COURSE OUTLINE

1. GENERAL

SCHOOL	**Exact Sciences**		
ACADEMIC UNIT/UNITS	**DEPARTMENT OF MATHEMATICS**		
TITLE OF MASTER'S DEGREE	Pure and Applied Mathematics		
LEVEL OF STUDIES	Post Graduate		
COURSE CODE	**PAM_26**	SEMESTER	2°
COURSE TITLE	Topics in Topology		

INDEPENDENT TEACHING ACTIVITIES *if credits are awarded for separate components of the course, e.g. lectures, laboratory exercises, etc. If the credits are awarded for the whole of the course, give the weekly teaching hours and the total credits*	WEEKLY TEACHING HOURS	CREDITS
Lectures	3	6
Add rows if necessary. The organisation of teaching and the teaching methods used are described in detail at (d).		

COURSE TYPE *general background, special background, specialised general knowledge, skills development*	Specialised general knowledge
PREREQUISITE COURSES:	<u>Suggested prerequisite knowledge:</u> Topology, Algebra
LANGUAGE OF INSTRUCTION and EXAMINATIONS:	Greek or/and English
IS THE COURSE OFFERED TO ERASMUS STUDENTS	YES
COURSE WEBSITE (URL)	

2. LEARNING OUTCOMES

Learning outcomes

The course learning outcomes, specific knowledge, skills and competences of an appropriate level, which the students will acquire with the successful completion of the course are described.

Consult Appendix A

- *Description of the level of learning outcomes for each qualifications cycle, according to the Qualifications Framework of the European Higher Education Area*
- *Descriptors for Levels 6, 7 & 8 of the European Qualifications Framework for Lifelong Learning and Appendix B*
- *Guidelines for writing Learning Outcomes*

Dimension Theory: Students will understand, through the first lessons, the reasons for creating and developing of dimensions theory. They will learn the three basic dimensions for topological spaces and theorems about the dimension of subspace, sum, product and compactification. They will see the development of this theory in the framework of partially ordered sets and graph theory and finally will understand the extension of this theory to the theory of dynamic systems by introducing and studying the Carathéodory Dimension, Hausdorff Dimension and Dimension Box.

Algebraic Topology: Students will understand the method of classification of topological spaces using the fundamental groups, their homology groups and Euler's characteristic. They will learn various methods of identifying these groups by using mappings such as: lifting of map, retraction, covering transformation. They will learn how fundamental groups and homology groups are related by the generalized Hurewicz's Theorem.

Special Issues of General Topology: Students will understand, through the first lessons, the most known ways of defining topologies on sets of open and closed subsets of a topological space. They will learn projective limits, their properties and applications. They will see the Čech-Stone compactifications and Wallman extensions. They will understand advanced issues of General Topology such as the Čech-complete spaces, the paracompact topological spaces, the Uniform spaces and the Proximity spaces. Finally, they will see theorems of metrizability of a topological space.

Introduction to Topological Groups and Topological Vector Spaces: Students will understand the notion of the topological group, the functions between topological groups, the separation axioms, the subgroups of topological groups and the product of topological groups. They will understand the notions of connectedness and compactness and the action of topological groups. Finally, the notion of topological vector space will be taught and the students will see basic results and theorems, related to these spaces.

Continuum Theory: Students will see, through the first lessons, examples of continuum and constructions of continuum, product of spaces, reverse limits of continuum and the embedding theorem of Anderson-Choquet. They will understand the quotient spaces of continuum and the upper-semicontinuous partitions. They will learn limitis of sets LimInf, LImSup, Lim, convergence theorems and boundary collision theorems. They will understand the continuum of Peano (locally connected continuum), the property S and the theorem of Hach-Mazurkiewicz. They will learn the notion of the curve and the classification of the curves. Finally, they will see the notion of the graph, the theorem of Kuratowski for graphs, the theorem of Jordan, the type of Euler, the dendrites and its basic properties.

General Competences

Taking into consideration the general competences that the degree-holder must acquire (as these appear in the Diploma Supplement and appear below), at which of the following does the course aim?

Search for, analysis and synthesis of data and information, with the use of the necessary technology	*Project planning and management*
Adapting to new situations	*Respect for difference and multiculturalism*
Decision-making	*Respect for the natural environment*
Working independently	*Showing social, professional and ethical responsibility and sensitivity to gender issues*
Team work	*Criticism and self-criticism*
Working in an international environment	*Production of free, creative and inductive thinking*
Working in an interdisciplinary environment	*……*
Production of new research ideas	*Others…*
	…….

Adapting to new situations
Decision-making
Working independently
Team work
Working in an international environment
Working in an interdisciplinary environment
Production of new research ideas
Production of free, creative and inductive thinking

3. CONTENT OF THE COURSE

Each year, one of the following subjects is taught:

Dimension Theory: Historical Review of Dimension Theory, Small and large inductive dimension, Covering dimension, The basic theorems of dimensional spaces n (embedding, union, product and compactification), Universal spaces, Euclidean spaces and Hilbert cube, Posets and Dimension Theory, The Dimension Theory in Graph Theory, Dynamic Systems and Dimensional Theory (Carathéodory Dimension, Hausdorff Dimension and Box Dimension).

Algebraic Topology: Topological quotient space. Homotopy, Fundamental group, Lifting of maps, Covering spaces, Covering transformations, Homology groups, Brouwer's fixed point theorem, The Borsuk-Ulam theorem, Classification of compact surfaces, The Euler-Poincaré characteristic.

Special Issues of General Topology: Function spaces, Toplogies on sets of open and closed subsets of a topological space, Projective limits, Čech-Stone compactifications and Wallman extensions, Čech-complete spaces, Paracompact topological spaces, Theorems of metrizability of a topological space, Uniform spaces, Proximity spaces.

