Joanna Mamona-Downs

Education

Ph.D. Mathematics Education, Faculty of Mathematical Sciences, University of Southampton U.K, 1987. Advisor: Brian Griffiths.

M.Sc. Mathematics Education, School of Education, University of Reading U.K., 1984.

B.S., Mathematics, University of Patras, Greece, 1976.

Academic Positions

2011-present**:**	Professor, Dept. of Mathematics, University of Patras, Greece.
2005- 2010:	Associate Professor, Dept. of Mathematics, University of Patras, Greece
1998-2005:	Associate Professor, Dept. of Educational and Social Policy, University of Macedonia, Greece.
1993-1998:	Assistant Professor, Dept. of International and European Studies, University of Macedonia, Greece.
1992-1993:	Lecturer (elected Assistant Professor in June 1993), Department of Education, University of Cyprus.
1993-1996:	Visiting Professor, Dept. of Education Sciences in Early Childhood, Democritus University of Thrace, Greece.
2013-2014:	Visiting Scholar, School of Mathematical and Statistical Sciences, Arizona State University, U.S. A.
2001-2002:	Visiting Scholar, Graduate School of Education University of Berkeley, U.S.A.

1988-1990: Postdoc Researcher –Visiting Lecturer at the Learning Research and Development Center & Department of Mathematics, University of Pittsburgh, U.S.A.

General research domain and research interests

Joanna Mamona-Downs researches in the field of Mathematical Education. She is mostly interested in examining the effectiveness of teaching practices and how students learn Mathematics at the tertiary level (as well as at the later secondary school years). This research interest area is often termed 'Advanced Mathematical Thinking' (or A.M.T.) by educators. The term is relative, in the sense that detailed cognitive analyses of students understanding of explicit concepts typically only involve fundamental ideas found in the first year of studying mathematics at University. However, the main aim of A.M.T. is identifying and describing general skills that students need to succeed in studying University level mathematics overall, and how these skills are acquired.

J. Mamona-Downs has a particular interest in investigating students' comprehension of the real line system, limiting processes and the fundamental concepts and theoretical constructs of Real Analysis. More generally, she is involved in studying students' evolving abilities to solve mathematical tasks, and how to design courses such that to encourage students to build up the necessary 'culture of mathematical thinking' that will qualify them as competent, active mathematicians on graduation. Also, she has worked on students' understanding of the character of proof and their difficulties in formulating, presenting and reading proof. In the course of her research directed to problem solving and proof, the tasks involved are posed within the mathematical topics of Real Analysis, Euclidean and Co-ordinate Geometry, Elementary Number theory and Combinatorics.

Publications

Journals

1. Mamona, J. (1990). «Sequences and Series – Sequences and Functions: Students' Confusions», International Journal of Mathematical Education in Science and Technology, Vol. 21, No 2, (p.p. 333-337).

The paper draws on a broader research that studies how sixth-form or first year Honors Mathematics students form basic concepts of Real Analysis. It focuses on how students relate sequences and series; also, if students accept sequences as functions. It presents vivid evidence of the students' confusion between sequences and series and their resistance to regarding a sequence in any sense as a function.

2. Downs, M. & Mamona-Downs, J. (1995). «Matrices – a Case of Abstraction», International Journal of Mathematical Education in Science and Technology, Vol. 26, No 2, (p.p. 267-271) (submitted 1993).

The paper discusses the significance of the matrix as a mathematical object. Its didactical aims are two-fold, one is to give a case in hand of a critical examination of a definition. The second is to give a case in hand where abstraction evolves naturally from a starting point involving concrete geometric objects (i.e. intersections of hyperplanes, or equivalently linear transformations of a real space).

3. Silver, E., Mamona-Downs, J. et al. (1996). «Posing Mathematical Problems in a Complex task- Environment: An exploratory Study», Journal for Research in Mathematics Education, Vol. 27, No 3, (p.p. 293-309) (submitted 1995).

The paper examines the problems posed by 53 middle school teachers and 28 prospective secondary teachers in a reasonably complex task setting. It makes inferences about cognitive processes used to generate the problems and to examine differences between problems posed prior to solving the given problem and those posed during or after solving. A sizable portion of the posed problems were produced in clusters of related problems, thereby suggesting systematic problem generation. The posed problems were not always ones that subjects could solve, nor were they always problems with 'nice' mathematical solutions.

