications

US7853355B1

United States

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Inventor: Waldemar Willemse, Clasina Aletta Willemse, Werner Waldemar Willemse

Current Assignee : McKesson Canada Corp

Worldwide applications

2006 • US

Application US11/482,889 events

2006-07-07 • Application filed by Individual

2006-07-07 • Priority to US11/482,889

2010-12-14 • Application granted

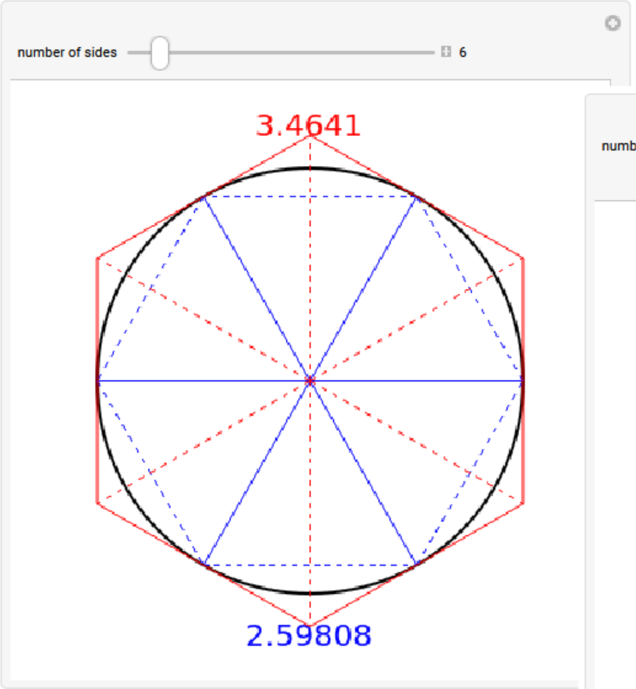
