

## COURSE OUTLINE

### 1. GENERAL

|                                                                                                                                                                                                                                                               |  |                                                                                                                                                     |                                     |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|-----------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------|
| SCHOOL                                                                                                                                                                                                                                                        |  | NATURAL SCIENCES                                                                                                                                    |                                     |
| DEPARTMENT                                                                                                                                                                                                                                                    |  | MATHEMATICS                                                                                                                                         |                                     |
| LEVEL OF COURSE                                                                                                                                                                                                                                               |  | UNDERGRADUATE                                                                                                                                       |                                     |
| COURSE CODE                                                                                                                                                                                                                                                   |  | MAT_AM434                                                                                                                                           | SEMESTER OF STUDIES 7 <sup>th</sup> |
| COURSE TITLE                                                                                                                                                                                                                                                  |  | DYNAMICAL SYSTEMS                                                                                                                                   |                                     |
| INDEPENDENT TEACHING ACTIVITIES<br><i>if credits are awarded for separate components of the course, e.g. lectures, laboratory exercises, etc. If the credits are awarded for the whole of the course, give the weekly teaching hours and the total credit</i> |  | TEACHING HOURS PER WEEK                                                                                                                             | ECTS CREDITS                        |
| Lectures and Tutorials                                                                                                                                                                                                                                        |  | 4                                                                                                                                                   | 6                                   |
|                                                                                                                                                                                                                                                               |  |                                                                                                                                                     |                                     |
| Add rows if necessary. The organisation of teaching and the teaching methods used are described in detail at (d).                                                                                                                                             |  |                                                                                                                                                     |                                     |
| COURSE TYPE<br><i>general background, special background, specialised general knowledge, skills development</i>                                                                                                                                               |  | Compulsory course for the specialization <i>Applied Mathematics</i><br>Elective course for each of the other specializations                        |                                     |
| PREREQUISITE COURSES:                                                                                                                                                                                                                                         |  | Recommended prerequisite knowledge: INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS, SECOND COURSE IN ORDINARY DIFFERENTIAL EQUATIONS               |                                     |
| TEACHING AND ASSESSMENT LANGUAGE:                                                                                                                                                                                                                             |  | Greek                                                                                                                                               |                                     |
| THE COURSE IS OFFERED TO ERASMUS STUDENTS                                                                                                                                                                                                                     |  | Yes                                                                                                                                                 |                                     |
| COURSE WEBPAGE (URL)                                                                                                                                                                                                                                          |  | <a href="http://www.math.upatras.gr/~weele/weeleteaching_WinterSemester.htm">http://www.math.upatras.gr/~weele/weeleteaching_WinterSemester.htm</a> |                                     |
|                                                                                                                                                                                                                                                               |  |                                                                                                                                                     |                                     |

### 2. LEARNING OUTCOMES

#### Learning outcomes

The course learning outcomes, specific knowledge, skills and competences of an appropriate level, which the students will acquire with the successful completion of the course are described.

Consult Appendix A

- Description of the level of learning outcomes for each qualifications cycle, according to the Qualifications Framework of the European Higher Education Area
- Descriptors for Levels 6, 7 & 8 of the European Qualifications Framework for Lifelong Learning and Appendix B
- Guidelines for writing Learning Outcomes

In this course the student acquires the essential knowledge and technical skills for analyzing Dynamical Systems, i.e. systems of coupled ordinary differential equations, with emphasis on nonlinear equations. The student becomes acquainted with the analytical tools to correctly interpret the dynamical behavior in phase space, which is one of the central concepts in the field of Dynamical Systems. The course is structured in such a way that the level of complexity is increased gradually: the student is first introduced to Dynamical Systems which have a two-dimensional phase space, and later to systems with three or more dimensions, where the dynamics can become chaotic. The notion of chaos (i.e. sensitive dependence on initial conditions, also called "the butterfly effect") and its ubiquity in the natural world, in economics, in the health sciences and innumerable other applications of practical interest, is one of the major new insights of Mathematics of the past 50 years.

After having successfully followed the course, the student will have acquired a sound working knowledge of Dynamical Systems and will be able to follow the new scientific developments in this active and highly relevant research field.

### General Abilities

*Taking into consideration the general competences that the degree-holder must acquire (as these appear in the Diploma Supplement and appear below), at which of the following does the course aim?*

*Search for, analysis and synthesis of data and information, with the use of the necessary technology  
Adapting to new situations*

*Decision-making*

*Working independently*

*Team work*

*Working in an international environment*

*Working in an interdisciplinary environment*

*Production of new research ideas*

*Project planning and management*

*Respect for difference and multiculturalism*

*Respect for the natural environment*

*Showing social, professional and ethical responsibility and sensitivity to gender issues*

*Criticism and self-criticism*

*Production of free, creative and inductive thinking*

*Others...*

- Finding, analyzing and combining facts and information using the most suitable technologies.
- Adapting to new situations.
- Decision making.
- Working and studying autonomously.
- Working in a team.
- Generating new research ideas.
- Stimulating free, creative and constructive thinking.