4. Mamona-Downs, J. (1997). (in Greek) « The role of variable in recognizing functions». Research Dimension of the Didactics of Mathematics Issue 2, (p.p.73-95).

5. Downs, M. and Mamona-Downs, J. (2000). “On Graphic Representation of Differentiation of Real Functions”. Themes in Education Vol. 1 (2), (p.p. 173-198).

The basic motive in establishing the concept of differentiation is to extend the idea of (constant) rate of change understood for linear functions to an idea of (instantaneous) rate of change for non-linear functions. However, in practice students rely heavily on an image of geometric tangent (i.e. a line that 'touches' but does not cut the graph curve locally). This paper describes this phenomenon and some of its disadvantages and advantages in terms of the students' cognition. In particular, the paper describes a 'dynamic model' of the limiting process inherent in the secant / tangent 'representation' of differentiation, and proposes that this model has the potential to help the student to mentally maneuver the notion of differentiation within certain type of heuristic argumentation.

6. Mamona-Downs, J. (2001). “Letting the Intuitive bear on the Formal; a Didactical Approach for the Understanding of the Limit of a Sequence”. Educational Studies in Mathematics, Vol. 48 (2-3), (p.p. 259-288).

This theoretical paper provides: (1) a presentation of some tasks that may be regarded as typical sources for forming students’ intuitions and understandings about limiting processes of real sequences, (2) an analysis of the formal definition of limit via identifying roles for each symbol that occurs in order to achieve a mental image firmly consonant with the definition, and (3) a description of how this mental imagery may be used to re-examine the validity of some intuitive beliefs. In particular, a persistent issue found in (1) is that the sources encourage an intuitive image of a sequence as having an ultimate term associated with the limit; it is this belief that is mostly discussed in (3).

7. Mamona-Downs, J. and Downs, M. (2004). "Realization of Techniques in Problem Solving: The Construction of Bijections for Enumeration Tasks". Educational Studies in Mathematics, Vol. 56, (p.p. 235-253).

The paper deals with a teaching approach aimed to help students to become aware of targeted techniques of particular significance in problem solving. The teaching approach is to present a series of tasks that can all be solved by applying the same technique. Two levels of prompting are used; first for the students to realize the solution without necessarily being cognizant of the technique, second for them to perform further mathematical modeling that should highlight the similarities in solution shared by all the tasks. In the fieldwork, such a teaching sequence is implemented for a technique involving enumeration via constructing a bijection. Certain factors in the students' behavior suggested that their realization of the technique was not as secure as desired. A modification of the teaching sequence is proposed to counter this.

8. Mamona-Downs, J. and Downs, M. (2005). "The identity of problem solving". Journal of Mathematical Behavior 24, (p. p. 385-401)

This paper raises issues motivated by considering the 'identity' of problem solving. In particular, the following themes are discussed: problem solving vis-à-vis proof; conceptualization; structure and representations; raising questions and Posing problems; the significance to problem solving of techniques; application of knowledge; exploration; the reading of mathematical texts as a problem- solving activity.

9. Cai, J., Mamona-Downs, J., Weber, K. (2005). "Mathematical problem solving: What we know and where we are going" Journal of Mathematical Behavior 24, (p. p. 217-220)

This paper introduces, summarizes and gives short critique on the papers published in a double special issue of the Journal of Mathematical Behavior on Problem Solving. The authors were invited Guest Editors of the issue.

10. J., Mamona-Downs (2008). " Procepts and Property- Based Thinking; to what extent can the two co-exist?" Mediterranean Journal for Research in Mathematics Education, 7, 2 (pp. 49-57).

The paper examines the relationship between procept and property-based thinking. At the first sight, property-based thinking is different to thinking in terms of procepts, as the former involves a property that must be a-priori associated with some category of objects, whereas for the procept the identity of the objects or conceptual input is negotiated through processes, and vice-versa. Despite this difference, the paper will argue through some examples that, to some degree, the two can be reconciled, and can be made to act productively in tandem. The illustrations concern the limit of a real sequence, the

Fundamental Theorem of Calculus and the prime decomposition of positive integers.