2010-12-14 • Publication of US7853355B1

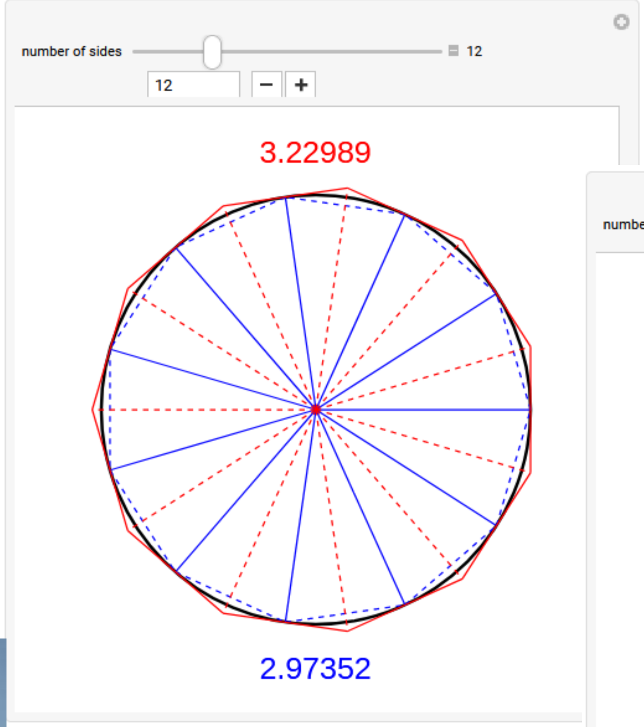
Status • Active

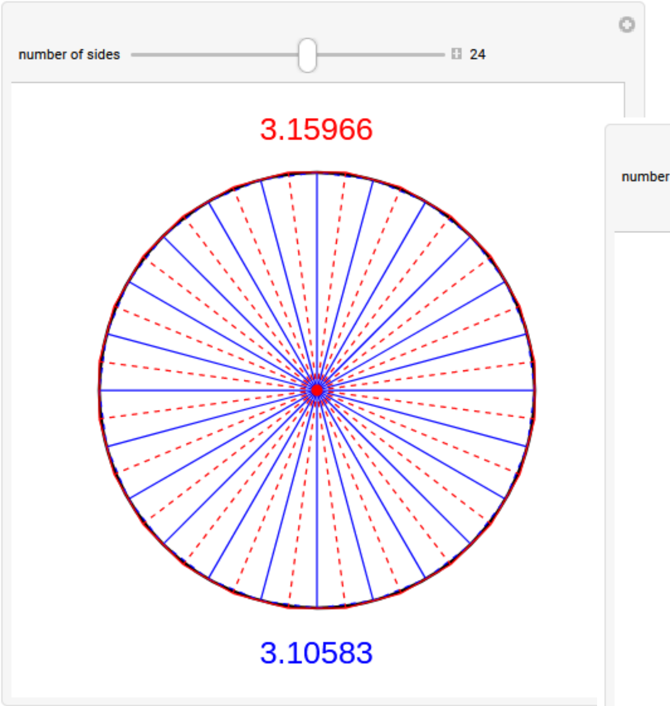
WOLFRAM Demonstrations Project

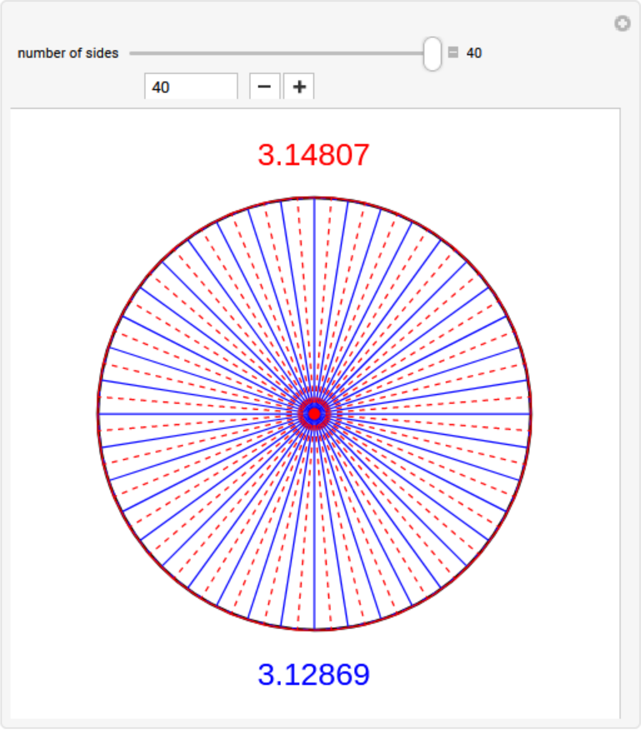
Archimedes' Approximation of Pi

https://demonstrations.wolfram.com/ArchimedesApproximationOfPi/





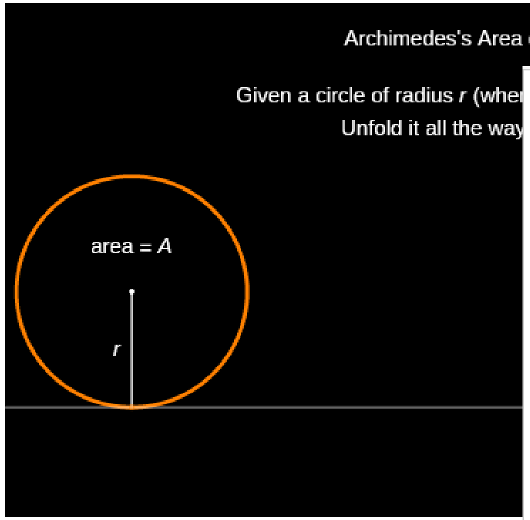


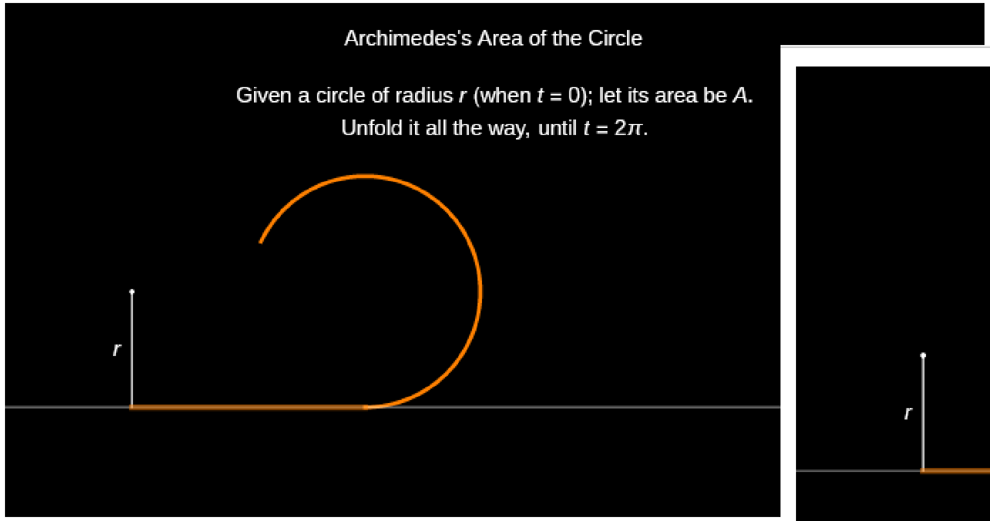


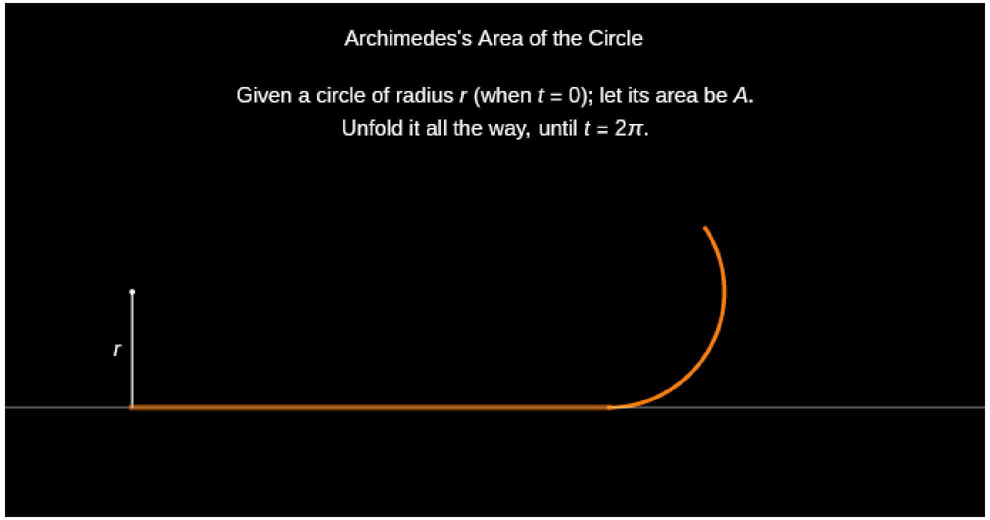