### 3. COURSE CONTENT

Autonomous systems of ODEs in the two-dimensional phase plane, equilibrium points and their stability properties, the importance of the nonlinear terms. Population dynamics of two competing species (Lotka-Volterra model) and other applications. Hamiltonian dynamical systems, gradient systems. Local vs. global stability, Lyapunov functions. Periodic solutions, limit cycles and the Poincaré-Bendixson theorem. The Van der Pol oscillator and other applications. The notion of structural stability/instability. Bifurcations of equilibrium points and periodic trajectories: saddle-node, transcritical, pitchfork and Hopf bifurcations. Systems of ODEs with a phase space of three or more dimensions, the appearance of chaotic behavior. The Lorenz attractor and other chaotic ("strange") attractors in phase space.

#### 4. TEACHING AND LEARNING METHODS - ASSESSMENT

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                                                                                                                                                                                                                                                     |                          |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------|
| <b>TEACHING METHOD</b><br><i>Face-to-face, Distance learning, etc.</i>                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Lectures (face to face)                                                                                                                                                                                                                                                             |                          |
| <b>USE OF INFORMATION AND COMMUNICATION TECHNOLOGIES</b><br><i>Use of ICT in teaching, laboratory education, communication with students</i>                                                                                                                                                                                                                                                                                                                                                                                        | <ul style="list-style-type: none"> <li>✓ Use of information and communication technologies in the classroom, the tutorial classes, and for communication with the students.</li> <li>✓ Website of the course.</li> <li>✓ Use of the Department's online platform MyMath.</li> </ul> |                          |
| <b>TEACHING ORGANIZATION</b><br><i>The manner and methods of teaching are described in detail.</i><br><br><i>Lectures, seminars, laboratory practice, fieldwork, study and analysis of bibliography, tutorials, placements, clinical practice, art workshop, interactive teaching, educational visits, project, essay writing, artistic creativity, etc.</i><br><br><i>The student's study hours for each learning activity are given as well as the hours of non-directed study according to the principles of the ECTS</i>        | <b>Activity</b>                                                                                                                                                                                                                                                                     | <b>Semester workload</b> |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Lectures                                                                                                                                                                                                                                                                            | 52                       |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                                                                                                                                                                                                                                                     |                          |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Solving suggested exercises                                                                                                                                                                                                                                                         | 28                       |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Self-study during the semester                                                                                                                                                                                                                                                      | 40                       |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                                                                                                                                                                                                                                                     |                          |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Preparation for the final examination                                                                                                                                                                                                                                               | 27                       |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Final examination                                                                                                                                                                                                                                                                   | 3                        |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                                                                                                                                                                                                                                                     |                          |
| <b>STUDENT ASSESSMENT</b><br><i>Description of the evaluation procedure</i><br><br><i>Language of evaluation, methods of evaluation, summative or conclusive, multiple choice questionnaires, short-answer questions, open-ended questions, problem solving, written work, essay/report, oral examination, public presentation, laboratory work, clinical examination of patient, art interpretation, other</i><br><br><i>Specifically-defined evaluation criteria are given, and if and where they are accessible to students.</i> | <b>Assessment Language:</b> Greek<br><b>Assessment Language for Erasmus students:</b> English                                                                                                                                                                                       |                          |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | <b>Assessment methods:</b> Written final examination (100%) including: <ul style="list-style-type: none"> <li>✓ Theory</li> <li>✓ Exercises</li> <li>✓ Applications of Dynamical Systems</li> </ul>                                                                                 |                          |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Minimum passing grade: 5<br>Maximum passing grade: 10                                                                                                                                                                                                                               |                          |

#### 5. RECOMMENDED LITERATURE

(in Greek)

- Μπούντης Αναστάσιος. *Μη Γραμμικές Συνήθειες Διαφορικές Εξισώσεις*. Εκδόσεις Α.Γ. Πνευματικός, 1997.
- Μπούντης Αναστάσιος. *Δυναμικά Συστήματα και Χάος*. Εκδόσεις Παπασωτηρίου, 1995.

(in English)

- Strogatz Steven H. *Nonlinear Dynamics and Chaos: with Applications to Physics, Biology, Chemistry and Engineering*. 2<sup>nd</sup> ed., Westview Press, 2014.
- Arrowsmith David K. and Place C.M. *Dynamical Systems: Differential Equations, Maps and Chaotic Behaviour*. Chapman & Hall, 1992.