11. K., Jones and J. Mamona-Downs (2008). "Brian Griffiths (1927-2008) his pioneering Contribution to Mathematics and Education". Educational Studies in Mathematics, Vol. 69 (3), (p.p. 283-286).

A paper in Educational Studies in Mathematics to honor Professor Brian Griffiths' contribution to Mathematics and Mathematics Education.

12. Mamona-Downs, J. (2010). " On Introducing a Set Perspective in the learning of limits of real sequences". International Journal of Mathematical Education in Science and Technology, 41(2), p.p. 277-291.

The paper consists of an integrated exposition concerning the connection of accumulation points with bounds and the significance of stressing this connection in educational terms. It starts by claiming that the identification of the similarities, differences and inter- play of allied concepts can act as a mutual enrichment of their understanding. It examines the case of limits of real sequences and limits / continuity of real functions, contrasting the set theoretical perspective inherent in the notions of bounds and accumulation points with the ordering inherent in sequences.

14. Eisenberg, T. & Engelbrecht, J., Mamona-Downs, J., (2010). "Advanced Mathematical Topics: Transitions, evolutions, and changes of foci." International Journal of Mathematical Education in Science and Technology, 41(2), pp. 139-141.

15. Mamona-Downs, J. & Downs, M. (2010). “The decimal system as a topic in transition from school to university”. CULM’s Newsletter, 1, pp. 27-34.

This paper aims to outline some of the mathematical options that exist to reinforce the understanding of infinite decimals, and how this understanding could fit in with an axiomatic approach. In particular, the concept of completeness of the real numbers is discussed. Hence we are treating a particular, but major, theme concerning the transition from school to university.

16. Mamona-Downs, J. & Papadopoulos, I. (2011). Problem-solving activity ancillary to the concept of area. Mediterranean Journal for Research in Mathematics Education, 10(1-2), 103-129. (Submitted 2009).

This paper concerns the results of the second stage of a two -tier program designed to enhance students' technique usage in area measurement. The first stage involves 11year old students; certain techniques were didactically introduced with the dual purpose of cementing the concept of area and area preservation, and of giving the students tools for explicit area measurement (either exact or estimates). The second stage deals with the development of the same techniques, but the focus is not now primarily on the direct enhancement of the central concept (area) but on the re- assessing, re-examining and adapting of the techniques themselves.

17. Mamona-Downs, J., Megalou, F. (2013) Students’ understanding of limiting behavior at a point for functions from R^2 to R**.** Journal of Mathematical Behavior, 32 (1) pp. 53-68.

The aim of this paper is to describe and analyze University students’ understanding of the limiting behavior of a function of R^2 to R and to discuss issues associated to it such as the neighborhood of a point, ‘directional approach’ to a point, etc. The purpose of the study is to help instructors to gain useful insights towards: i) students' thoughts about the concept of the limit of a function of two variables and its relation with the concept of the limit of a function of one variable, ii) students’ ability to apply different methods in order to find limits of functions of R^2 to R, and iii) their understanding of equivalence (or not) between the different definitions (and or necessary conditions) of the concept.

18. Mamona-Downs, J. & Downs, M. (2013). “Problem Solving and its elements in forming Proof”. *The Mathematics Enthusiast*, Vol. 10 (1), pp 137-162.

The character of the mathematics education traditions on problem solving and proof are compared, and aspects of problem solving that occur in the processes of forming a proof, which are not well represented in the literature, are portrayed.

Chapters in Books

1. Mamona-Downs, J. (1990). «Calculus-Analysis: A Review of recent Educational Research», in R. Cantoral, F. Cordero, R.M. Farfan. C. Imaz (Eds.), Calculus-Analysis in Mathematical Education Research (p.p. 11-36), Editions of Universidad Autonoma del Estado de Mexico.

The chapter gives an overview of the didactics of Calculus and Analysis at the pre-university and the early - university levels. It concentrates on the long-standing debate on use of infinitesimals, limits and concepts arising from these as differentials, tangents, differentiation, integration. The paper concentrates on the following research approaches to the subject:

(i) Concept Images, (ii) Epistemological Obstacles, (iii) Non-Standard Analysis and Infinitesimal Models, Differentials, Phenomenology, Objective Testing, Graphic Calculus.