WOLFRAM Demonstrations Project

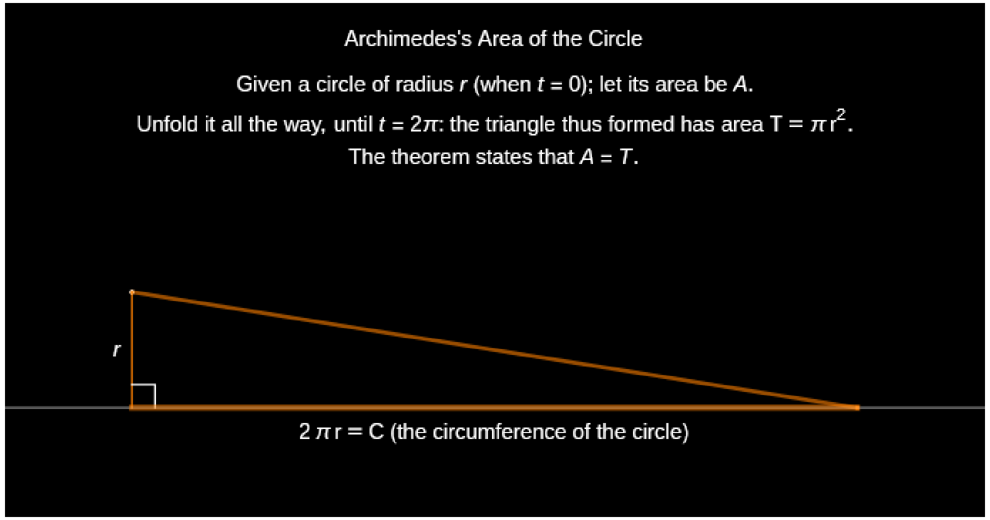
Archimedes's Method for Determining the Area of a Circle

https://demonstrations.wolfram.com/ArchimedessMethodForDeterminingTheAreaOfACircle/





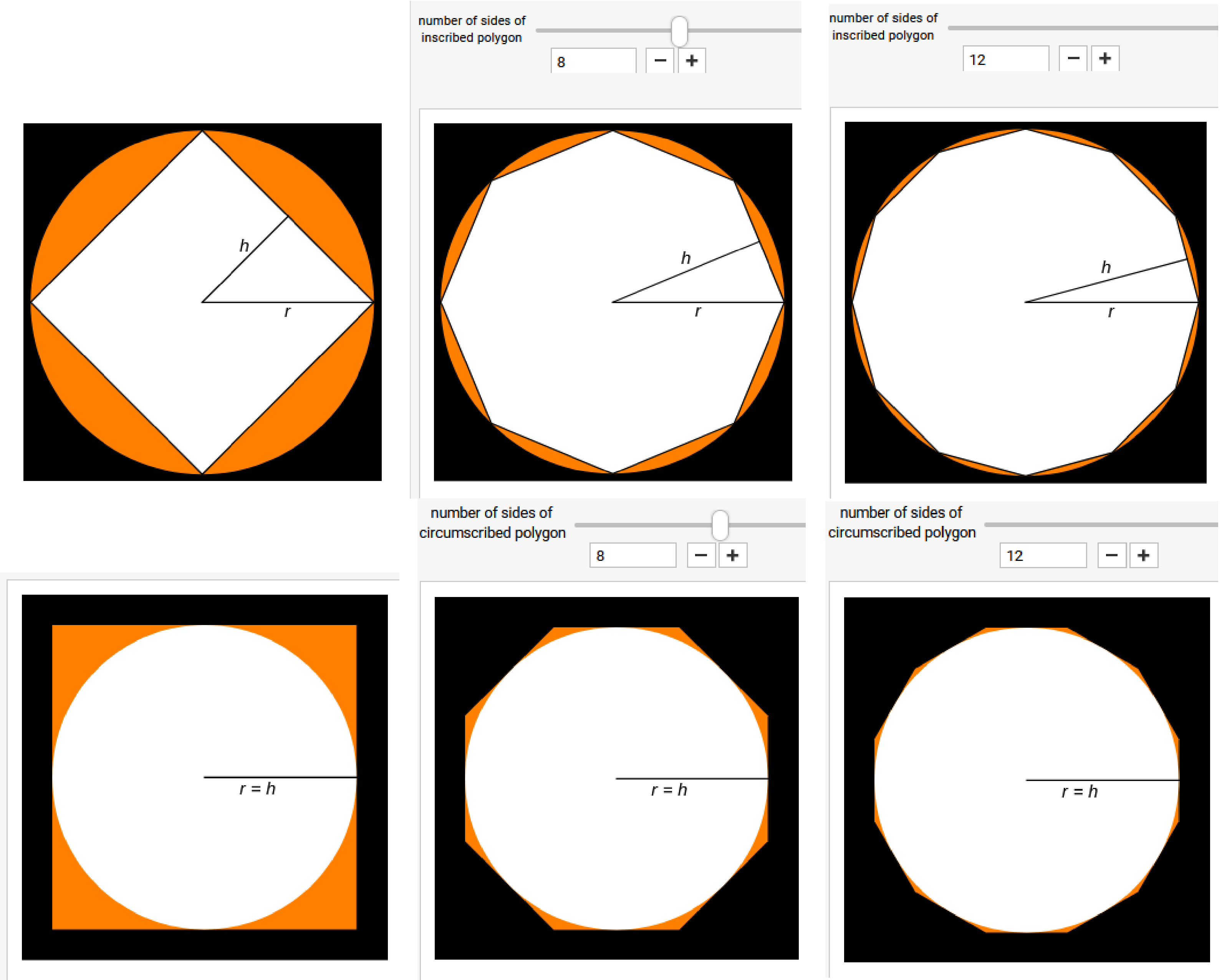




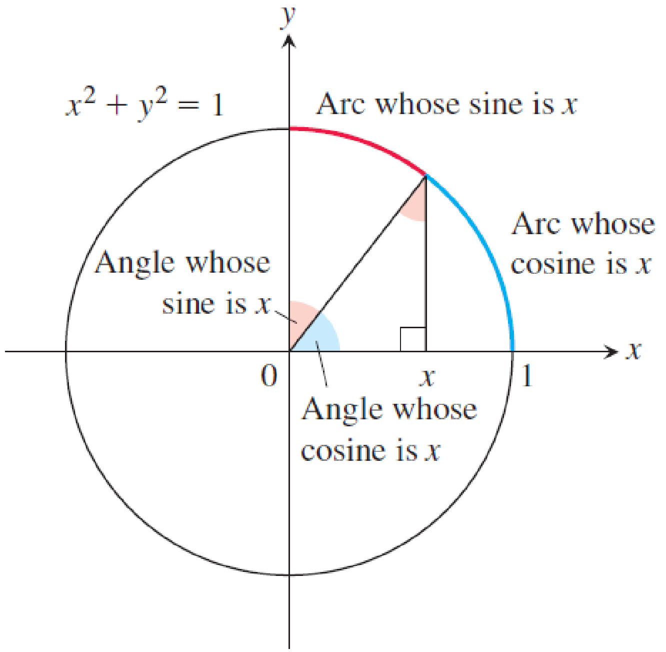
WOLFRAM Demonstrations Project https://demonstrations.wolfram.com/ArchimedessMethodForDeterminingTheAreaOfACircle/