Introduction to Topological Groups and Topological Vector Spaces: Basic notions and examples of topological groups, Functions between topological groups, Separation axioms, Subgroups of topological groups, Product of topological groups, Connectedness and Compactness, Action of topological groups, Basic notions and examples of the theory of topological vector spaces, Subspaces-products-quotient spaces, Convex and bounded sets, Seminorms and norms, Topological linear spaces with norm, Metrizability of topological linear spaces, Locally convex spaces, Continuous linear functions, Dual spaces. **Continuum Theory:** Examples of continuum and nested constructions of continua, Product of spaces, Reverse limits of continua and Anderson-Choquet embedding theorem, Quotient spaces of continuum and upper-semicontinuous partitions, Limitis, LimInf, LImSup and Lim of sets and convergence theorems, The boundary bumping theorems, Subcontinuum of convergence and existence theorems, Peano Continuum (locally connected continuum), Property S and Hach-Mazurkiewicz Theorem, the notion of curve and classification of curves, Graphs and Kuratowski Graph Theorem, Theorem of Jordan and Euler's formula for Graphs, Dendrites and its basic properties.

4. TEACHING and LEARNING METHODS - EVALUATION

DELIVERY *Face-to-face, Distance learning, etc.*	Face-to-face
USE OF INFORMATION AND COMMUNICATIONS TECHNOLOGY *Use of ICT in teaching, laboratory education, communication with students*	
TEACHING METHODS *The manner and methods of teaching are described in detail.* *Lectures, seminars, laboratory practice, fieldwork, study and analysis of bibliography, tutorials, placements, clinical practice, art workshop, interactive teaching, educational visits, project, essay writing, artistic creativity, etc.* *The student's study hours for each learning activity are given as well as the hours of non-directed study according to the principles of the ECTS*	see table below
STUDENT PERFORMANCE EVALUATION *Description of the evaluation procedure* *Language of evaluation, methods of evaluation, summative or conclusive, multiple choice questionnaires, short-answer questions, open-ended questions, problem solving, written work, essay/report, oral examination, public presentation, laboratory work, clinical examination of patient, art interpretation, other* *Specifically-defined evaluation criteria are given, and if and where they are accessible to students.*	**Evaluation language:** Greek **Evaluation language for Erasmus students:** English **Evaluation Methods:** Problem solving during final writing exam. Minimum passing grade: 5 Maximum passing grade: 10

Activity	*Semester workload*
Lectures	39
Bibliography study and analysis	60
Project	18
Homework	30
Final exam	3
Total	***150***

5.RECOMMENDED BIBLIOGRAPHY

Indicative Bibliography for «Dimension Theory»

[1] Aarts, J. M. and T. Nishiura (1993), *Dimension and Extensions,* Elsevier.

[2] Engelking R. (1995), *Theory of Dimensions: Finite and Infinite,* Sigma Series in Pure Mathematics 10; Heldermann Verlag.

[3] Nagata Jun-iti (1983), *Modern dimension theory,* Sigma Series in Pure Mathematics 2; Heldermann Verlag; Revised ed.
[4] Pesin Y.B. (1997), *Dimension Theory in Dynamical Systems*, University of Chicago Press.
[5] Trotter W.T., (1992), *Combinatorics and Partially Ordered Sets: Dimension Theory,* Johns Hopkins University Press.

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Indicative Bibliography for «Special Issues of General Topology»

[1] Arkhangel'skiĭ, A. V., Ponomarev, V. I., *Fundamentals of general topology. Problems and exercises*, Translated from the Russian by V. K. Jain. With a foreword by P. Alexandroff [P. S. Aleksandrov]. Mathematics and its Applications. D. Reidel Publishing Co., Dordrecht, 1984.
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[4] Dugundji, James, *Topology*, Reprinting of the 1966 original. Allyn and Bacon Series in Advanced Mathematics. Allyn and Bacon, Inc., Boston, Mass.-London-Sydney, 1978.
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[6] Isbell, J. R., *Uniform spaces*, Mathematical Surveys, No. 12 *American Mathematical Society, Providence, R.I.* 1964.
[7] James, I. M., *Introduction to uniform spaces*, London Mathematical Society Lecture Note Series, 144. Cambridge University Press, Cambridge, 1990.
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[9] Willard, Stephen, *General topology,* Reprint of the 1970 original [Addison-Wesley, Reading, MA; MR0264581]. Dover Publications, Inc., Mineola, NY, 2004.

Indicative Bibliography for «Introduction to Topological Groups and Topological Vector Spaces»

[1] Arhangel'skii Alexander, Tkachenko Mikhail, *Topological groups and related structures,* Atlantis Studies in Mathematics, 1. Atlantis Press, Paris; World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2008.
[2] Bourbaki, Nicolas, *General topology,* Chapters 1–4. Translated from the French. Reprint of the 1989 English translation. Elements of Mathematics (Berlin). Springer-Verlag, Berlin, 1998.
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[4] Narici, Lawrence, Beckenstein Edward. *Topological vector spaces,* Second edition. Pure and Applied Mathematics (Boca Raton), 296. CRC Press, Boca Raton, FL, 2011.
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[6] Schaefer, H. H., Wolff, M. P., *Topological vector spaces,* Second edition. Graduate Texts in Mathematics, 3. Springer-Verlag, New York, 1999.

Indicative Bibliography for «Continuum Theory»

[1] S. Nadler, *Continuum Theory: An Introduction*, M. Dekker, 1992.
[2] A. Illanes, S. Masias, and W. Lewis*, Continuum Theory*, Marcel Dekker, Inc., New York – Basel, 2002.