2. Silver, E. & Mamona, J. (1990). «Stimulating Problem Posing In Mathematics Instruction», in G. Blume and M.K. Heid (Eds), Implementing New Curriculum and Evaluation Standards, (p.p. 1-7). University Park, PA: Pennsylvania Council of Teachers of Mathematics.

The chapter discusses aspects of the rationale for including problem - posing activities in pre-college mathematics classes, examples of appropriate tasks that might be used to encourage problem posing, and an analysis of the relation between problem posing and problem solving.

3. Mamona-Downs, J. & Downs, M. (2002). "Advanced Mathematical Thinking with a special reference to Reflection on Mathematical Structure". In Lyn English (Chief Ed.) Handbook of International Research in Mathematics Education, Lawrence Erlbaum Ass., N. J. (p.p. 165 – 195).

This chapter puts forward the notion of *Reflection on Mathematical Structure* as a significant characterization of the work done at AMT. Its major importance lies in its allowing mathematical understanding that may be independent of continuous conceptual thought. Within the RMS milieu, it introduces the term '*decentralized notions'*, which constitute standard ways of thinking in advanced mathematics and having roles cutting through mathematical theories. Examples are: decomposition, symmetry, order (in the sense of arrangement), similarity, projection, equivalence, inverse, dual, canonical forms), It is claimed that the acquisition of decentralized notions is essential for the mathematical progress of young mathematicians.

4. Mamona-Downs, J. & Downs, M. (2008). "Advanced Mathematical Thinking and the role of Mathematical Structure". In Lyn English (Chief Ed.) Handbook of International Research in Mathematics Education, Routledge, Taylor & Francis Group, New York & London, (p.p. 154 – 175).

This chapter tries to analyze the apparent chasm that exists between school mathematics and university mathematics, especially as practiced in mathematics departments. Without belittling factors concerning social and institutional changes, it claims that these seem secondary to concerns in having to cope with a fundamental transformation of the character of mathematics itself. It raises the topic of mathematical structure as the best medium to judge what Advanced Mathematical Thinking is and what it is not.

5. Mamona-Downs, J. & Downs, M. (2016). Mathematical Structure, Proof, and Definition in Advanced Mathematical Thinking. In Lyn English and David Kirshner (Eds.) Handbook of International Research in Mathematics Education, Routledge, New York, (p.p. 239 – 256).

This chapter is directed to themes explicitly concerning proof and definition at the AMT level examined in the light of the underlying mathematical structures. We contend that the most vibrant areas in the A.M.T. research today concern: guided re-invention of proof and conceptually based definitions, the need for students to read given proofs in a proactive manner, and students to experience the interplay (or even interchange) between definitions and what is proved.

6. Mamona-Downs, J. (2013). Expectations according to a mathematics educator from a mathematics department. In Michael N. Fried & Tommy Dreyfus (Eds.), Mathematics & Mathematics Education: Searching for Common Ground. New York: Springer, Advances in Mathematics Education series.

This paper advocates the following aims concerning the collaboration between research mathematicians and mathematics educators at the AMT level:

1. To persuade mathematicians that the educator does have a role in improving university mathematics instruction.
2. A.M.T. educators to pursue regular communication with lectures, taking an active role; not to regard this exchange merely as an opportunity to research how mathematicians work, but also to relate the gains gotten from this co-operation, with input from both sides.
3. To document the results of the above communication in a way that both educators and mathematicians can digest.

7. Poulos, A., & Mamona-Downs, J., (2018). Gifted students' approaches when solving challenging mathematical problems. In Mihaela Singer, (Ed.) *Activities for, and Research On, Mathematically Gifted Students,* Springer, p.p. 309-342.

The chapter presents the solving approaches of three young gifted mathematicians when trying to resolve a problem of characterization in the milieu of Euclidean Geometry. The goal was to compare the various methods the solvers employed, and their transitions from geometrical to algebraic means and vice versa. In the end the students developed a computer program in order to proceed with the solution, something that brought to the surface questions about the 'rigidity' and legitimacy of it. The paper offers a detailed microscopic analysis of students attempts.

Book (in Greek)

Mamona-Downs, J., Papadopoulos, I. (2017). (in Greek) *Problem Solving in Mathematics.* Crete University Press.