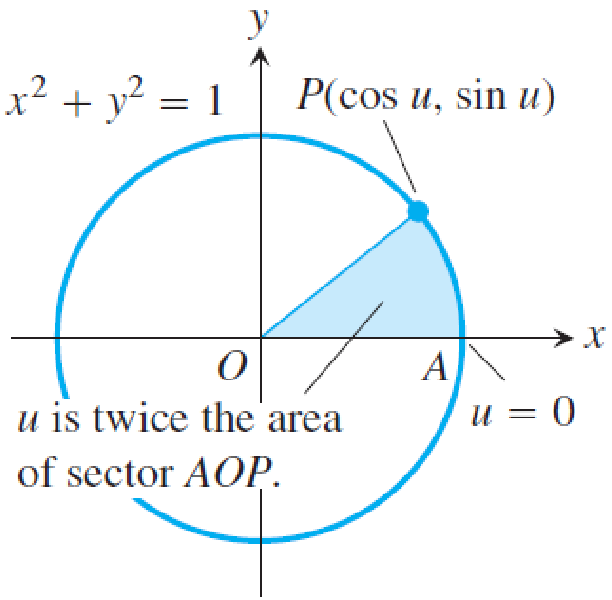
Archimedes's proof proceeds through a double *reductio ad absurdum*. Assume, first, that A > T, so that A - T > 0. In the image on the left there is a polygon, inscribed in a circle of radius *r*. If *h* is the apothem of the polygon, and *Q* is its perimeter, then its area is $\frac{1}{2} h Q$, (in the figure, a polygon with 4 sides).

Increasing the number of sides of the polygon, a number *m* will be found such that the difference between the area of the circle and the area of the inscribed polygon, say, P_m, satisfies $A - P_m < A - T$, so $T < P_m$. But $R_m = \frac{1}{2} h Q > \frac{1}{2} r C = T$, since $h < r$ and $Q < C$. That is, $T > P_m$, which contradicts the result $T < P_m$, which follows from the assumption A > T. This assumption must then be rejected.



Proofs without words





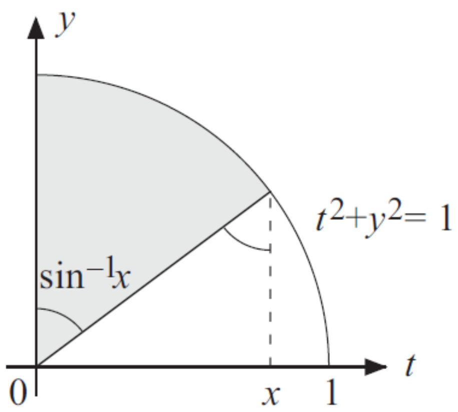


Figure 9.1. The derivative of the arcsine

The area of the shaded region can be computed in two ways:

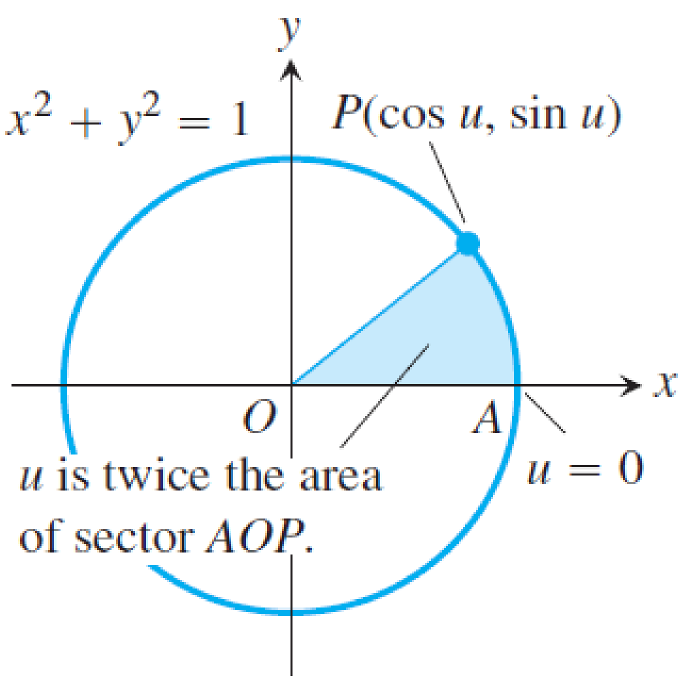
$$\frac{1}{2}\sin^{-1}x = \int_0^x \sqrt{1-t^2}dt - \frac{1}{2}x\sqrt{1-x^2}.$$