Peer-reviewed Conference Proceedings

1. Silver E. & Mamona, J. (1988). «Problem Posing by Middle School Mathematics Teachers». In C. A. Maher, G.A. Goldin & R.B. Davis (Eds), Proceedings of the 11th Annual Meeting of the PME-NA (I p.p. 263-269). New Brunswick, NJ.

The paper presents the analysis of problem posing and conjecturing by Middle School Mathematics Teachers. The findings suggest that the teachers could generate reasonable, interpretable conjectures and problems related to changing the conditions implicit in the task environment.

2. Mamona-Downs, J. (1990). «Pupils' Interpretations of the Limit Concept; A Comparison Study between Greeks and English». In G. Booker, P. Cobb & T. N. de Mendicuti (Eds.), Proceedings of the 14th Annual Conference of the P.M.E. International (I p.p. 69-76), Mexico.

The paper presents the examination of responses of English and Greek students at pre-university stage on the nature of limits on the real line. It was found that the English have a psychology of the 'continuum' closer to the Leibniz-Cauchy model than to that of Weierstrass; the Greeks mostly accept the Weierstrass model but not without conflict with the 'dynamic' approach, suggesting that the latter is closer to their intuition.

3. Mamona-Downs, J. (1993). «On Analysing Problem Posing». In I. Hirabayashi, N. Nohda, K. Shigematsu & Fou-Lai Lin (Eds.), Proceedings of the 17th Annual Conference of the P.M.E. International (III p.p. 41-49), Tsukuba, Japan.

The paper describes a novel theoretical framework to analyze results of problem posing activity according to the independence from the original problem and to the suitability for a satisfying mathematical solution to ensue. Data from a fieldwork undertaken are analyzed under this framework. A greater dependence on the original problem than desired was shown.

4. Patronis, T. & Mamona-Downs, J. (1994). «On Students' Conceptions of the Real Continuum». I n J. da Ponte & J. Matos (Eds), Proceedings of the 18th Annual Conferences of the P.M.E. International (I p. 63), University of Lisboa, Portugal.

This oral communication deals with the students' understanding of the real continuum through perceived properties of subsets of the reals designed to provoke reactions on this issue. The representations that the students made in response include: the decimal system; images related to limiting processes; recursive argumentation depending on a sense of ordering; the existence of suprema; the notion of nested intervals.

5. Mamona-Downs, J. & Downs, M. (1995). «Common Sense, Area and the Fundamental Theorem of Calculus». In Christine Keitel (Chief Editor) Mathematics (Education) and Common Sense. Proceedings of the CIEAEM 47 Conference (p.p. 166-170), Freie Universitat Berlin, Germany.

The paper examines aspects of the role of common sense in Mathematics in regard with the concept of area. It comments on topics such as the difference between conceptual common sense and common sense reasoning, the role of common sense in 'meaning' in Mathematics and the particular place of common sense in numbers, measures and in Calculus.

6. Mamona-Downs, J. (1996). «On the Notion of Function». In L. Puig & A. Gutierrez (Eds.) Proceedings of the 20th Annual Conference of the P.M.E. International, University of Valencia, Spain (III p.p. 321-328).

The paper concerns certain broad topics about functions and potential problems students might have with them. The focus is on the more creative aspects, e.g. identifying, forming and using functions, rather than analyzing given functions. The statement of the Fundamental Theorem of Calculus is used as a running illustration of many issues brought in.

7. Mamona-Downs, J. (1997). «Students Dependence on Symbolic Variables in Functions». In Er. Pehkonen (Ed.) Proceedings of the 21st Annual Conference of the P.M.E. International, University of Helsinki, Lahti, Finland (I p. 245).

This oral communication describes a pilot study given to first year university students (studying Economics). All the problems given involved functions extracted from geometrical or more general physical contexts, but none require forming explicit algebraic expressions. The aim of the study was to ascertain how effective this solving experience would be in broadening the students' comprehension of the significance and character of functions.

8. Mamona-Downs, J. & Downs, M. (1999). “Reinforcing Teacher’s Understanding of Limiting Processes by Considering Sequences of Plane Figures”. In O. Zaslavsky (Ed.) Proceedings of the 23rd Annual Conference of the PME International, (I p. 356), Haifa, Israel.

The issues raised in this communication are the following: (1) If the cognitive problems students have with limits of real sequences can be alleviated (or change) with limits of figures. (2) In teachers training, the introduction of a 'parallel' concept (new to the teachers) may prompt better understanding towards these problems of students in the original concept. (3) The exercise of forming definitions may provide a way of partially dissipating the 'Platonic' bias towards Mathematics.

9. Mamona-Downs, J. (2002). “Accessing Knowledge for Problem Solving”. Plenary Lecture in the Proceedings of the 2nd International Conference on the Teaching of Mathematics (at the undergraduate level), (electronic form), Hersonissos Crete.

This paper studies the modes of thought that occur during the act of solving problems in mathematics. It examines the two main instantiations of mathematical knowledge, the conceptual and the structural, and their role in the afore said act. It claims that awareness of mathematical structure is the lever that educes mathematical knowledge existing in the mind in response to a problem-solving activity, even when the knowledge evoked is far from being evidently connected with the activity. For didactical purposes, it proposes the consideration of mathematical techniques to facilitate the accessing of pertinent knowledge. All the assertions above are substantiated by close examination of some exemplars taken from various mathematical topics, and the presentation of fieldwork results.

10. Mamona-Downs, J. & Downs, M. (2002). “Promoting students’ awareness in applying bijections in enumeration tasks”, in A. Cockburn, E. Nardi (Eds.) Proceedings of the 26rd Annual Conference of the PME International, (I p. 295), Norwich, England.

This oral communication presents a study based on the notion of 'inert knowledge' due to A. N. Whitehead. The mathematical concept employed is the one of bijection, preserving set order. The study has two aims. The first is to provide some evidence that the knowledge of the preservation of set order under bijective correspondence is inert. The second proposes a general framework concerning developing techniques to address inert knowledge. An important factor in a technique is a ‘cue’, that acts to release’ inert knowledge.

11. Mamona-Downs, J. & Downs, M. (2003). “Broadening Teachers’ Experience of the Notion of Convergence via Plane Figures”, in A. Gagatsis, & S. Papastavridis (Eds.) Proceedings of the 3rd Mediterranean Conference on Mathematical Education, (p.p.647–655), Athens, Greece.

First, the paper proposes that the topic of limits of sequences of plane figures may be a good candidate for inclusion in pre-service or in-service training of secondary school mathematics teachers. Next, it presents the major issues and possible approaches concerning this topic (on the mathematical level). Finally, a description is given of fieldwork conducted in a workshop type environment involving 18 teachers; here the reactions of the participants are noted as they were confronted with some of the issues and approaches above.

12. Downs, M., Mamona-Downs, J. (2004). "Correspondences, Functions and Assignations Rules". Proceedings of the 28th Annual Conference of the PME International, Vol.2 (p.p. 303-310) Bergen, Norway.

In this paper a theoretical position is put forward that, in cognitive terms, a differentiation should be made between a correspondence and a function. Important in understanding this difference is the role of an assignation rule; the correspondence acts as a way to identify a rule in context, whilst the function accommodates the rule in a more formal framework providing a secure base for argumentation. This perspective is used to interpret some students' behavior in a task where the identification of a particular relationship is crucial for its solution.

13. Cai, J. and Mamona-Downs (2004) "Problem-solving in mathematics education" Proceedings of ICME 10.

This paper summarizes the issues raised at the Topic Study Group on Problem Solving, ICME 10. The primary concerns were: (1) To understand the complex cognitive processes involved in Problem Solving; (2) To explore the actual mechanisms in which students learn and make sense of mathematics through Problem Solving, and how this can be supported by the teacher; and (3) To identify future directions of problem-solving research, including the usage of information technology. A more specific aim of the group was concerned with determining the scope of problem solving.

14. Mamona-Downs, J., Meehan, M., Monaghan, J. (2005), "Synopsis of the activities of Working Group 14 'Advanced Mathematical Thinking'".

Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education, (electronic form), Sant Feliu de Guixols, Spain).

In this paper an introduction is given for the educational issues covered, and a rationale why they are important to examine. The issues are educational frameworks concerning dualities in mathematical thinking; teaching Calculus/ Real Analysis and Vector Spaces; institutional factors when learning mathematics at tertiary level; the linkage between proof and problem solving.