Multiplication by 2 and differentiation yields

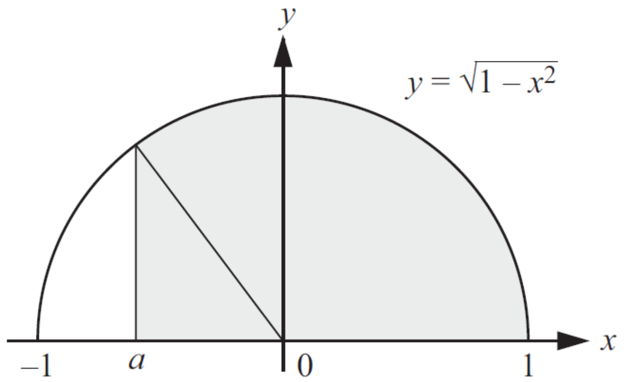
$$\frac{d}{dx}\sin^{-1}x = 2\sqrt{1-x^2} - \left(\sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}}\right) = \frac{1}{\sqrt{1-x^2}}$$

Proofs without words

$$\int_a^1 \sqrt{1-x^2}\,dx = \frac{\cos^{-1} a}{2} - \frac{a\sqrt{1-a^2}}{2}, \quad a \in [-1, 1]$$

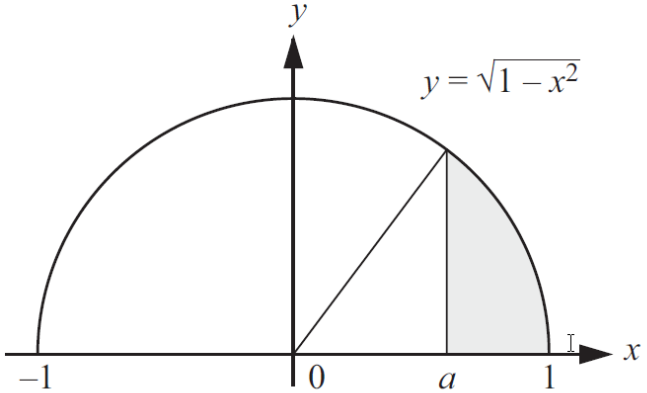


I. $a \in [-1, 0]$



$$\int_a^1 \sqrt{1-x^2}\,dx = \frac{\cos^{-1} a}{2} + \frac{(-a)\sqrt{1-a^2}}{2}$$

II. $a \in [0, 1]$

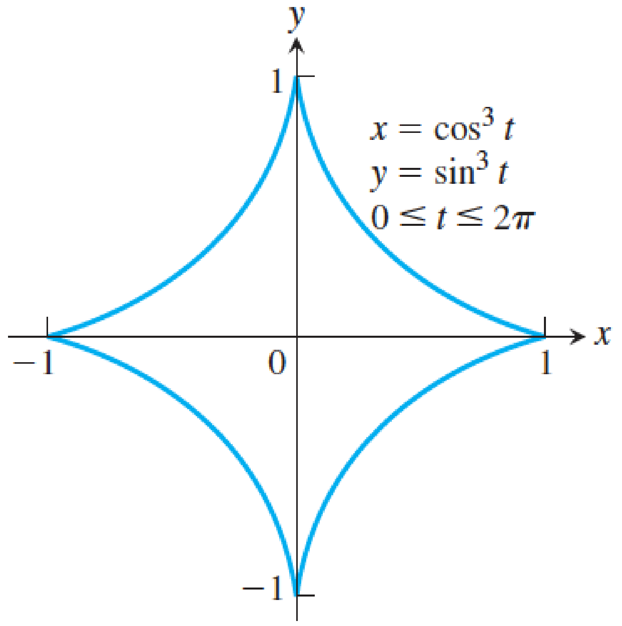


$$\int_a^1 \sqrt{1-x^2}\,dx = \frac{\cos^{-1} a}{2} - \frac{a\sqrt{1-a^2}}{2}.$$

Ασκήσεις

$$(x, y) = (f(t), g(t))$$

$$x^{2/3} + y^{2/3} = 1$$



αστροειδής

the length of the astroid

$$x = \cos^3 t, \qquad y = \sin^3 t$$

$$\left(\frac{dx}{dt}\right)^2 = [3\cos^2 t(-\sin t)]^2 = 9\cos^4 t \sin^2 t$$

$$\left(\frac{dy}{dt}\right)^2 = [3\sin^2 t(\cos t)]^2 = 9\sin^4 t \cos^2 t$$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{9\cos^2 t \sin^2 t \underbrace{(\cos^2 t + \sin^2 t)}_{1}}$$

$$= \sqrt{9\cos^2 t \sin^2 t}$$

$$= 3|\cos t \sin t| \qquad \cos t \sin t \geq 0 \text{ for } 0 \leq t \leq \pi/2$$

$$= 3\cos t \sin t.$$

$$\text{Length of first-quadrant portion} = \int_0^{\pi/2} 3\cos t \sin t \, dt$$

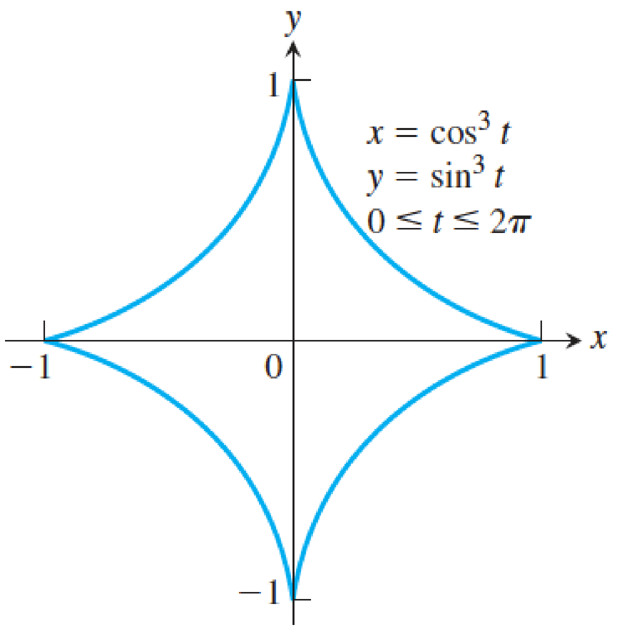
$$= \frac{3}{2}\int_0^{\pi/2} \sin 2t \, dt \qquad \cos t \sin t = (1/2)\sin 2t$$

$$= -\frac{3}{4}\cos 2t\Big]_0^{\pi/2} = \frac{3}{2}. \qquad L = 4(3/2) = 6.$$

Ασκήσεις

the area enclosed by the astroid

$$(x, y) = (f(t), g(t)) \qquad x = \cos^3 t, \qquad y = \sin^3 t, \qquad 0 \le t \le 2\pi.$$



αστροειδής

$$A = 4\int_0^1 y\,dx \qquad \text{4 times area under } y \text{ from } x = 0 \text{ to } x = 1$$

$$= 4\int_0^{\pi/2} (\sin^3 t)(3\cos^2 t \sin t)\,dt \qquad \text{Substitution for } y \text{ and } dx$$

$$= 12\int_0^{\pi/2} \left(\frac{1 - \cos 2t}{2}\right)^2 \left(\frac{1 + \cos 2t}{2}\right) dt \qquad \sin^4 t = \left(\frac{1 - \cos 2t}{2}\right)^2$$

$$= \frac{3}{2}\int_0^{\pi/2} (1 - 2\cos 2t + \cos^2 2t)(1 + \cos 2t)\,dt \qquad \text{Expand squared term.}$$

$$= \frac{3}{2}\int_0^{\pi/2} (1 - \cos 2t - \cos^2 2t + \cos^3 2t)\,dt \qquad \text{Multiply terms.}$$

$$= \frac{3}{2}\left[\int_0^{\pi/2} (1 - \cos 2t)\,dt - \int_0^{\pi/2} \cos^2 2t\,dt + \int_0^{\pi/2} \cos^3 2t\,dt\right]$$

$$= \frac{3}{2}\left[\left(t - \frac{1}{2}\sin 2t\right) - \frac{1}{2}\left(t + \frac{1}{4}\sin 2t\right) + \frac{1}{2}\left(\sin 2t - \frac{1}{3}\sin^3 2t\right)\right]_0^{\pi/2}$$

$$= \frac{3}{2}\left[\left(\frac{\pi}{2} - 0 - 0 - 0\right) - \frac{1}{2}\left(\frac{\pi}{2} + 0 - 0 - 0\right) + \frac{1}{2}(0 - 0 - 0 + 0)\right]$$

$$= \frac{3\pi}{8}.$$

```
> integrate(f,lower=0,upper=pi/2)$value
[1] 0.09817477
> f<-function(t){3*(sin(t))^4*(cos(t))^2}
> integrate(f,lower=0,upper=pi/2)$value
[1] 0.2945243
> 4*integrate(f,lower=0,upper=pi/2)$value
[1] 1.178097
> 3*pi/8
[1] 1.178097
```

χρονοβόρο

DEFINITION The **volume** of a solid of integrable cross-sectional area $A(x)$ from $x = a$ to $x = b$ is the integral of A from a to b,

$$V = \int_a^b A(x)\,dx.$$

Calculating the Volume of a Solid

1. *Sketch the solid and a typical cross-section.*
2. *Find a formula for* $A(x)$, the area of a typical cross-section.
3. *Find the limits of integration.*
4. *Integrate* $A(x)$ to find the volume.

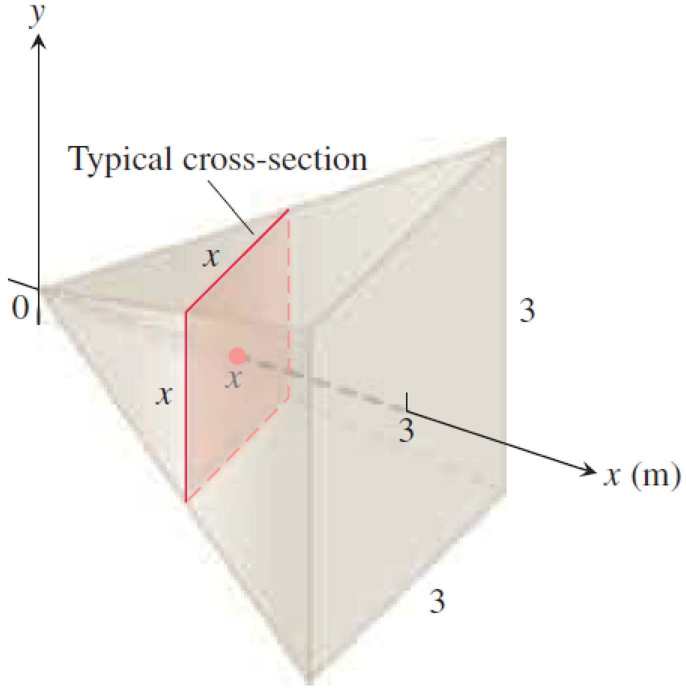


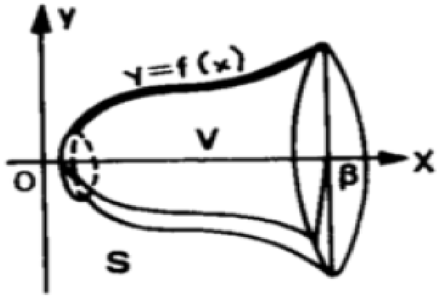
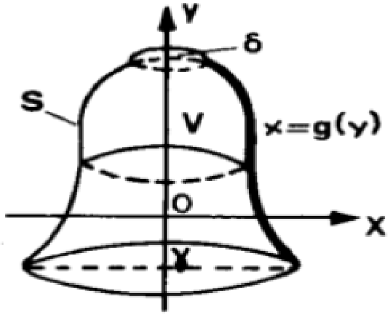
FIGURE The cross-sections of the pyramid in Example 1 are squares.