15. Downs, M., Mamona-Downs, J. (2005). “The Proof Language as a Regulator of Rigor in Proof, and its effect on Student Behavior”. Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education, (electronic form), Sant Feliu de Guixols, Spain).

This paper discusses the character of the language in which formal proof is set, and the difficulties for students to appreciate its exact form, and why it is needed. It describes the effect that these difficulties have on student attitude towards proof, and how it influences student behavior whilst generating proofs. This is placed in a perspective of what extra demands there are in producing proofs over those that occur in general problem solving.

16. Mamona-Downs, J. & Papadopoulos, I. (2006). "The problem-solving element in young students' work related to the concept of area". Proceedings of the 30th Annual Conference of the PME International, Vol.4 (p.p. 121 - 128)

The focus of this paper is on the problem-solving skills that may accrue from exposition to tasks related to the calculation of area. In particular, the working of two 7th grade students on one specific task is examined vis-à-vis certain executive control issues about the selecting, handling and adaptation from a body of previously known methods concerning area determination.

17. Παπαδοπουλος, Ι. & Μαμωνά - Downs I. (2006). "Υιοθέτηση στρατηγικών επίλυσης προβλήματος· η περίπτωση της μέτρησης του εμβαδού". Πρακτικά του Πανελληνίου Συνεδρίου Μαθηματικής Παιδείας, σ.461-470, Πάτρα.

18. Mamona-Downs, J. (2007. "Synopsis of the activities of Working Group 14 CERME-5 on the theme of 'Advanced Mathematical Thinking'". Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education, (electronic form), Larnaca, Cyprus.

The paper describes the themes that were discussed in the Group 14. The themes are: The nature of Advanced Mathematical Thinking; educational models of mathematical reasoning; the role of entities and constructs in mathematics; students' generation of examples and counter-examples; the difference between 'vernacular logic' and 'mathematical logic; the relation between the mathematics educator and the mathematician.

19. Downs, M., Mamona-Downs, J. (2007). "Local and Global Perspectives in Problem Solving". Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education, (electronic form), Larnaca, Cyprus.

This paper will raise issues concerning the interaction between local and global foci realized in the working mathematical environment. These issues are illustrated by suitably tailored tasks and presented solutions. Predicted difficulties for students in effecting switches in argumentation from local to global perspectives or vice-versa are considered, as well as the consequences on students' general problem-solving ability if they are not overcome. Pedagogical measures are mentioned.

20. Mamona-Downs, J. (2008). "Mathematical Creativity, Structure and Control" Proceedings of the Fifth International Conference on Creativity in Mathematics and the Education of Gifted Students, p.p. 405-407. Haifa, Israel.

This paper examines the role of mathematical creativity in the sphere of non-procedural work. The notion of 'creativity' is an elusive one and it is wide open to interpretation, as it is evident in the literature. In the paper an attempt is made to define or characterize mathematical creativity, guided whether it is appropriate to invoke the term when 'imagination' plays a role. It brings evidence of this happening if a new perception of the task environment is effected.

21. Mamona-Downs, J. (2008). "On Development of Critical Thinking and Multiple Solution Tasks". Proceedings of the International Research Workshop of the Israel Science Foundation p.p. 77-79. Haifa, Israel.

The paper examines the development of critical thinking in mathematics through experiencing multiple solution tasks. 'Critical thinking' is taken in a restricted sense to mean the general structural appraisement of a completed solution or a solution attempt with an eye either to improve the solution or to provide alternative solution approaches. The rationale is that the original construction will give concrete and traceable points of reference in how a student crafts a second construction from the first. A further line of appraisement concerns the comparison of several existing solutions.

22. Mamona -Downs J. & Downs M. (2008). "On Students' appreciation of the relationship between bounds and limits". Proceedings of ICME 11, electronic form.

This paper examines how well students can combine working on a real sequence and its underlying set. The cognitive interest is by considering the underlying set, we are denying a main conceptional aspect concerning limits, i.e. the ordering implicit in sequences. The results are achieved by observing students' difficulties on the following proposition: 'If (a_n) is a convergent sequence and the supremum of the underlying set A of (a_n) is not an element of A, then the limit of (a_n) is sup(A).