1. *A sketch.* We draw the pyramid with its altitude along the x-axis and its vertex at the origin and include a typical cross-section (Figure). Note that by positioning the pyramid in this way, we have vertical cross-sections that are squares, whose areas are easy to calculate.
2. *A formula for* $A(x)$. The cross-section at x is a square x meters on a side, so its area is

$$A(x) = x^2.$$

3. *The limits of integration.* The squares lie on the planes from $x = 0$ to $x = 3$.
4. *Integrate to find the volume*:

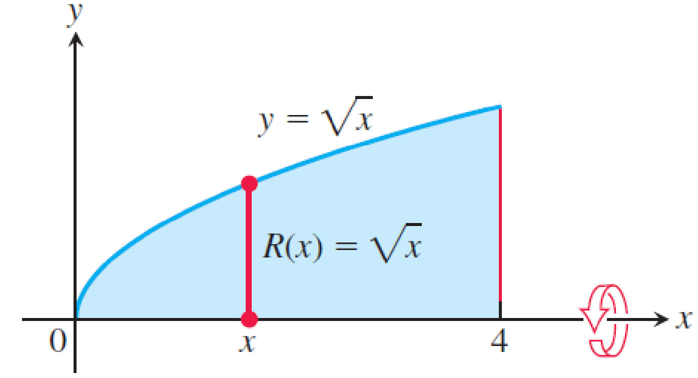
$$V = \int_0^3 A(x)\,dx = \int_0^3 x^2\,dx = \frac{x^3}{3}\bigg]_0^3 = 9\text{ m}^3.$$

Εφαρμογή	Σχήμα	Τύπος
Όγκος και Επιφάνεια εκ περιστροφής της καμπύλης y=f(x) από x=α μέχρι x=β γύρω από τον άξονα των x		$V=\pi\int_{\alpha}^{\beta} y^2\,dx$ $S=2\pi\int_{\alpha}^{\beta} y\sqrt{1+(y')^2}\,dx$
Όγκος και Επιφάνεια εκ περιστροφής της καμπύλης x=g(y) από y=γ μέχρι y=δ γύρω από τον άξονα των y		$V=\pi\int_{\gamma}^{\delta} x^2\,dy$ $S=2\pi\int_{\gamma}^{\delta} x\sqrt{1+(x')^2}\,dy$

Volume by Disks for Rotation About the x-Axis

$$V = \int_a^b A(x)\,dx = \int_a^b \pi\,[R(x)]^2\,dx.$$

This method for calculating the volume of a solid of revolution is often called the **disk method** because a cross-section is a circular disk of radius $R(x)$.



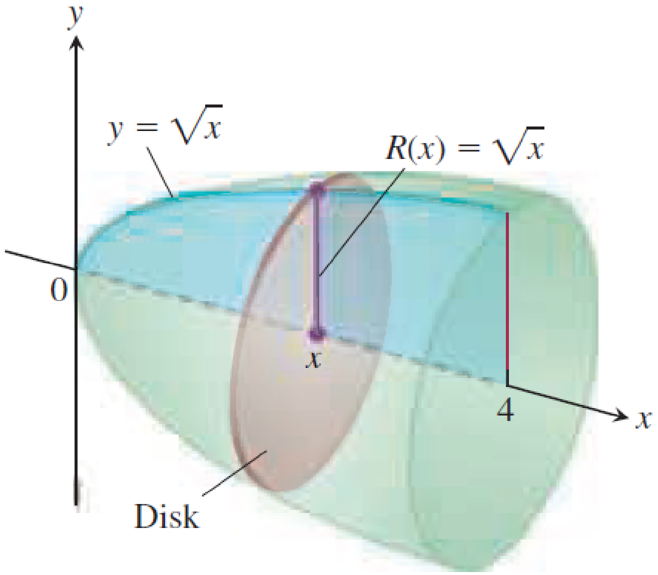
(a)

The region between the curve $y = \sqrt{x}$, $0 \le x \le 4$, and the x-axis is revolved about the x-axis to generate a solid. Find its volume.

We draw figures showing the region, a typical radius, and the generated solid (Figure 6.8). The volume is

$$\begin{aligned} V &= \int_a^b \pi\,[R(x)]^2\,dx \\ &= \int_0^4 \pi\,[\sqrt{x}]^2\,dx \\ &= \pi\int_0^4 x\,dx = \pi\frac{x^2}{2}\Big]_0^4 = \pi\frac{(4)^2}{2} = 8\pi. \end{aligned}$$

Radius $R(x) = \sqrt{x}$ for rotation around x-axis.

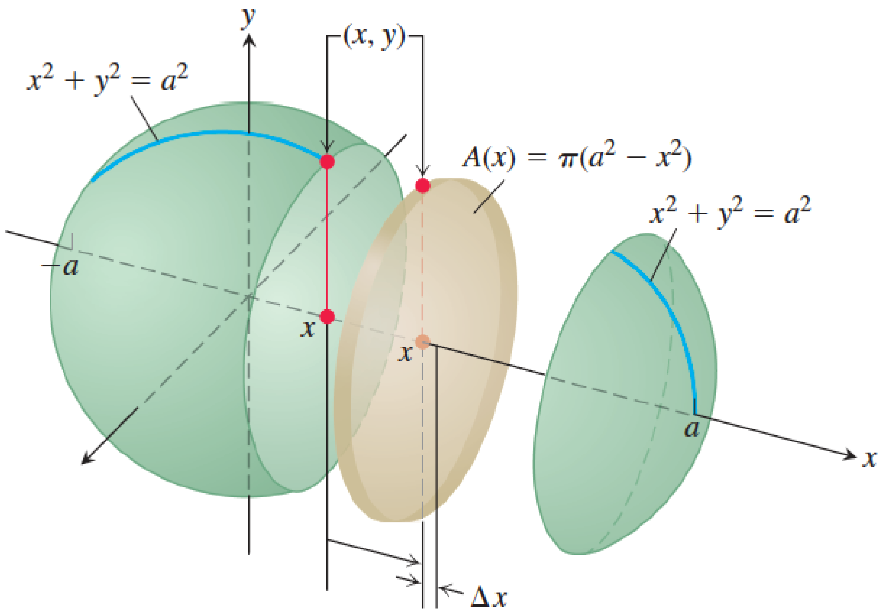


Volume by Disks for Rotation About the x-Axis

The circle

$$x^2 + y^2 = a^2$$

is rotated about the x-axis to generate a sphere. Find its volume.



ΣΧΗΜΑ Η σφαίρα που παράγεται με περιστροφή του κύκλου $x^2 + y^2 = a^2$ γύρω από τον άξονα x. Η ακτίνα είναι $R(x) = y = \sqrt{a^2 - x^2}$ (Παράδειγμα 5).

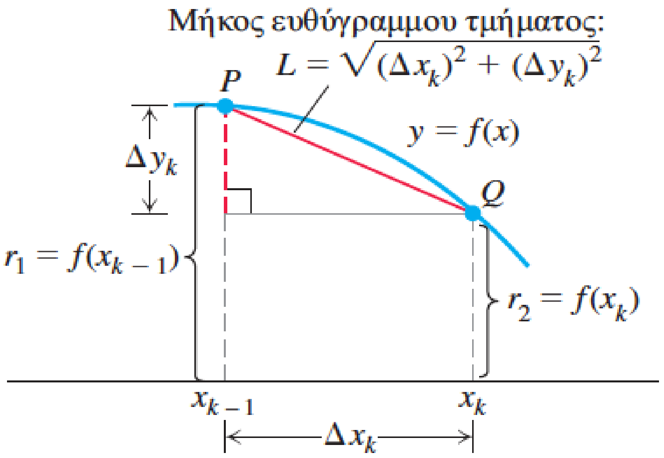
We imagine the sphere cut into thin slices by planes perpendicular to the x-axis
The cross-sectional area at a typical point x between $-a$ and a is

$$A(x) = \pi y^2 = \pi(a^2 - x^2).$$

$R(x) = \sqrt{a^2 - x^2}$ for rotation around x-axis.

$$V = \int_{-a}^{a} A(x)\,dx = \int_{-a}^{a} \pi(a^2 - x^2)\,dx = \pi\left[a^2x - \frac{x^3}{3}\right]_{-a}^{a} = \frac{4}{3}\pi a^3.$$

Defining Surface Area



ΣΧΗΜΑ Οι διαστάσεις που σχετίζονται με το τόξο και το ευθύγραμμο τμήμα PQ.

DEFINITION If the function $f(x) \geq 0$ is continuously differentiable on $[a, b]$, the **area of the surface** generated by revolving the graph of $y = f(x)$ about the x-axis is

$$S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_a^b 2\pi f(x)\sqrt{1 + (f'(x))^2}\, dx.$$

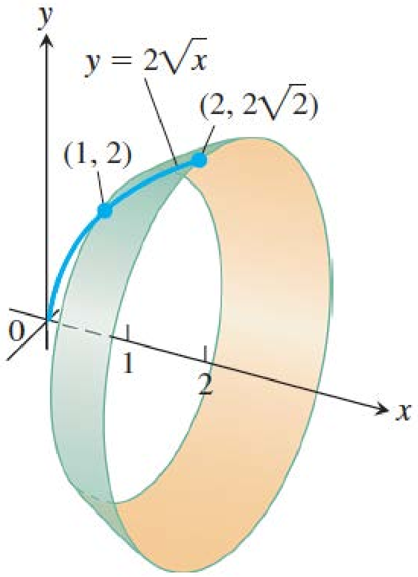
$$a = 1, \qquad b = 2, \qquad y = 2\sqrt{x}, \qquad \frac{dy}{dx} = \frac{1}{\sqrt{x}}$$

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + \left(\frac{1}{\sqrt{x}}\right)^2}$$

$$= \sqrt{1 + \frac{1}{x}} = \sqrt{\frac{x+1}{x}} = \frac{\sqrt{x+1}}{\sqrt{x}}$$

$$S = \int_1^2 2\pi \cdot 2\sqrt{x}\frac{\sqrt{x+1}}{\sqrt{x}}\, dx = 4\pi \int_1^2 \sqrt{x+1}\, dx$$

$$= 4\pi \cdot \frac{2}{3}(x+1)^{3/2}\Big]_1^2 = \frac{8\pi}{3}\left(3\sqrt{3} - 2\sqrt{2}\right).$$



Βιβλιογραφία

- Ανδρεαδάκης, Κατσαργύρης, Μέτης, Μπρουχούτας, Παπασταυρίδης, Πολύζος. *ΜΑΘΗΜΑΤΙΚΑ Γ΄ Τάξης Γενικού Λυκείου Ομάδας Προσανατολισμού Θετικών Σπουδών & Σπουδών Υγείας και Σπουδών Οικονομίας & Πληροφορικής,* 22-0273 Αναθεωρημένη έκδοση
- Ζαφειροπούλου-Καρατζόγλου (2020). *Σημειώσεις Εφαρμοσμένων Μαθηματικών για το Τμήμα Φαρμακευτικής*
- Μωυσιάδης (2016). *Ανώτερα Μαθηματικά, Αφοι Κυριακίδη Εκδόσεις α.ε.*
- Nelsen (2015). *Cameos for Calculus: Visualization in the First-Year Course,* The Mathematical Association of America
- Nelsen (*Proofs Without Words II,* The Mathematical Association of America
- [Thomas], Hass, Heil, Weir (2018). *THOMAS ΑΠΕΙΡΟΣΤΙΚΟΣ ΛΟΓΙΣΜΟΣ*, ITE-ΠΑΝΕΠΙΣΤΗΜΙΑΚΕΣ ΕΚΔΟΣΕΙΣ ΚΡΗΤΗΣ