23. Mamona -Downs J. & Downs M. (2009). "Necessary Realignments from Mental Argumentation to Proof presentation". Proceedings of CERME 6, electronic form.

This paper deals with students' difficulties in transforming mental argumentation into proof presentation. A teaching / research tool is put forward, where the statement of a task is accompanied by a given written piece of argumentation suggesting a way to resolve the task intuitively. The student must convert this into an acceptable mathematical form. Three illustrative examples are given.

24. Mamona-Downs, J. (2009). "Research and development in the teaching and learning of advanced mathematical topics". Proceedings of ICME 11, electronic form.

25. Mamona-Downs, J. (2009). «Enhancement of Students' Argumentation through exposure to other approaches». Proceedings of PME 33 International, Vol. 4. pp. 89-96, Thessaloniki, Greece.

The paper discusses and illustrates the advantages of making available to students the work of their peers that yield a result in another form. It is claimed that *reflection* on the *structural differences* inherent can give students a channel to strengthen the exposition that they originally gave.

26. Mamona-Downs, Joanna (2010). "On the Communication of Proof". Plenary Lecture. 'Proceedings of the Encontro de Investigação em Educação Matemática 2010.' Edited by Sociedade Portuguesa de Investigação em Educação Matemática, Costa da Caparica, Lisbon.

The paper discusses the role of articulation in fostering the processes of solving a mathematical task. Articulation is taken as indicating that a phase of argumentation has been enunciated, and by its enunciation, is settled on. In particular, a general framework is put forward where acts of articulation determine four stages in the making of the solution. Two different models are made within this framework, and illustrations are given.

27. Mamona-Downs-Downs, J., et al. (2011). (in Greek). Classroom activities facilitating 'Proof'. ENEDIM Proceedings, (electronic form).

28. Mamona-Downs, J. & Downs M. L. N. (2011). Proof: a game for pedants? Proceedings of CERME 7, p.p. 213- 223.

This paper examines the types of argument that are deemed acceptable at tertiary level mathematics and under which circumstances, and why the expectancy that a tight proof is required is sometimes relaxed. It analyses the status of proof in cases where mathematical modeling takes place, and on tasks whose informal resolution rests on two or more mathematical milieu. On occasion, can the insistence on a proof be regarded as pedantry?

29. Mamona-Downs, J. (2012). Do students write down the output of their thought, or write to expound? In Avgerinos, P. & Gagatsis, A. (Eds.) Research on Mathematical Education and Mathematics Applications, Edition of Mathematics Education and Multimedia Lab., pp. 35-46.

The paper considers the place of the term 'presentation' in the mathematical discourse. It puts forwards the following research questions: What constitutes and motivates a presentation? Are students concerned, and able, to write out their solutions in the form of a presentation? What are the cognitive advantages and disadvantages in (the process of) making a presentation? Is the presentation for the satisfaction of the individual, or for purposes of communication with others? Some light on these research questions is thrown by commenting on selected extracts from project work done by undergraduate students attending a course on Problem Solving.

30. Mamona-Downs, J. (2014). Reconciling two non-equivalent definitions for the limit of two-variable real functions. *Talk presented at the MAA Join*.

jointmathematicsmeetings.org/amsmtgs/2160_abstracts/1096-l1-2705.pdf

In the talk a teaching sequence was proposed to elicit student comparison of two given candidate definitions for the same mathematical concept i.e. the limit for a function mapping (a subset of) $\mathbf{R}^2$ into **R**. The two definitions given are not logically equivalent, but students are guided to make an additional condition for one of them such that to result in a third definition that *is* equivalent with the first.

31. Mamona-Downs, J. (2017). We define, we solve, we prove, we develop theories ...Facets of Mathematical Education. (in Greek). Plenary Lecture. ENEDIM Proceedings 2017, (http://enedim7.gr) p.p. 52-65.

32. – 33. Two EME Annual Conferences (Corfu, Chios)

34. Mamona-Downs J. & Kourouniotis C. (2018). The ε-δ definition for one-variable real function revisited. In E.Bergqvist, M.Österholm, C.Granberg & L.Sumpter (Eds.). *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 5, p. 110). Umea, Sweden: